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Matching the Nagy-Soper parton shower at NLO

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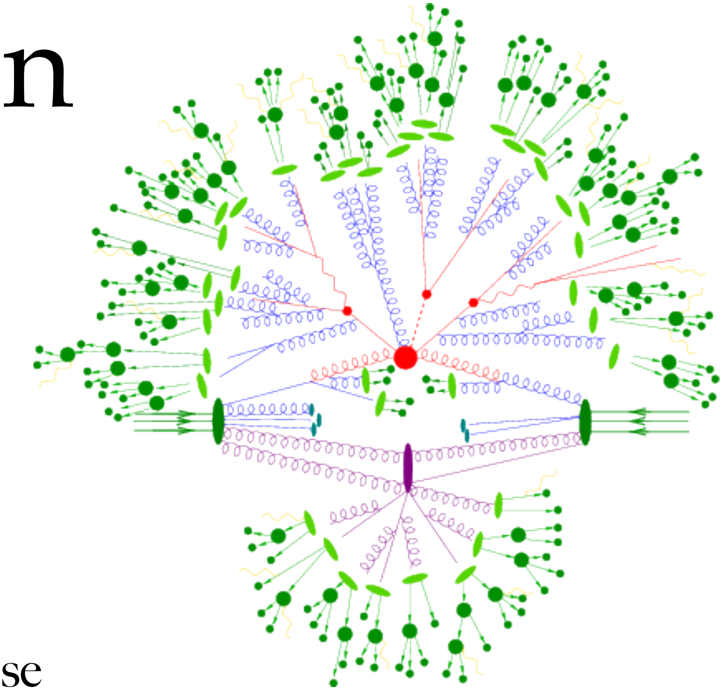
Motivation



[Sherpa homepage]

❑ *Monte Carlo Event Generators*

- Provide simulations of LHC collisions
- Present in all experimental analyses
- Widely used to make predictions
- Need to be improved as data become more precise

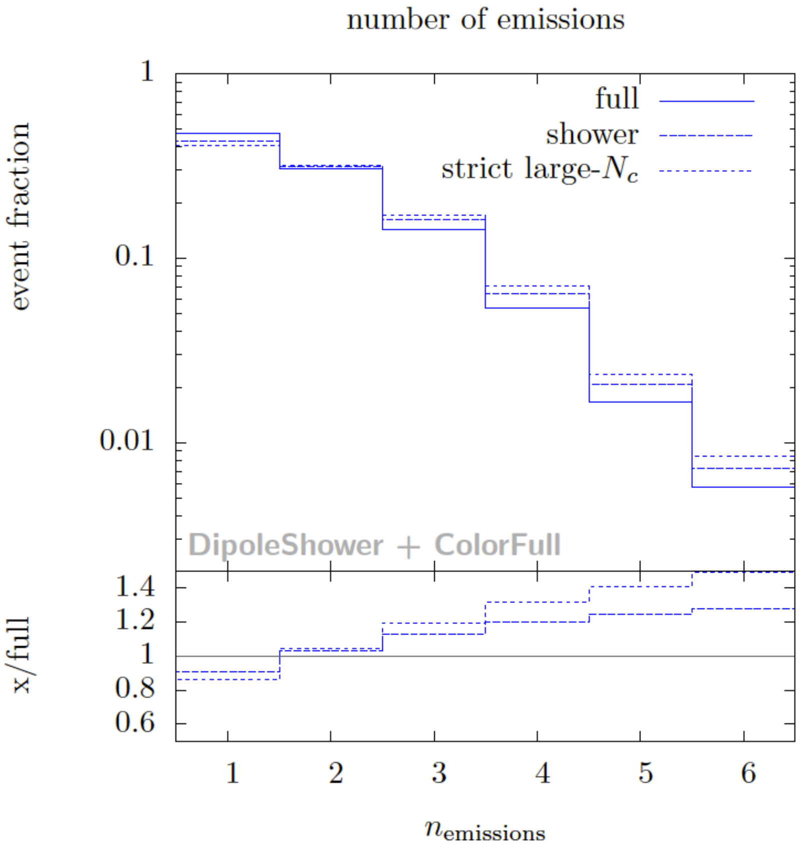
❑ *Improving theoretical prediction to reach better accuracy*

- Fixed order: (NLO, NNLO, N^3LO) QCD, NLO EW
- Matching parton shower to fixed order calculation (LO, NLO, NNLO)
- Merging matched calculation for different jet multiplicities (LO & NLO)
- Current parton showers with LC/LL accuracy
- Improvement on the PS front by inclusion of subleading effects

Subleading Effects

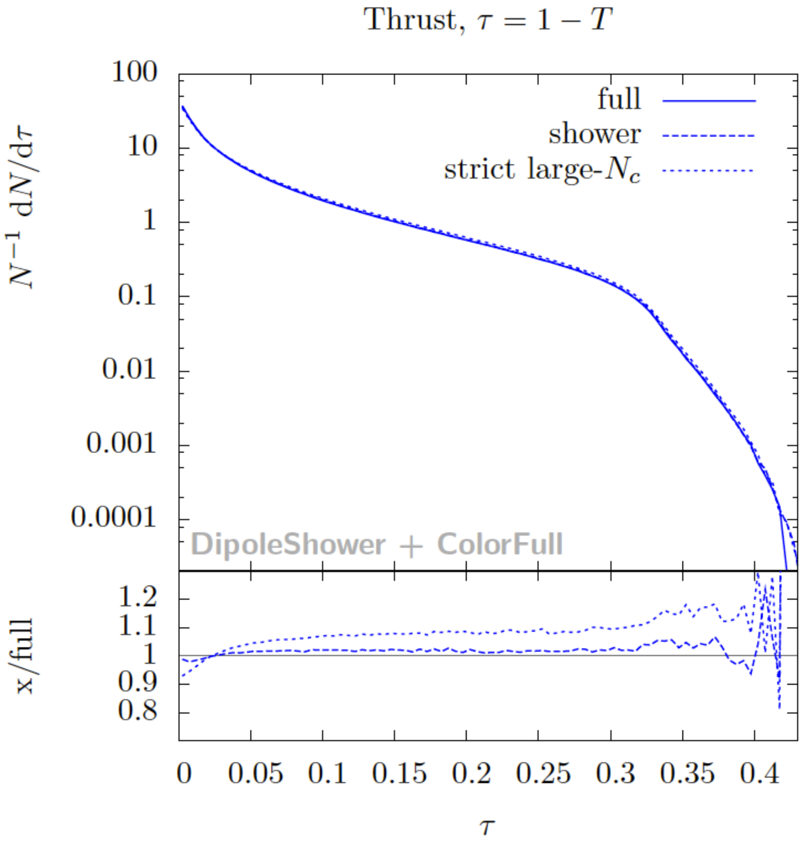
- ❑ Subleading N_C contributions visible for tailored observables
  - Sensitive to soft (wide angle) splitting

$e^+e^- \to$ jets



$C_F = 4/3$

$C_F = 3/2$



[Plätzer, Sjödahl '12]

Outline of the Talk

❑ *Nagy-Soper PS concept allows for parton state evolution to include both spin and color correlations*

❑ Nagy-Soper PS in a nutshell
  - **DEDUCTOR (**LC+ & Spin averaged)

[Nagy, Soper '07 '08 '12 '14]

❑ Matching Nagy-Soper parton shower and NLO calculation (MC@NLO)
  - **HELAC-NLO+DEDUCTOR** *[Czakon, Hartanto, Kraus, MW '15]*

❑ First study for $\mathbf{pp \to t\bar{t}j}$ production at the LHC (LC & Spin averaged)

❑ Comparison with other frameworks

Nagy-Soper Parton Shower

[Nagy, Soper '07 '08 '12 '14]

- ☐ Cross section for an inclusive observable F

$$\sigma[F] = \sum_m \frac{1}{m!} \int [d\{p,f\}_m] \left\langle \mathcal{M}(\{p,f\}_m) \right| F(\{p,f\}_m) \left| \mathcal{M}(\{p,f\}_m) \right\rangle \frac{f_a(\eta_a,\mu_F^2) f_b(\eta_b,\mu_F^2)}{4 n_c(a) n_c(b) \times \mathrm{flux}}$$

- ☐ Quantum density matrix

$$\rho(\{p,f\}_m) \sim \left| \mathcal{M}(\{p,f\}_m) \right\rangle \left\langle \mathcal{M}(\{p,f\}_m) \right|$$

- ☐ $\left| \mathcal{M}(\{p,f\}_m) \right\rangle$ is a vector in color ⊗ spin space

- ☐ Perturbative evolution is described by a unitary operator $U(t_F, t_0)$ obeying

$$\frac{dU(t,t_0)}{dt} = [\mathcal{H}_I(t) - \mathcal{V}(t)] U(t,t_0)$$

- ☐ $\mathcal{H}_I(t)$: resolved emission; $\mathcal{V}(t)$: unresolved/virtual emission
- ☐ Can be decomposed into color diagonal and off-diagonal parts

$$\mathcal{V}(t) = \mathcal{V}_E(t) + \mathcal{V}_S(t)$$

Nagy-Soper Parton Shower

- ❑ Solution of evolution equation

$$U(t,t_0) = N(t,t_0) + \int_{t_0}^{t} d\tau\ U(t,\tau)\left[\mathcal{H}_I(\tau) - \mathcal{V}_S(\tau)\right] N(\tau,t_0)$$

- ❑ Sudakov form factor

$$N(t,t_0) = \exp\left(-\int_{t_0}^{t} d\tau\ \mathcal{V}_E(\tau)\right)\ , \qquad \mathcal{V}(t) = \mathcal{V}_E(t) + \mathcal{V}_S(t)$$

- ❑ Exponentiation of $\mathcal{V}(t)$ can be difficult in the case of non-trivial color evolution

[Plätzer, Sjodahl '12]

  - ▪ Only the color diagonal part $\mathcal{V}_E(t)$ is exponentiated
  - ▪ Color off-diagonal part $\mathcal{V}_S(t)$ is treated perturbatively

- ❑ Expectation value of observable F including shower effects

$$\sigma[F] = (F|\rho(t_F)) = (F|U(t_F,t_0)|\rho(t_0))$$

- ❑ t_F → scale at which parton emission can not be described perturbatively

Features of Nagy-Soper PS

- Splitting functions are different from Altarelli-Parisi
- Massive initial state charm and bottom quarks
- Constructed to include full spin evolution and full color evolution *[Nagy, Soper '08 '12]*
- Ordering variable Λ_l *[Nagy, Soper '14]*

$$\Lambda_l^2 = \frac{|(\hat{p}_l \pm \hat{p}_{m+1})^2 - m_l^2|}{2p_l \cdot Q_0} Q_0^2, \qquad e^{-t} = \frac{\Lambda_l^2}{Q_0^2}$$

- PDFs are evolved according to shower splitting functions *[Nagy, Soper '14]*
- Public code: **DEDUCTOR** *[Nagy, Soper '14]*
  - LC+ approximation *[Nagy, Soper '12]*
    - Full color for collinear and soft-collinear limits
    - LC for pure soft limits
  - Spin averaged evolution

Matching Inclusive Processes

- NLO density matrix

$$|\rho) = \underbrace{|\rho_m^{(0)})}_{\text{Born}, \, \mathcal{O}(1)} + \underbrace{|\rho_m^{(1)})}_{\text{Virtual}, \, \mathcal{O}(\alpha_s)} + \underbrace{|\rho_{m+1}^{(0)})}_{\text{Real}, \, \mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^2)$$

- Shower evolution on the NLO density matrix expanded to ***O(α$_s$)***

$$|\rho(t_F)) = U(t_F, t_0)|\rho) \approx |\rho) + \int_{t_0}^{t_F} d\tau \, [\mathcal{H}_I(\tau) - \mathcal{V}(\tau)] \, |\rho_m^{(0)}) + \mathcal{O}(\alpha_s^2)$$

- Modify density matrix to remove double counting (MC@NLO approach)

[Frixione, Webber ‘02]

$$|\bar{\rho}) \equiv |\rho) - \int_{t_0}^{t_F} d\tau \, [\mathcal{H}_I(\tau) - \mathcal{V}(\tau)] \, |\rho_m^{(0)}) + \mathcal{O}(\alpha_s^2)$$

- For an infrared safe observable F we have

$$\begin{aligned} \bar{\sigma}[F] = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m) \left[(\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + \int_{t_0}^{t_F} d\tau (\Phi_m|\mathcal{V}(\tau)|\rho_m^{(0)}) \right] \\ + \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1}) \left[(\Phi_{m+1}|\rho_{m+1}^{(0)}) - \int_{t_0}^{t_F} d\tau (\Phi_{m+1}|\mathcal{H}_I(\tau)|\rho_m^{(0)}) \right] \end{aligned}$$

Matching Inclusive Processes

- Shower kernels are used to define subtraction terms of IR singularities, $t_F \to \infty$

$$\int_{t_0}^{\infty} d\tau\ \mathcal{H}_I(\tau) = \sum_l \mathbf{S}_l \int_0^{\infty} d\tau\ \delta(\tau - t_l)\Theta(\tau - t_0) = \sum_l \mathbf{S}_l \Theta(t_l - t_0)$$

$$\int_{t_0}^{\infty} d\tau\ \mathcal{V}(\tau) = \sum_l \int d\Gamma_l\ \mathbf{S}_l \Theta(t_l - t_0) \equiv \mathbf{I}(t_0) + \mathbf{K}(t_0)$$

- Matched cross section including shower evolution

$$\begin{aligned} \bar{\sigma}[F]^{PS} &= \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m)(\Phi_m|S) \\ &+ \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1})(\Phi_{m+1}|H) \end{aligned}$$

$$(\Phi_m|S) \equiv (\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + (\Phi_m|[\mathbf{I}(t_0) + \mathbf{K}(t_0) + \mathbf{P}]|\rho_m^{(0)})$$

$$(\Phi_{m+1}|H) \equiv (\Phi_{m+1}|\rho_{m+1}^{(0)}) - \sum_l (\Phi_{m+1}|\mathbf{S}_l|\rho_m^{(0)})\Theta(t_l - t_0)$$

- Matching in two steps: generation of $(\Phi_m|S)$ and $(\Phi_{m+1}|H)$
- Application of $U(t_F, t_0)$

Matching Exclusive Processes

❑ Inclusion of generation cuts

$$
\begin{aligned}
\bar{\sigma}[F]^{PS} &= \frac{1}{m!}\int [d\Phi_m](F|U(t_F,t_0)|\Phi_m)(\Phi_m|S)F_I(\{\hat{p},\hat{f}\}_m) \\
&+ \frac{1}{(m+1)!}\int [d\Phi_{m+1}](F|U(t_F,t_0)|\Phi_{m+1})(\Phi_{m+1}|H)F_I(\{p,f\}_{m+1})
\end{aligned}
$$

❑ Expanding the evolution operator

$$
\begin{aligned}
\bar{\sigma}[F]^{PS} &\approx \frac{1}{m!}\int [d\Phi_m](F|\Phi_m)(\Phi_m|\left[|\rho_m^{(0)}) + |\rho_m^{(1)}) + \mathbf{P}|\rho_m^{(0)})\right] F_I(\{\hat{p},\hat{f}\}_m) \\
&+ \frac{1}{(m+1)!}\int [d\Phi_{m+1}](F|\Phi_{m+1})(\Phi_{m+1}|\rho_{m+1}^{(0)})F_I(\{p,f\}_{m+1}) \\
&+ \int \frac{[d\Phi_m]}{m!}\frac{[d\Phi_{m+1}]}{(m+1)!}\int_{t_0}^{t_F} d\tau\ (F|\Phi_{m+1})(\Phi_{m+1}|\mathcal{H}_I(\tau)|\Phi_m) \\
&\quad \times(\Phi_m|\rho_m^{(0)})\left[F_I(\{\hat{p},\hat{f}\}_m) - F_I(\{p,f\}_{m+1})\right] + \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

Matching Exclusive Processes

- Mismatch is cured by enforcing the subtraction terms to fulfill $F_I(\{\hat{p}, \hat{f}\}_m)$

$$(\Phi_{m+1}|H) \to (\Phi_{m+1}|\tilde{H}) \equiv (\Phi_{m+1}|\rho^{(0)}_{m+1}) - \sum_{l} (\Phi_{m+1}|\mathbf{S}_l|\rho^{(0)}_m)\Theta(t_l - t_0) F_I(Q_l(\{p, f\}_{m+1}))$$

- $F_I(Q_l(\{p, f\}_{m+1})) = F_I(\{\hat{p}, \hat{f}\}_m)$ and Q_l is inverse momentum mapping

- After modification we have

$$\bar{\sigma}[F]^{PS} \approx \sigma^{NLO} + \int \frac{[d\Phi_m]}{m!} \frac{[d\Phi_{m+1}]}{(m+1)!} \int_{t_0}^{t_F} d\tau \; (F|\Phi_{m+1})(\Phi_{m+1}|\mathcal{H}_I(\tau)|\Phi_m)$$
$$\times (\Phi_m|\rho^{(0)}_m)\Big[1 - F_I(\{p, f\}_{m+1})\Big] F_I(\{\hat{p}, \hat{f}\}_m) + \mathcal{O}(\alpha_s^2)$$

- The double counting is removed if $\Big[1 - F_I(\{p, f\}_{m+1})\Big] F(\{p, f\}_{m+1}) = 0$

$$F_I(\{p, f\}_{m+1}) = 1 \text{ for } F(\{p, f\}_{m+1}) \neq 0$$

Generation cuts more inclusive than the cuts on the final observable

Summary of Ambiguities

- ☐ ***Parton masses***
  - Nagy-Soper parton shower treats bottom and charm quarks as massive
  - NLO calculation treats them as massless (bottom in 5FS)
  - Masses for the relevant quarks introduced by the on-shell projection

- ☐ ***Parton distribution functions***
  - PDFs are evolved differently in the NLO calculation and in the shower
  - NLO calculation: NLO PDFs are used
  - Parton Shower: PDFs are evolved using Nagy-Soper splitting kernels
  - The presence of quark masses
  - The evolution is of higher order → NLO accuracy is maintained if the evolutions share a common point e.g. at the low scale

- ☐ ***Initial shower time***
  - The choice of t_0 in the parton evolution is arbitrary
  - Requirement: NLO prediction recovered for hard emissions
  - Different choices of t_0 can achieve this
  - We pick one possible choice of t_0 but others are possible
  - Vary t_0 to study the uncertainty of NLO+PS matching systematic

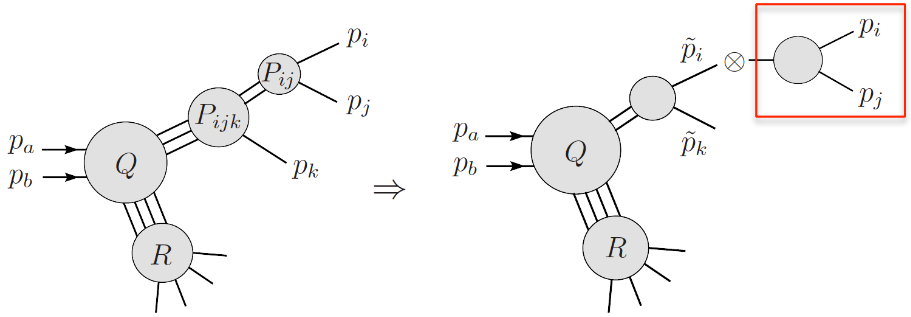
Implementation

❑ Nagy-Soper subtraction scheme in **HELAC-DIPOLES**

[Bevilacqua, Czakon, Kubocz, MW ‘13]

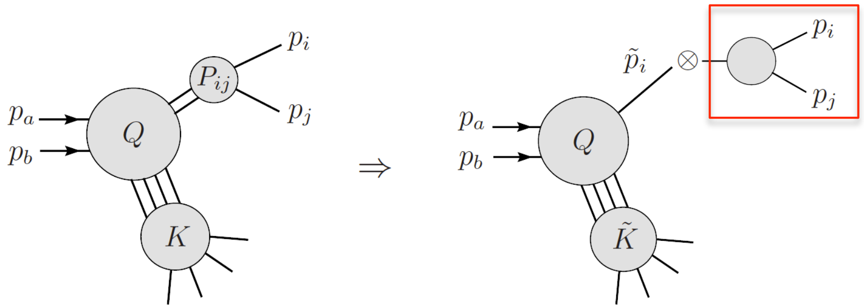
Catani-Seymour

- Easier dipole integration
- n^3 growth of subtraction terms



Nagy-Soper

- More complex dipole integration
- n^2 growth of subtraction terms



❑ Due to differences in

- splitting functions, momentum mappings, dipole phase space factorization

❑ Extensively tested for various processes, e.g. study of pp → bbbb

[Bevilacqua, Czakon, Krämer, Kubocz, MW ‘13]

Modifications in HELAC-DIPOLES

- ❑ *Momentum mapping for initial state splitting*
  - Implementation of subtraction scheme in **HELAC-DIPOLES** based on the first Nagy-Soper parton shower paper *[Nagy, Soper '07]*
  - **DEDUCTOR** uses revised momentum mapping for initial state splitting
  - Improves log resummation for certain observables *[Nagy, Soper '10]*
  - Now implemented in **HELAC-DIPOLES**

- ❑ Event samples are generated using **HELAC-1LOOP** and **HELAC-DIPOLES**

- ❑ Supply leading color and unpolarized events to **DEDUCTOR**

Interface to DEDUCTOR

- ❑ Use reweighting for m-parton samples
  - ▪ Generate unweighted LO events, then reweight according to

$$\omega_i(\{p,f\}_m) = 1 + \frac{(\{p,f\}_m|\rho_m^{(1)})}{(\{p,f\}_m|\rho_m^{(0)})} + \frac{(\{p,f\}_m|\mathbf{I}(t_0)+\mathbf{K}(t_0)+\mathbf{P}|\rho_m^{(0)})}{(\{p,f\}_m|\rho_m^{(0)})}$$

- ❑ Use unweighting for (m + 1)-parton samples
  - ▪ Pick the most probable diagonal color flow for each event
  - ▪ Store the generated events in the LHE file format

- ❑ Interface to **DEDUCTOR**: implementation of LHE file reader
- ❑ On-shell projection for charm and bottom quarks
- ❑ Translate color flow in the LHE file to internal representation of **DEDUCTOR** in terms of color strings

ttj production at LHC

- NLO calculations available *[Dittmaier, Uwer, Weinzierl '07; Melnikov, Schulze '10]*
- NLO+PS using **POWHEG** method *[Kardos, Papadopoulos, Trocsanyi '11]* *[Alioli, Moch, Uwer '11]*

- $\sqrt{\mathbf{s}} = \mathbf{8}\ \mathrm{TeV}, \mathbf{m_t} = \mathbf{173.5}\ \mathrm{GeV},\ \mathbf{m_b} = \mathbf{4.75}\ \mathrm{GeV},\ \mathbf{m_c} = \mathbf{1.4}\ \mathrm{GeV}$
- MSTW2008NLO PDF sets, provided in PS at $\mu_\mathbf{F} = \mathbf{1}\ \mathrm{GeV}$

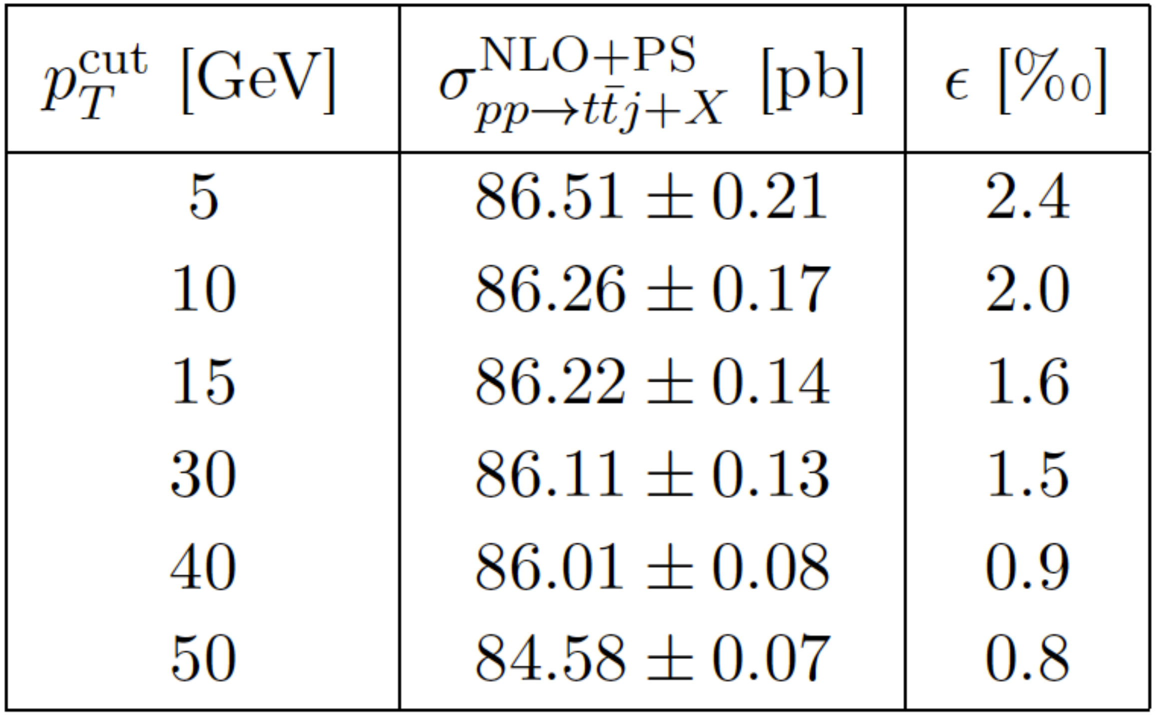
  $\mathbf{p_t^{gen}} > \mathbf{30}\ \mathrm{GeV},\ \mathbf{p_T} > \mathbf{50}\ \mathrm{GeV},\ |\mathbf{y_j}| < \mathbf{5}\ \mathrm{GeV},\ \mu_\mathbf{R} = \mu_\mathbf{F} = \mathbf{m_t}$
- anti-k_T jet algorithm with R=1

- LC and spin averaged shower evolution, full correlation in the subtraction
- Top decays, hadronization and multiple interactions are not included
  - **HELAC-NLO** + **DEDUCTOR** v1.0.0 is compared to
  - NLO calculation (from **HELAC-1LOOP** and **HELAC-DIPOLES**)
  - aMC@NLO + (**Pythia8** and **Pythia6Q**) (from **MadGraph5_aMC@NLO**)
  - **POWHEG** + **Pythia8** (from **POWHEG-BOX**)

[Czakon, Hartanto, Kraus, MW '15]

Generation Cut

- **HELAC-NLO+DEDUCTOR**



p_T^{cut} [GeV]	$\sigma^{\text{NLO+PS}}_{pp\to t\bar{t}j+X}$ [pb]	ϵ [‰]
5	86.51 ± 0.21	2.4
10	86.26 ± 0.17	2.0
15	86.22 ± 0.14	1.6
30	86.11 ± 0.13	1.5
40	86.01 ± 0.08	0.9
50	84.58 ± 0.07	0.8

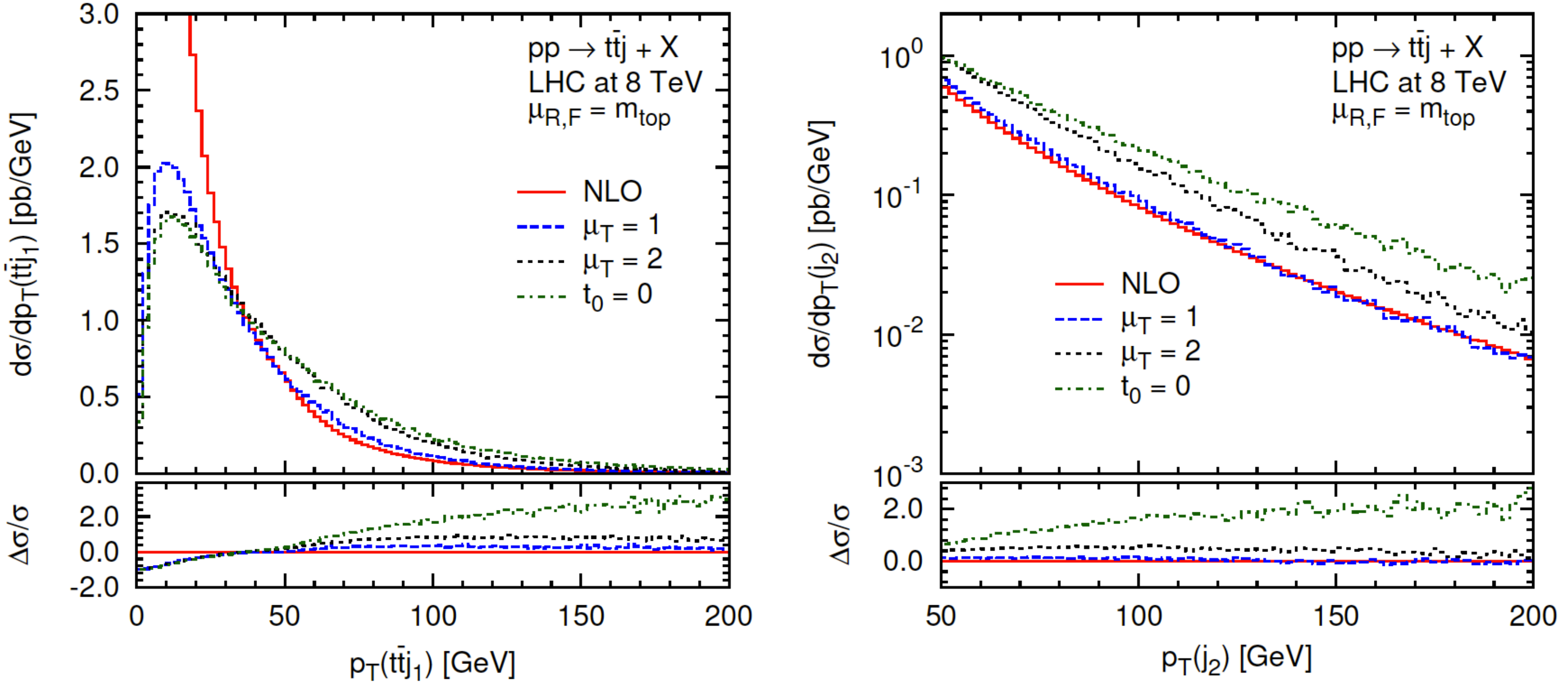
- Total cross section together with statistical and relative errors for different values of the generation cut

[Czakon, Hartanto, Kraus, MW '15]

Initial Shower Time

- ☐ Virtuality rescaling parameter $\mu_{\mathbf{T}}$

$$e^{-t_0} = \min_{i \neq j} \left\{ \frac{2p_i \cdot p_j}{\mu_T^2 Q_0^2} \right\}$$

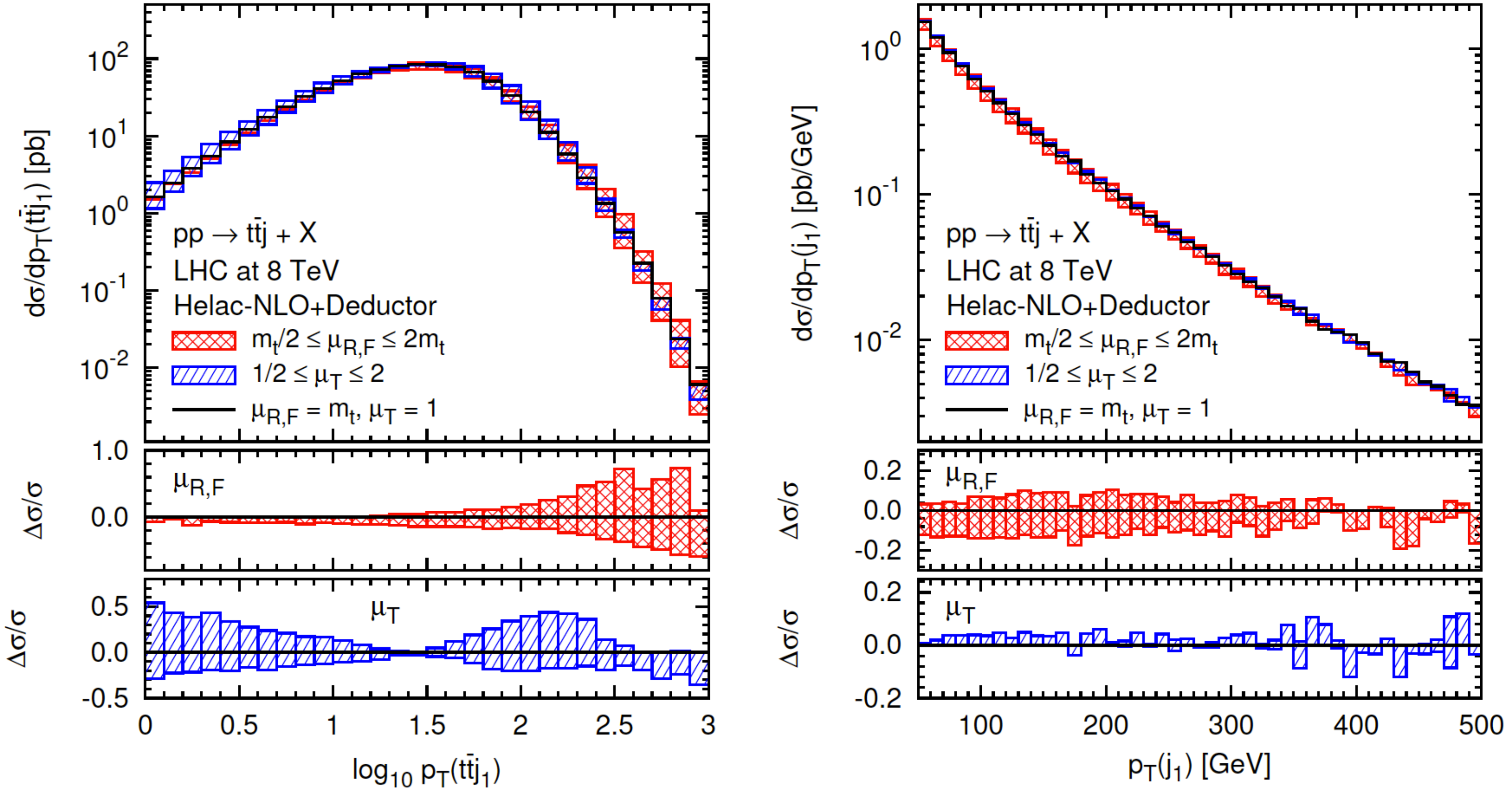


- ☐ Higher $\mu_{\mathbf{T}}$ value → larger correction in the high-p_T tail
- ☐ To recover the NLO prediction, we set $\mu_{\mathbf{T_0}} = \mathbf{1}$

[Czakon, Hartanto, Kraus, MW '15]

Uncertainties

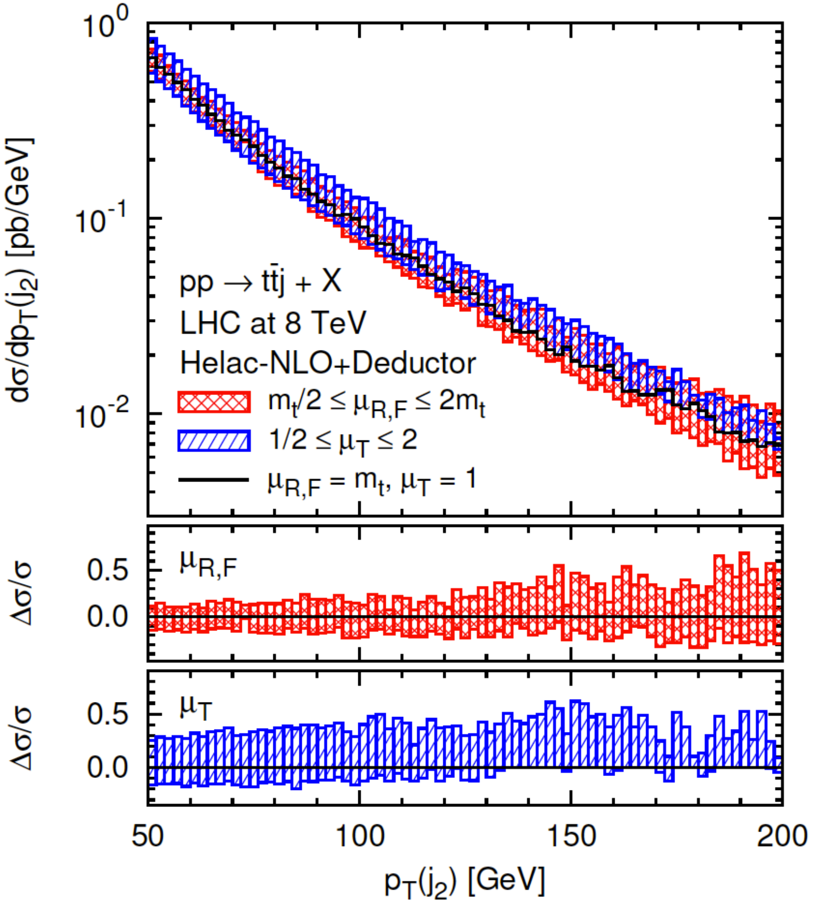
[Czakon, Hartanto, Kraus, MW '15]

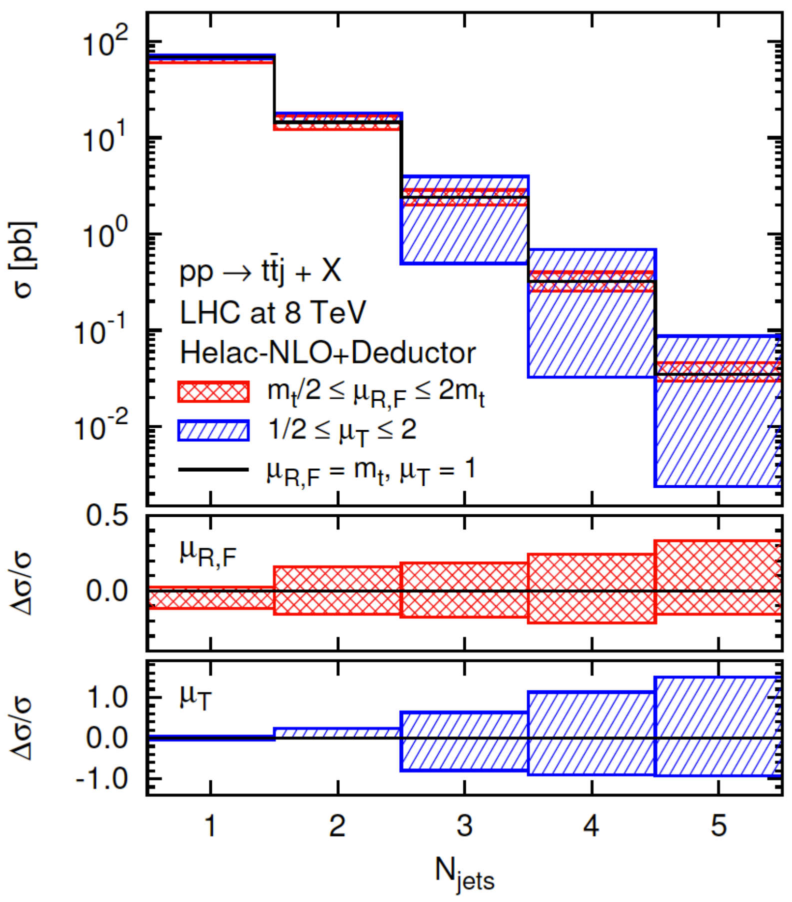


Scale uncertainties: $m_t/2 < \mu_{R,F} < 2m_t$
PS initial conditions: $1/2 < \mu_T < 2$

Uncertainties

[Czakon, Hartanto, Kraus, MW '15]





Scale uncertainties: $m_t/2 < \mu_{R,F} < 2m_t$
PS initial conditions: $1/2 < \mu_T < 2$

Comparison

LHC @ 8 TeV

- **HELAC-NLO**

$$\sigma^{\rm NLO}_{pp\to t\bar{t}j+X} = 86.04\,^{+5.10\;(\;+6\%)}_{-11.41\;(-13\%)}\ {\rm pb}$$

- **HELAC-NLO+DEDUCTOR**

$$\sigma^{\rm NLO+PS}_{pp\to t\bar{t}j+X} = 86.11\,^{+4.38\;(\;+5\%)}_{-10.88\;(-13\%)}\ [{\rm scales}]\ ^{+0.80\;(+1\%)}_{+2.17\;(+3\%)}\ [{\rm PS\ time}]\ {\rm pb}$$

- **Others**

$$\sigma^{\rm NLO+PS}_{pp\to t\bar{t}j+X}(\text{aMC@NLO+PYTHIA6Q}) = 84.85\,^{+8.95\;(+11\%)}_{-13.75\;(-16\%)}\ [{\rm scales}]\ {\rm pb}$$

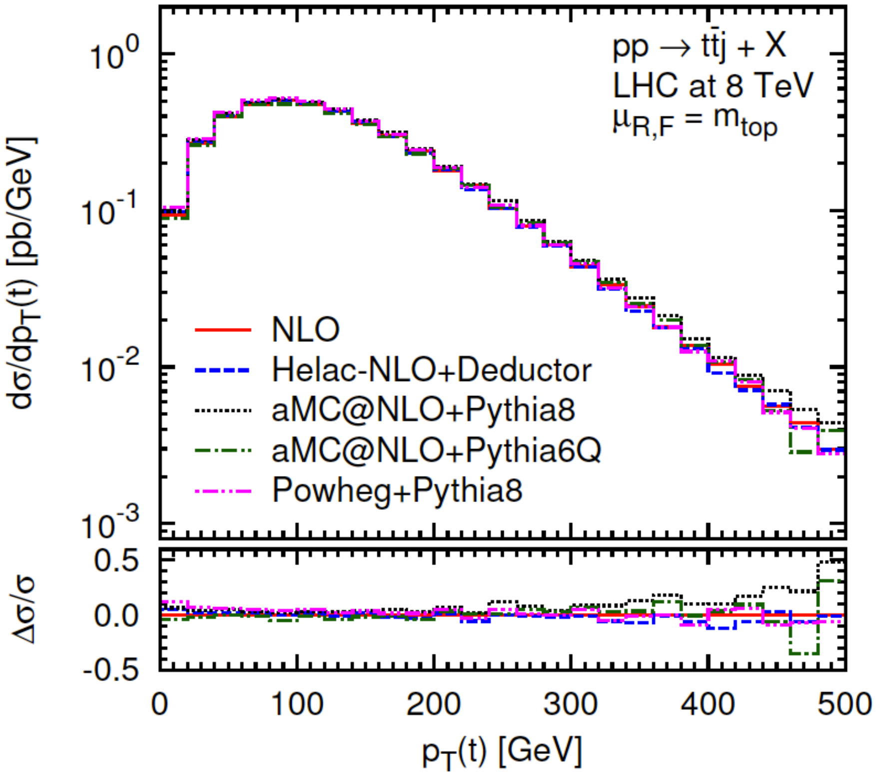
$$\sigma^{\rm NLO+PS}_{pp\to t\bar{t}j+X}(\text{aMC@NLO+PYTHIA8}) = 89.55\,^{+8.44\;(\;+9\%)}_{-15.41\;(-17\%)}\ [{\rm scales}]\ {\rm pb}$$

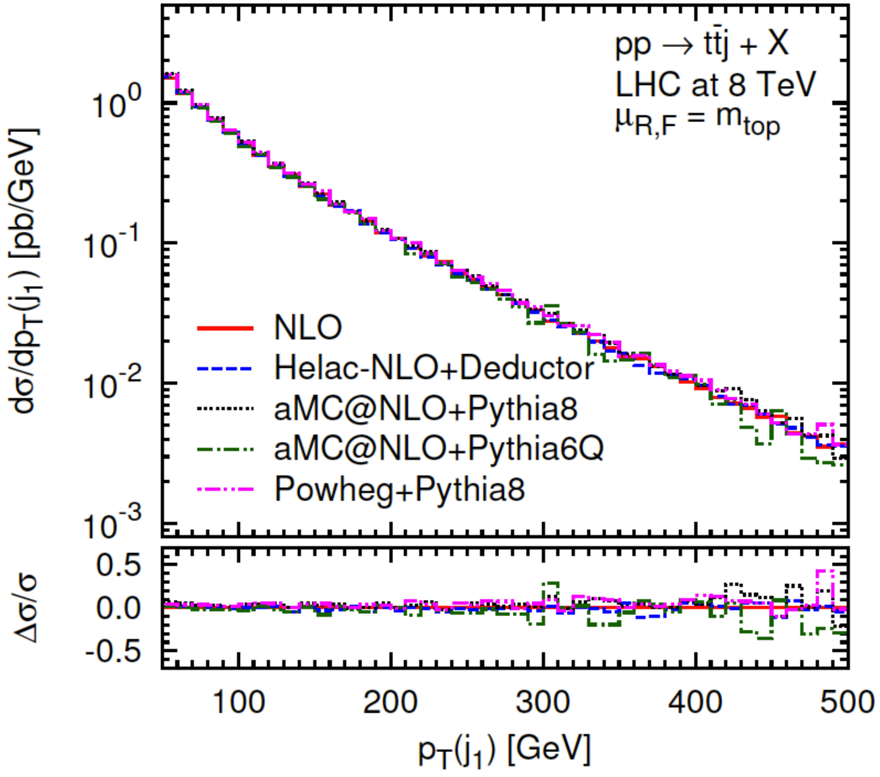
$$\sigma^{\rm NLO+PS}_{pp\to t\bar{t}j+X}(\text{POWHEG+PYTHIA8}) = 89.12\,^{+26.22\;(+29\%)}_{-8.96\;(-10\%)}\ [{\rm scales}]\ {\rm pb}$$

[Czakon, Hartanto, Kraus, MW ’15]

Comparison

❑ Differences between: matching procedures and showers





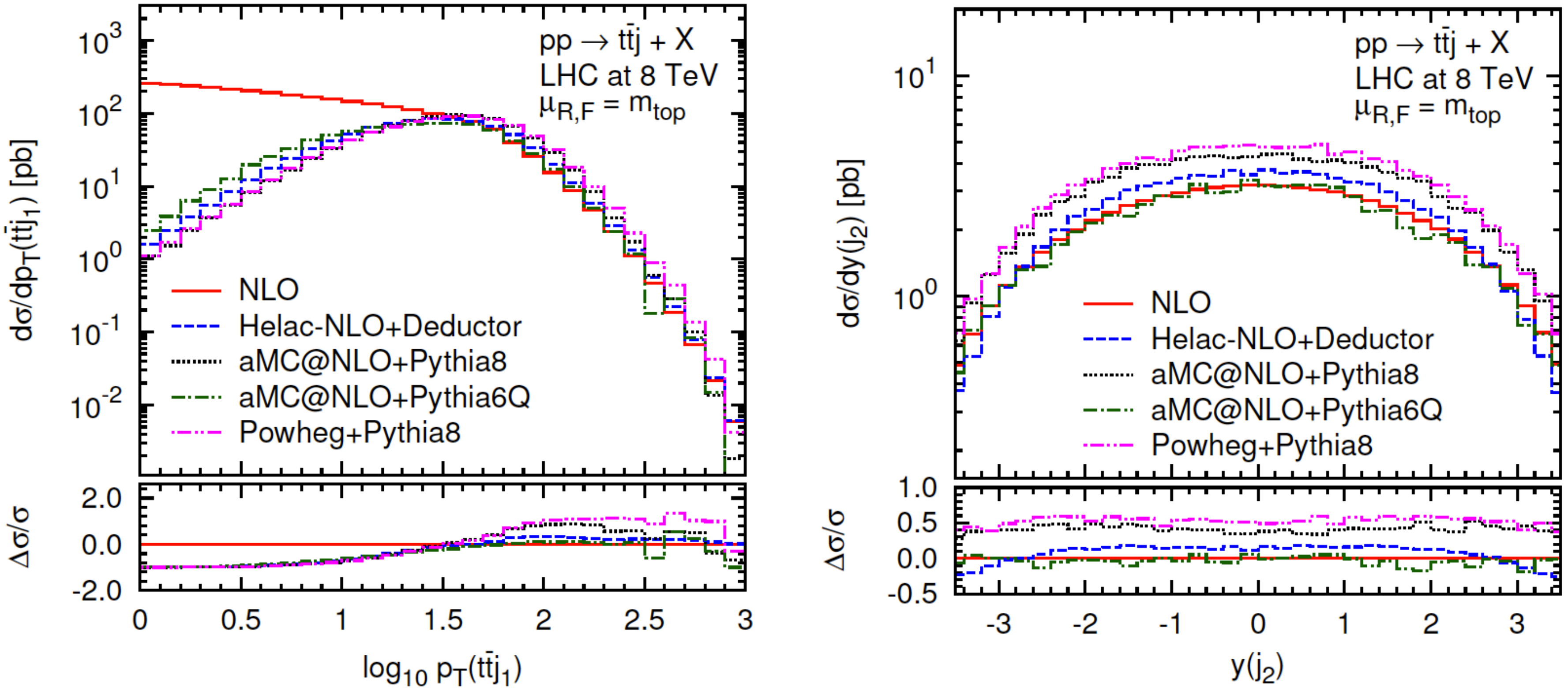
❑ Agreement between different predictions for inclusive distributions

[Czakon, Hartanto, Kraus, MW '15]

Comparison

- ❑ Shower sensitive obervables

[Czakon, Hartanto, Kraus, MW ’15]



- ❑ Helac-NLO+Deductor preserves NLO spectrum
- ❑ aMC@NLO+Pythia6Q recovers NLO results, produces softer emission
- ❑ Pythia8 (with MC@NLO and POWHEG matching) overshoots NLO at high-p_T

Summary

☐ *Already done*

- NLO matching scheme for the Nagy-Soper parton shower (MC@NLO approach)
- Implementation in HELAC-NLO framework
- LC and spin averaged
- ttj production at the LHC studied using **HELAC-NLO+DEDUCTOR**
- Comparison to other generators performed

☐ *Need to be added*

- In **DEDUCTOR**
  - Resonance decays
  - Non-perturbative effects
  - Go beyond LC+
  - Spin correlation
- In **HELAC-NLO**
  - Full treatments of color and spin correlation in the matching implementation