Kilogram as a relic in physics:

- In search of a universal definition of the unit of mass -

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National standard K20 stored in USA

Instead of Introduction…

- ⌘ Activities of every day life affected directly or indirectly by mass measurements
- ⌘ Direct or indirect impact on trade and commerce
- ⌘ Impact the scientific community
- ⌘ Ensure equity and equivalence in trade and manufacturing at the national and international levels

Uniform standards are needed !!!

History Lesson…

- ⌘ The metric system was developed from 1790 onwards by French scientists commissioned by Louis XVI to create a unified and rational system of measures
- ⌘ Original definition from 7 April 1795 proposed by French National Assembly

The kilogram is the mass of one cubic decimeter of water at the temperature of maximal density (4°C)

- ⌘ 22 June 1799 – an all-platinum prototype (the Kilogram of the Archives) was manufactured and deposited in the Archives of the French Republic in Paris

The kilogram is the mass of the Kilogram of the Archives

- ⌘ 20 May 1875 - The International Bureau of Weights and Measures (BIMP) & The International Committee for Weights and Measures (CIPM)
- ⌘ Followed by establishing foundations of the International System of Units (SI)

The Kilogram of the Archives

7 April 1795

22 June 1799

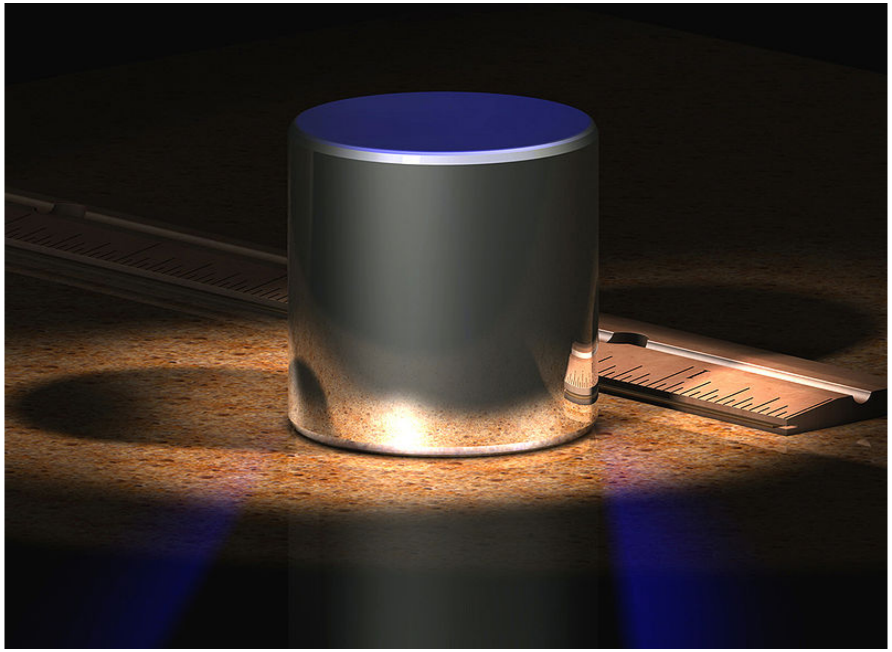
Photo: TJ Quinn

Le Grand K

- International prototype of the kilogram (IPK) is an artifact whose mass defines at present the SI unit of mass (definition established in 1901)

> ***The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram***

- Sanctioned in 1889 - *Le Grand K - KIII*
- Cylinder with diameter and height of about 39.17 mm
- Made of 90% platinum and 10% iridium
- Stored in a vault at BIPM outside Paris for more than 120 years with six copies
- Kept in air under three bell jars
- The basis for more than 80 copies distributed around the world



Pt-10Ir

Vault located in the basement of the BIPM's Pavillon de Breteuil in Sèvres

National prototype of the kilogram

The national prototype of the kilogram of Germany - No. 52 –

Germany (52, 55 and 70)

⌘ Maintained under two bell jars at standard ambient conditions at the German National Metrology Institute in Braunschweig

⌘ About every 10 years tested against the international prototype at the BIPM

⌘ The secondary standards of stainless steel are compared with the national prototype once a year

What about Other Units of Mass ?

- ⌘ The avoirdupois (or international) pound is used in both the Imperial system and U.S. customary units

- ⌘ Defined as exactly 0.45359237 kg, making one kilogram approximately equal to 2.2046 avoirdupois pounds

- ⌘ Other traditional units of mass around the world are also defined in terms of Kg

The IPK the primary standard for virtually all units of mass on Earth !!!

Problems…

- ⌘ The unit of mass is only available at the BIPM

- ⌘ Prototypes serving as national standards of mass must be returned periodically to the BIPM for calibration

- ⌘ High costs: BIPM: ~ 70 employees, budget ~ 10^6 EUR/year

- ⌘ Transport: constantly in danger of being damaged or destroyed

- ⌘ Long-term instability of the unit of mass ~ 30-50 μg over the last century

- ⌘ 1999: BIMP - Recommendation for redefinition of kilogram in terms of fundamental physics constants - Planck constant ***h***

Periodic verification

- ⌘ Simultaneous recalibration of all National Prototypes of the Kilogram at BIPM
  - ★ 1889, 1939 - 1946, 1988 – 1992
  - ★ Mass standards are stored in ambient air
  - ★ They accumulate contaminants and must be cleaned
  - ★ Surface contamination approaching 1 µg per year for national prototypes

- ⌘ 1939 - 1946: "the BIPM cleaning method”
  - ★ rubbing with chamois cloth that has been soaked in a mixture of equal proportions of diethyl ether and ethanol
  - ★ followed by steam cleaning with bi-distilled water
  - ★ 7 - 10 days to stabilize before calibration

- ⌘ New definition since 1989:

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram just after cleaning and washing using BIPM method

Mass Measurements

- ⌘ Balance reading = the difference between the gravitational and buoyant forces
- ⌘ Balance reading allows for the determination of the mass value
- ⌘ Reference ***R*** & unknown ***X***

$$\mathbf{m_R - \rho_a V_R = C_R}$$
$$\mathbf{m_X - \rho_a V_X = C_X}$$

$$\mathbf{m_X = m_R - \rho_a (V_R - V_X) - C}$$

$$\mathbf{C = (C_R - C_X)}$$

- ⌘ $\mathbf{m_R}$, $\mathbf{m_X}$, V_R, V_X – mass and volume for the reference ***R*** and the unknown ***X***
- ⌘ $\mathbf{C_R}$ and $\mathbf{C_X}$ – balance reading for the reference ***R*** and the unknown ***X***
- ⌘ ρ_a – assumption that air density does not change during the measurement
- ⌘ The simplest and most fundamental mass measurement process

Overview of Mass Standards

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Standard	Role	Date of first use at the BIPM	Date of last cleaning and washing prior to this work
IPK	International prototype of the Kilogram (IPK)	1889	1992
K1	Official copies of the IPK	1889	1992
7		1925	1992
8 (41)		1905	1992
32		1905	1992
43		1939	1992
47		1939	1992
25	Working standards reserved for special use	1958	2008
73		1988	2008
9	Working standards	1889	2003[a]
31		1889	2003
42′		1953	1976
63		1974	1982
77		1992	2004
88		2004	2003
91		2004	2004
650		1979	2001

All of these standards are made from Pt-Ir (90% to 10%)
2013-2014 a new campaign - after ~ 22 years
Masses of BIMP standards are compared to each other and to the IPK

Mass losses of the IPK

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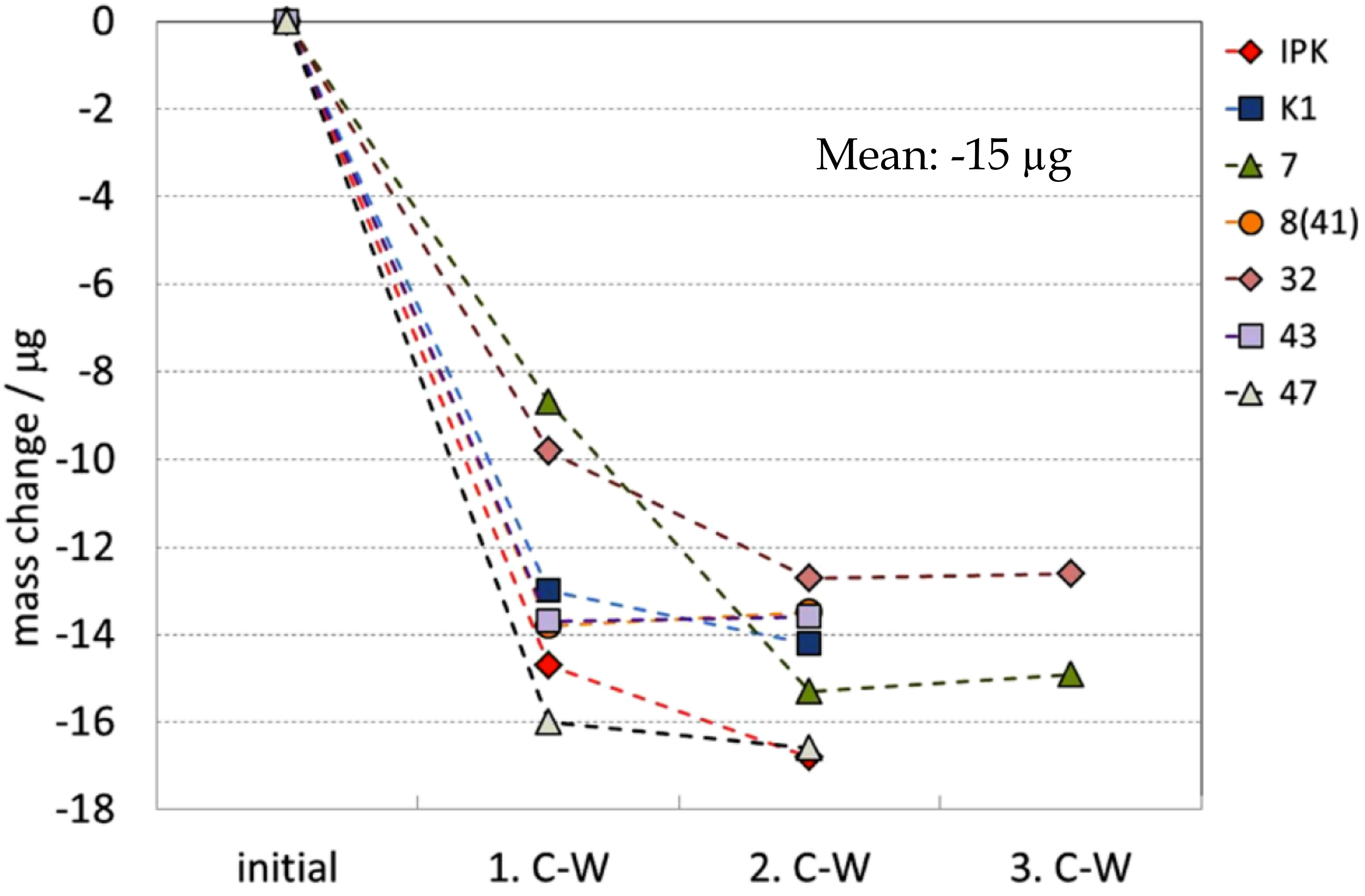

Mass losses of the IPK and its six official copies after cleaning and washing
Contamination rates 0.6 – 0.8 µg/yr $^{-1}$

Stability of the IPK

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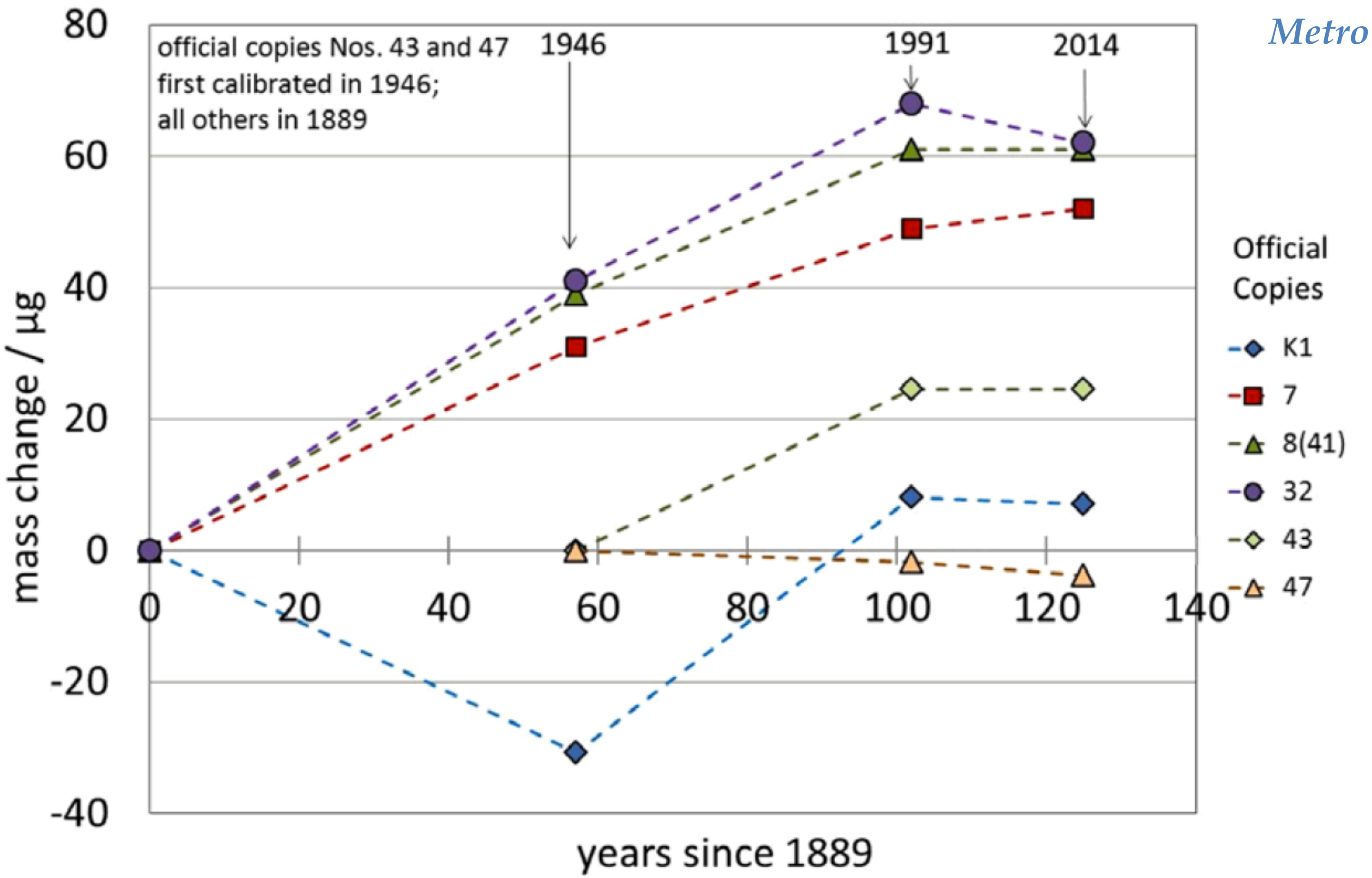

Evolution in mass: results of comparisons between the official copies and the IPK
some divergence with time → 50 µg in 100 years
On average a change by only 1 µg since the 3rd PV in 1991

Mass increase of the IPK

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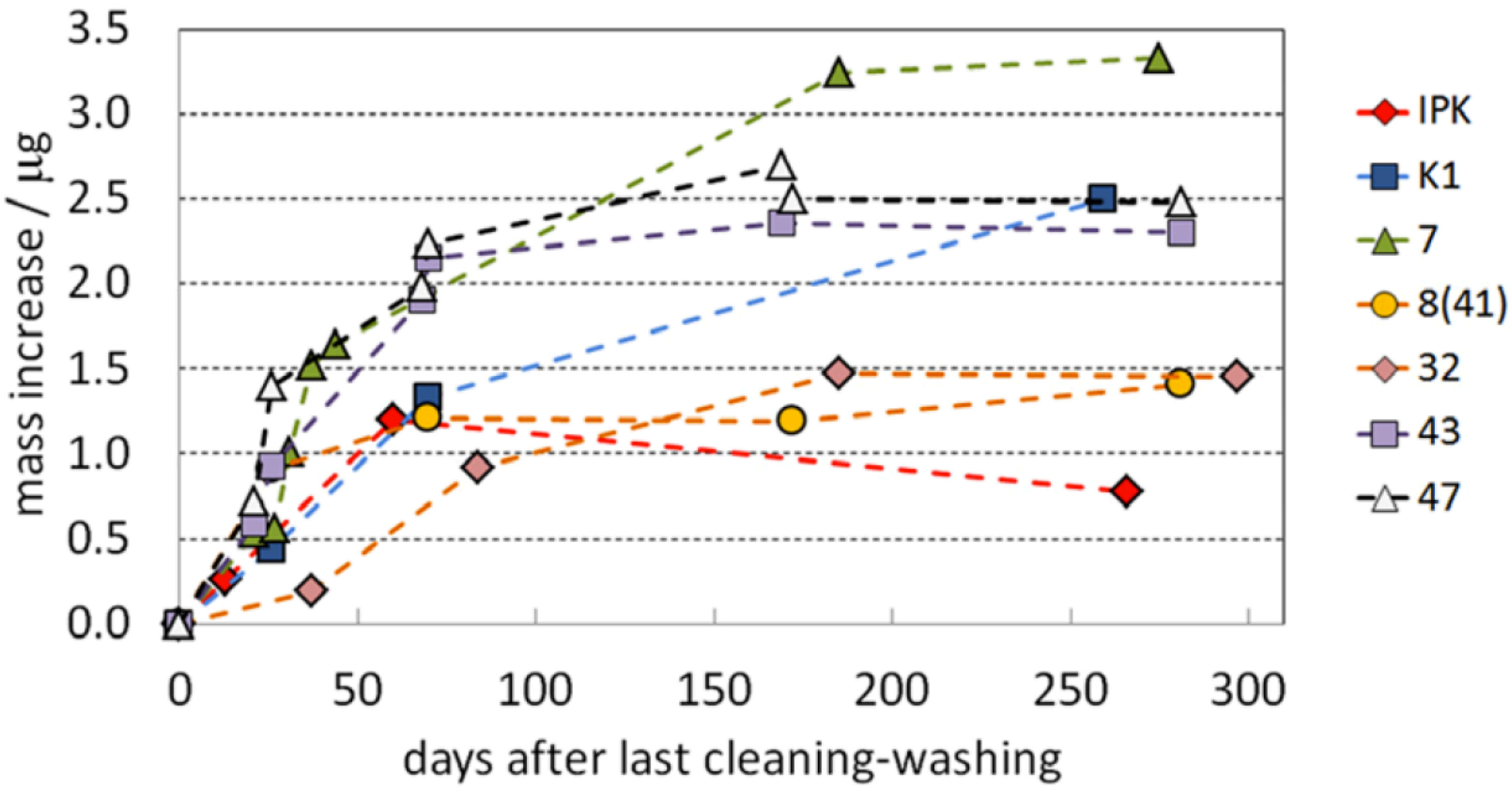


Mass increase of the IPK and its six official copies after the last cleaning and washing operation

Problems Continue…

- ⌘ Environment effects, mechanical wear and surface effects like adsorption & absorption of atmospheric contamination results in IPK gaining mass over time
- ⌘ Instabilities in definition of kg propagate to other SI base units…

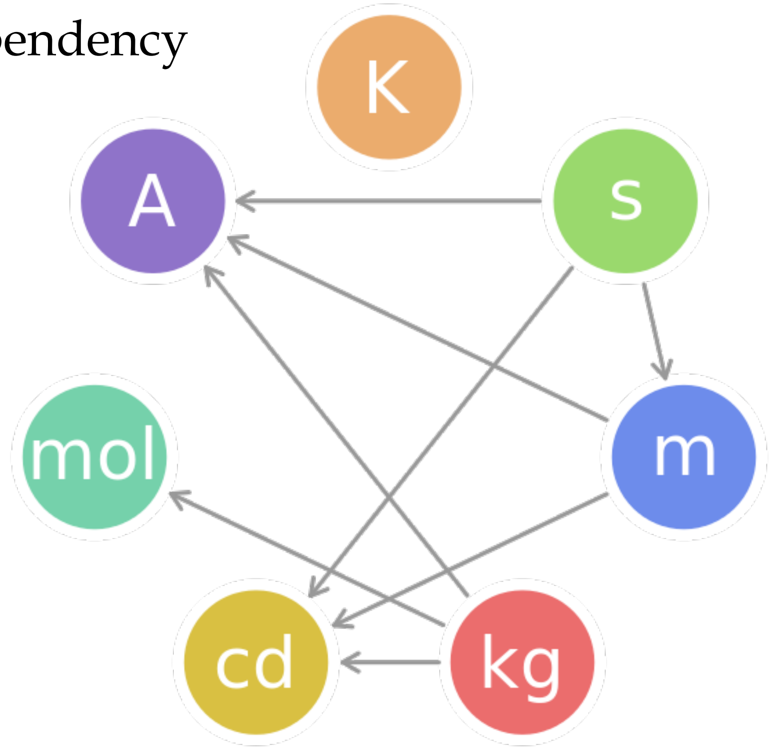
- ⌘ Precision of comparisons can be done at the level 10^{-10} - 10^{-12} !!!
- ⌘ Limitation lies within the artifact definition itself

As of 2015 the kilogram remains the only SI base unit defined by an artifact

Dependence of the SI on the IPK

⌘ The instability in the definition of the kilogram propagates to other SI base units that are tied to the kilogram

⌘ The seven SI base units and their interdependency
- ★ kelvin (temperature)
- ★ second (time)
- ★ metre (distance)
- ★ kilogram (mass)
- ★ *candela (luminous intensity)*
- ★ *mole (amount of substance)*
- ★ *ampere (electric current)*



⌘ SI derived units
- ★ Unit of force - newton [N]
- ★ Unit of pressure - pascal [Pa]
- ★ Unit of energy - joule [J]
- ★ Unit of power – watt [W]
- ★ Units of electricity – coulomb [C], volt [V], tesla [T], weber [Wb], ohm [Ω]
- ★ …

Towards a New Definition of Kg

Metrologia 51 (2014) R21

⌘ ***Long term solution:***

- ★ Kg defined with respect to universal constant of nature
- ★ Practical realization of Kg that can be reproduced in different laboratories
- ★ Specifications/Conditions

At least 3 experiments with relative standard uncertainties below 5 × 10^{-8}
One of these results should have uncertainty below 2 × 10^{-8}
Only two types of experiments relevant: Watt balance & Avogadro project

⌘ ***Experimental Groups:***

- ★ NIST – National Institute of Standards and Technology, USA
- ★ NPL – National Physical Laboratory, UK
- ★ NRC - Institute for National Measurements Standards, Canada
- ★ CODATA – Committee on Data for Science and Technology, France
- ★ METAS – Federal Institute of Metrology, Switzerland
- ★ IAC – International Avogadro Coordination Project – France, Italy, Belgium, USA, Australia, Japan, UK, Germany

The watt balance

Metrologia 51 (2014) S 5
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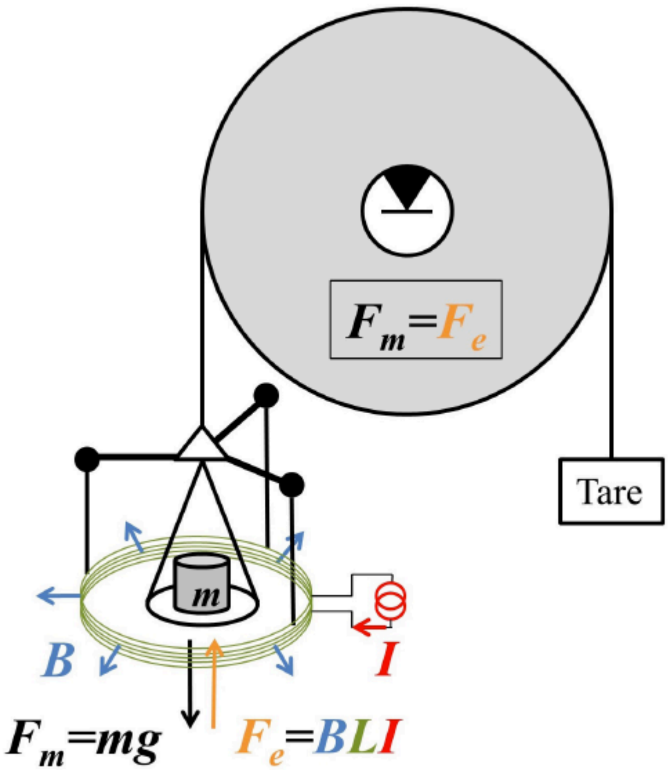
- ⌘ Proposed by B. P. Kibble in 1975
- ⌘ Consists of two parts to relate mechanical power to electrical power both in watts
- ⌘ Electrical power can be defined in terms of quantum effects



Force mode based on Lorentz Force

F_m – gravitational force on a mass m
F_e – electromagnetic force generated by a coil currying a current I in the magnetic field
B – magnetic field

L – wire length

$$F_m = F_e$$

$$mg = BLI$$

The watt balance

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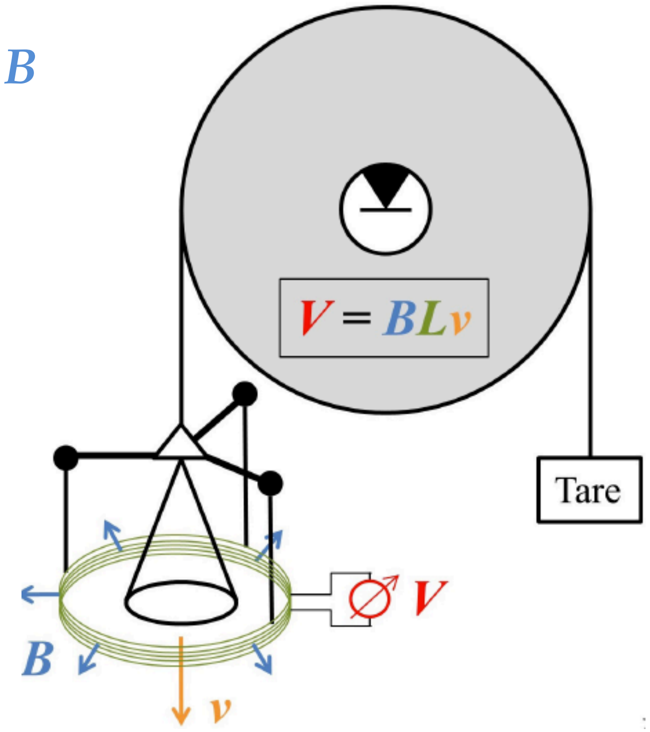
Velocity mode based on Faraday's Law of Induction

v – vertical speed of a coil in a magnetic field B
V - induced voltage
L – coil (wire) length

Induced voltage related to velocity
via flux integral BL

$$V = BLv$$



⌘ Connection from mass to h via electrical quantities & quantum physical effects

★ *Josephson effect & Quantum Hall effect*

$$mg\,v = V\,I$$

Josephson Effect

⌘ Observed in Josephson junctions (JJ)

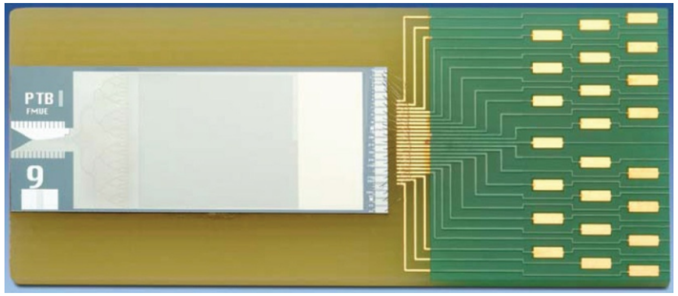
★ 2 superconductive materials separated by non-superconducting barrier

⌘ At superconducting temperatures while irradiating the junction with an electromagnetic field at microwave frequency *f* a current is forced through this junction

⌘ … and a voltage will develop across the junction

$$\mathbf{V} = \frac{\mathbf{h}}{\mathbf{2e}}\mathbf{f} \equiv \mathbf{K_J^{-1} f}$$

single junction 6 µm × 20 µm



⌘ Josephson constant $\mathbf{K_J = 2e/h}$

⌘ One junction delivers small voltage ~ 37 µV

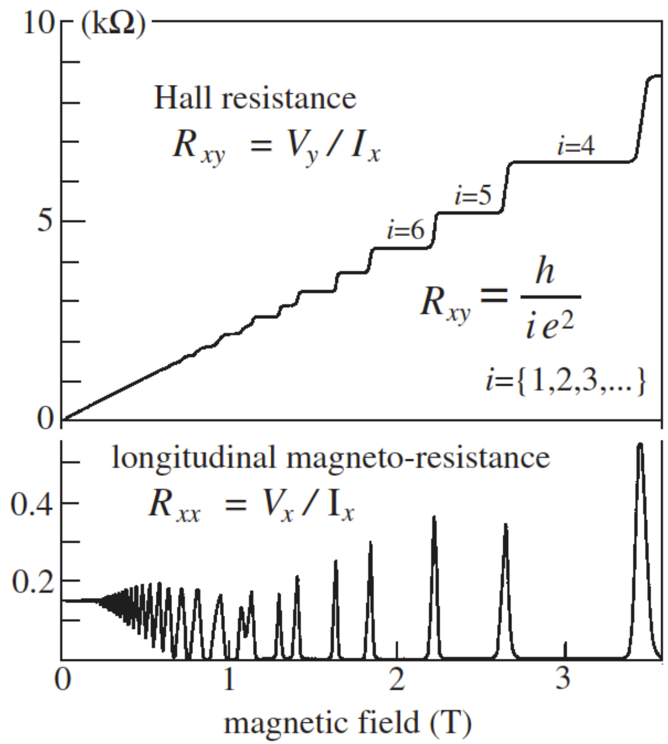
⌘ Practical voltage standard: 250 000 junctions connected in series on a single chip

⌘ Any voltage up to 10 V with uncertainty nV

Quantum Hall Effect

- ⌘ Special case of Hall effect for 2-dimensional electron system
- ⌘ Current currying conductor immersed in a magnetic field
- ⌘ Voltage V_H occurs perpendicular to B and I
- ⌘ At low temperatures (at liquid helium temperature T =4.2 K) and high magnetic fields (several tesla) Hall resistance R_H is quantized

$$\mathbf{R_H = \frac{V_H}{I} = \frac{1}{i}\frac{h}{e^2} \equiv \frac{1}{i} R_K}$$



- ⌘ von Klitzing constant $\mathbf{R_K = h/e^2}$
- ⌘ 100 Ω precision resistor with relative uncertainty 10^{-9}

Combining Two Effects

⌘ Instead of directly measure P=V I current $\boldsymbol{I}$ driven through precisely calibrated resistor $\boldsymbol{R}$ producing voltage drop V_R

$$\mathbf{P = V\,I = \frac{V\,V_R}{R} = mgv}$$

gravimeter
&
interferometer

⌘ Both voltages V and V_R are measured by comparing to Josephson voltage standard
- ★ Expressed in terms of f and K_J

⌘ Resistance is measured by comparing to a quantum Hall resistance standard
- ★ Expressed in terms of R_K

$$\mathbf{P = mgv = C\,f_1\,f_2\frac{h^2}{4e^2}\frac{e^2}{h} = C\frac{f_1\,f_2}{4}h}$$

In Practice

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- ⌘ Different system of units used for electrical measurements since 1990
- ⌘ Called conventional electrical units
- ⌘ based on the so-called "conventional values" of Josephson constant and von Klitzing constant & agreed by the International Committee for Weights and Measures (CIPM) - $K_{J\text{-}90}$ = ***483 597.9 GHz V^-1^*** and $R_{K\text{-}90}$ = ***25 812. 807 Ω*** (exact)

- ⌘ All electrical measurements calibrated in conventional units
- ⌘ By comparing electrical power in conventional units to mechanical power in SI units ***h*** can be determined
- ⌘ h_{90} is the conventional value of the Planck constant

$$\frac{(\mathrm{mgv})_{\{SI\}}}{(\mathrm{V\,I})_{\{90\}}} = \frac{\mathrm{W}_{\{90\}}}{\mathrm{W}_{\{SI\}}} = \frac{\mathrm{h}}{\mathrm{h}_{\{90\}}} \qquad \mathrm{h} = \mathrm{h}_{90}\frac{(\mathrm{mgv})_{\{SI\}}}{(\mathrm{V\,I})_{\{90\}}}$$

$$\mathrm{h}_{90} \equiv \frac{4}{\mathrm{K}_{\mathrm{J}-90}^{2}\,\mathrm{R}_{\mathrm{K}-90}} = 6.626\;068\;854\cdots \times 10^{-34}\,\mathrm{Js}$$

Results

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- ⌘ Measured values of the Planck constant
- ⌘ Lowest uncertainty 1.8×10^{-8} reported to date (March 2014)

NIST: $h = 6.626\,069\,79\,(30) \times 10^{-34}$ J s	*Relative uncertainty 4.5×10^{-8}*
NRC: $h = 6.626\,070\,34\,(12) \times 10^{-34}$ J s	*Relative uncertainty 1.8×10^{-8}*

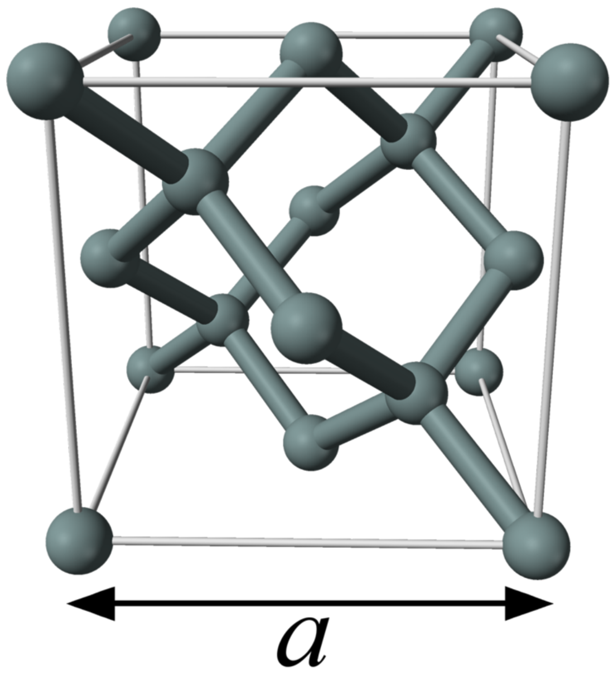
$$\mathbf{h} = \mathbf{h}_{90} \frac{\mathbf{R}_{\{90\}}}{\mathbf{V}_{\{90\}}\,\mathbf{V}_{\mathbf{R}\,\{90\}}}\,\mathbf{mgv}$$

$$\mathbf{h}_{90} \equiv \frac{4}{\mathbf{K}_{\mathbf{J}-90}^{2}\,\mathbf{R}_{\mathbf{K}-90}}$$

Avogadro Project

Metrologia 48 (2011) S1
PRL 106 (2011) 030801

- Avogadro constant N_A links atomic and macroscopic properties of matter
- Counting atoms in 1 kg single-crystal spheres made of ^{28}SI (silicon-28)
- Free of imperfections, mono-isotopic and chemically pure
- X-ray crystallography
- Avogadro constant - ratio of the molar volume V_{mol} to atomic volume V_a



Silicon unit cell

$$N_A = \frac{V_{mol}}{V_a}$$

$$V_a = \frac{V_{cell}}{n} = \frac{a^3}{n}$$

n =8 is the number of atoms per unit cell of volume V_{cell} of silicon crystal ^{28}SI

Avogadro Project

Metrologia 48 (2011) S1
PRL 106 (2011) 030801

⌘ Molar volume V_m

$$V_m = \frac{M_m}{\rho}$$

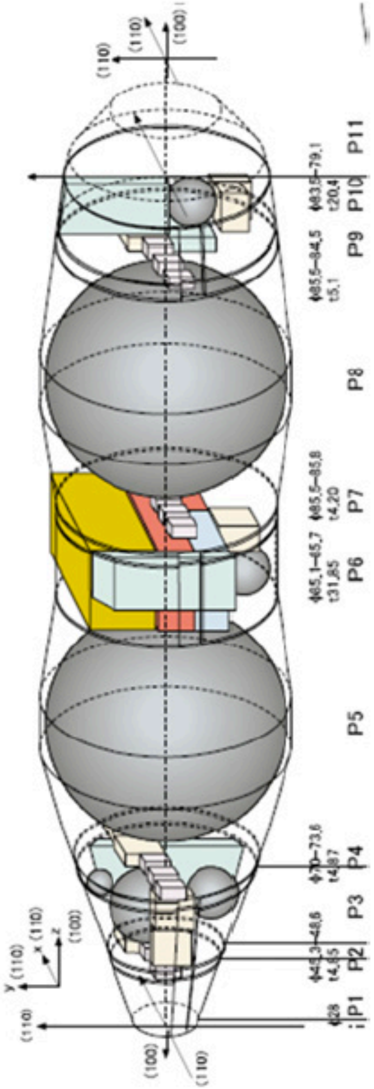
⌘ N_A measurement based on

$$N_A = \frac{n\,M_m}{\rho\,a^3}$$

⌘ Molar mass via gas mass spectrometry
⌘ X-ray interferometer measures a
⌘ Density from mass and volume of sphere
  - diameter measurements of sphere
⌘ IPK & optical interferometer

1 Kg single-crystal silicon sphere the roundest man-made object in the world

Avogadro Project



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PRL 106 (2011) 030801

The float-zone 28 Si crystal & its cutting plan

To determine density two spheres (AVO28-S5 and AVO28-S8) were manufactured from the two bulges

Avogadro Project

Metrologia 48 (2011) S1
PRL 106 (2011) 030801

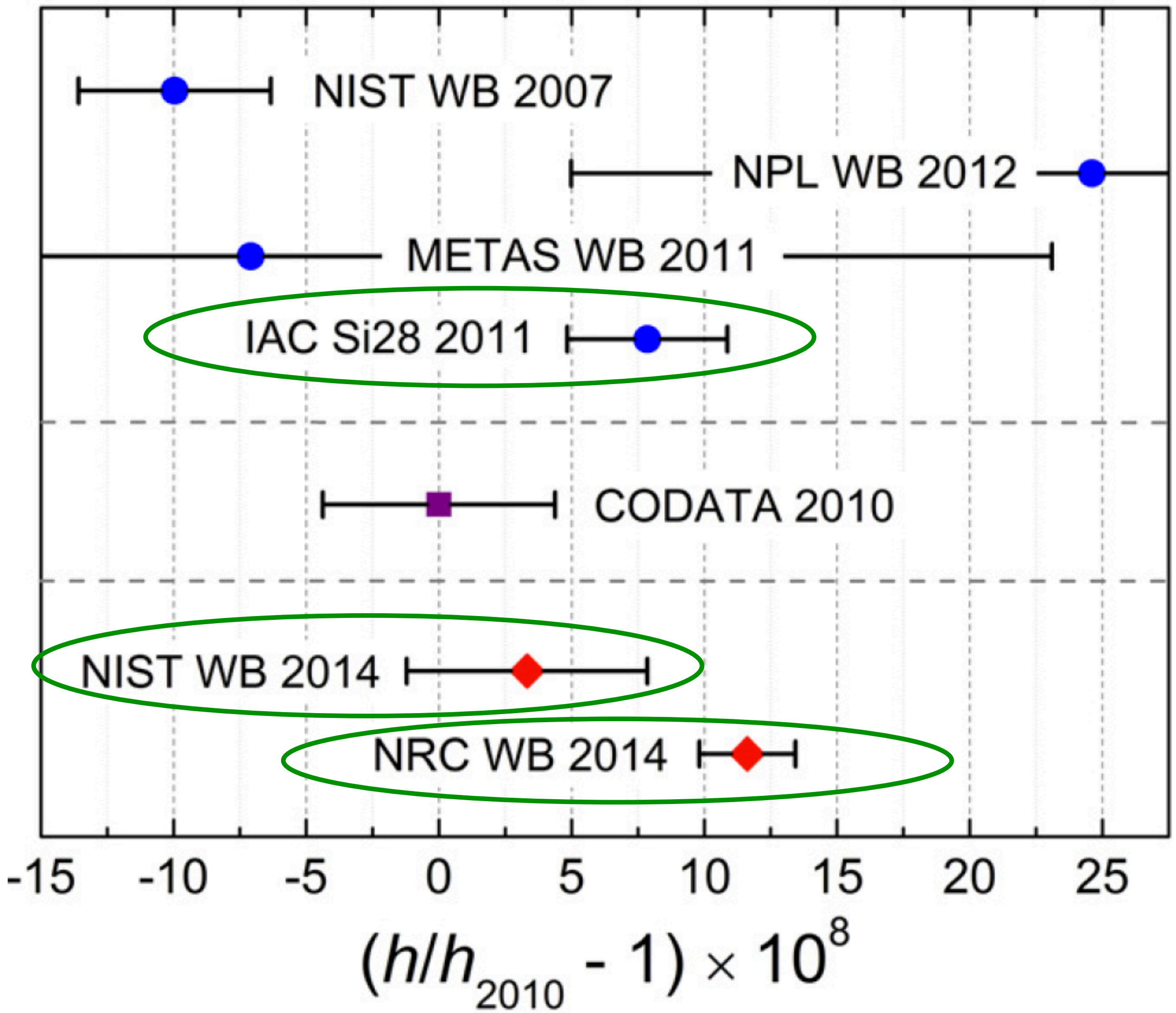
Quantity	Unit	AVO28-S5	AVO28-S8
a	pm	543.099 6240(19)	543.099 618 5(20)
m	g	1000.087 560(15)	1000.064 543(15)
V	cm^3	431.059 059(13)	431.049 110(10)
ρ	kg/m^3	2320.070 855(76)	2320.071 007(63)
M	g/mol	27.976 970 26(22)	27.976 970 29(23)
N_A	10^{23} mol^{-1}	6.022 140 91(21)	6.022 140 71(18)

$N_A = 6.022\ 140\ 78\ (18) \times 10^{23}\ mol^{-1}$ — *Relative uncertainty* 3×10^{-8}

$N_A h = 3.990\ 312\ 717\ 6\ \ J\ s\ mol^{-1}$ — *Relative uncertainty* 7×10^{-10}

Results of determination of *h*

Metrologia 51 (2014) R21



- ⌘ Watt balance (WB)
- ⌘ Avogadro project (IAC)
- ⌘ *Significant discrepancies*
- ⌘ *Postponed till data are consistent*
- ⌘ *Could be adopted in 2018*

Summary and Outlook

- ⌘ Activities of everyday life affected directly or indirectly by mass measurement
- ⌘ Uniform standards are needed
- ⌘ In 2015 the kilogram remains as the only SI base unit defined by artifact
- ⌘ In constant danger of being destroyed and damaged

- ⌘ Instabilities in definition of Kg propagate to other SI base units
  - ★ ampere, mole and candela
- ⌘ Propagates to to derived quantities
  - ★ density, force, pressure

- ⌘ Redefinition of Kg in terms of fundamental constant of nature ***h***
- ⌘ Standards need to be met: relative uncertainties below 2×10^{-8}
- ⌘ Two types of experiments can reach required precision
  - ★ watt balance and Avogadro project

- ⌘ Significant discrepancies between available experimental values as of 2014
- ⌘ Redefinition of the unit of mass postponed till data are consistent
- ⌘ Could be adopted in 2018…

SI Base Units

metre [m] – distance

The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second

second [s] – time

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom

ampere [A] - electric current

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length

SI Base Units

kelvin [K] - temperature

The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water

mole [mol] - amount of substance

1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12

2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles

candela [cd] – luminous intensity

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian

SI Derived Units

Name	Symbol	Quantity	SI Base Unit
newton	N	force	$1\ N = 1\ kg\ m\ s^{-2}$
pascal	Pa	pressure	$1\ Pa = kg\ m^{-1}\ s^{-2}$
joule	J	energy	$1\ J = 1\ kg\ m^{2}\ s^{-2}$
watt	W	power	$1\ W = kg\ m^{2}\ s^{-3}$
coulomb	C	electric charge	$1\ C = s\ A$
volt	V	voltage	$1\ V = kg\ m^{2}\ s^{-3}\ A^{-1}$
tesla	T	magnetic field	$1\ T = kg\ s^{-2}\ A^{-1}$
weber	Wb	magnetic flux	$1\ Wb = kg\ m^{2}\ s^{-2}\ A^{-1}$
ohm	Ω	electrical resistance	$1\ \Omega = 1\ kg\ m^{2}\ s^{-3}\ A^{-2}$