NLO QCD corrections to off-shell $ttj$ production at the LHC

Malgorzata Worek

Plan

- Motivation for $ttj$ production at the LHC
- Motivation for top-quark off-shell effects based on $tt$ production
- Status of theoretical predictions for $ttj$
- Complete off-shell effects with HELAC-NLO for $ttj$
- Results for LHC @ 8 TeV
- Summary & Outlook

Collaborators:
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Motivation for ttj

- @ LHC tops are produced with large energies & high transverse momenta
- Increase probability for additional (hard) radiation of gluons $\rightarrow ttj$ final state
- How big is the contribution of $ttj$ in the inclusive $tt$ sample?

- NNLO $tt$ cross section for $m_t = 173.2$ GeV @ LHC$_{13}$ TeV with CT14 PDF set:

$$\sigma(tt) = 807 \text{ pb}$$

<table>
<thead>
<tr>
<th>Jet $p_T$ cut [GeV]</th>
<th>$\sigma(ttj)$ [pb]</th>
<th>$\sigma(ttj)/\sigma(tt)$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>$296.97 \pm 0.29$</td>
<td>37</td>
</tr>
<tr>
<td>60</td>
<td>$207.88 \pm 0.19$</td>
<td>26</td>
</tr>
<tr>
<td>80</td>
<td>$152.89 \pm 0.13$</td>
<td>19</td>
</tr>
<tr>
<td>100</td>
<td>$115.60 \pm 0.14$</td>
<td>14</td>
</tr>
<tr>
<td>120</td>
<td>$89.05 \pm 0.10$</td>
<td>11</td>
</tr>
</tbody>
</table>

$\sigma(tt)$ = 807 pb

TOP++, Czakon, Mitov ‘14

HELC-NLO, G. Bevilacqua et al., ‘13
Motivation for $ttj$

- Background to SM Higgs production in VBF: $qq \rightarrow Hqq \rightarrow WWqq$
- 2 tagging jets: $\Delta y_{jj} = |y_{j1} - y_{j2}| > 4$ & $y_{j1} \times y_{j2} < 0$
- $\downarrow$ tt background: $tt \rightarrow WWbb$ & $\uparrow$ ttj background: $ttj \rightarrow WWbbj$

![Graphs showing dijet $y$ distributions for LO and NLO calculations with HELAC-NLO]

- Forward/backward light-jets
- B-jets from tops are central
Motivation for ttj

- Background to supersymmetric particle production
- Top decays into W and b-quark → SM: $t \rightarrow Wb \approx 100\%$
- **Decay channels:** di-leptons ($Br = 4\%$), lepton+jet ($Br = 30\%$), all-jets ($Br = 46\%$)
  - **ttj signature:** jets, charged leptons & $p_T^{miss}$ from invisible neutrinos
  - **Typical signals:** jets, charged leptons & $p_T^{miss}$ due to escaping lightest supersymmetric particle (neutralino)

Chain decays of gluino
Motivation for $ttj$

- Background to production of top flavor violating resonances $pp \rightarrow Mt \rightarrow t\bar{t}j$

$\tilde{t} = t$ for $M = W', Z'_H$ and $\tilde{t} = \bar{t}$ when $M = \phi^a$ (color triplet or sextet)

- $W'$ signal: $W' \rightarrow \tilde{t}q$
- Production processes: $pp \rightarrow W't \rightarrow t\bar{t}j$

$m_{W'} \in \{200, \ldots, 600\}$ GeV

$\sigma_{7\text{TeV}} \in \{40, \ldots, 4\}$ pb

- **ATLAS:** $m_{W'} > 430$ GeV

$\mathcal{L}_{W'} = \frac{1}{\sqrt{2}} \bar{d} \gamma^\mu g_R P_R t W'_\mu + \text{H.c.},$

$\mathcal{L}_{Z'_H} = \frac{1}{\sqrt{2}} \bar{u} \gamma^\mu g_R P_R t Z'_{H\mu} + \text{H.c.},$

$\mathcal{L}_\phi = \bar{t} c T^a_f (g_L P_L + g_R P_R) u \phi^a + \text{H.c.},$

M. I. Gresham, I.-W. Kim, K. M. Zurek '11

arXiv:1209.6593
NWA Vs. Off-shell Effects

- NWA $\rightarrow$ Tops are restricted to on-shell states
- Approximation is controlled by the ratio: $\Gamma_t/m_t \approx 10^{-2}$
- Contributions from diagrams involving two top-quark resonances

Should be accurate for sufficiently inclusive observables

Indeed $\rightarrow$ top-quark off-shell effects for $\sigma$ at few % level for:

- $pp \rightarrow tt$ A. Denner et al. '11, G. Bevilacqua et al. '11, A. Denner et al. '12
  R. Frederix '14, F. Cascioli et al. '14, G. Heinrich et al '14
- $pp \rightarrow ttH$ A. Denner, R. Feger '15
- $pp \rightarrow ttj$ G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek '16
NWA Vs. Off-Shell Effects

- Larger impact on differential distributions
- Full NWA (tt) versus full calculation (WWbb) for $p_T(bb)$

...also diagrams with one or no top-quark resonances
NWA Vs. Off-Shell Effects

- Full NWA ($tt$) versus full calculation ($WWbb$) for $M_{e+b}$

\[ \frac{d\sigma}{dM_{e+b}} \left[ \text{fb/GeV} \right] \]

\[ pp \to \nu_e e^+ \mu^- \bar{\nu}_\mu b \bar{b} + X \]

\[ \sqrt{s} = 7 \text{ TeV} \]

- If both top and $W$ decay on-shell \rightarrow end-point given by sharp cut

\[ M_{\ell b} = \sqrt{m_t^2 - m_W^2} \approx 152 \text{ GeV} \]

- Additional radiation & off-shell effects introduce smearing
Theoretical Predictions for \( ttj \)

- NLO QCD corrections to on-shell \( ttj \) production
  
  \( S. \) Dittmaier, P. Uwer, S. Weinzierl ’07 ’09

- NLO QCD correction to on-shell \( ttj \) production with LO decays
  
  K. Melnikov, M. Schulze ’10

- NLO QCD corrections to \( ttj \) in NWA (with jet radiation in top-quark decays)
  
  K. Melnikov, M. Schulze ’12

- NLO QCD corrections to \( ttj \) with full top-quark and W off-shell effects
  
  G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek ’16

- NLO QCD correction to on-shell \( ttj \) production + PS
  
  ★ POWHEG + PYTHIA, no spin correlations
  
  A. Kardos, C. G. Papadopoulos, Z. Trocsanyi ’11

  ★ POWHEG + PYTHIA/HERWIG with spin-correlations @ LO
  
  S. Alioli, S. Moch, P. Uwer ’12

  ★ MC@NLO + DEDUCTOR, without top-quark decays
  
  M. Czakon, H. B. Hartanto, M. Kraus, M. Worek ’15
Off-Shell Effects for \( ttj \)

\[
pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X
\]

- \( ttj \) with leptonic decays at \( \mathcal{O}(\alpha_s^4 \alpha^4) \)
- \( 2 \rightarrow 5 \) process from the QCD point of view
- Diagrams with complete off-shell effects for top & W
gauge boson for \( gg \) initial state:
  - \( \star \) \textit{LO}: 508
  - \( \star \) \textit{Real emission}: 4447
Off-Shell Effects for $ttj$

- $gg$ channel comprises 39 180 one-loop diagrams $\rightarrow$ according to QGRAF

  P. Nogueira ‘93

- The most complicated ones are 1155 hexagons & 120 heptagons
- Tensor integrals up to rank six

NWA for $ttj$ (on-shell top-quark production) – up to pentagons !

Full calculations for $ttj$ – up to heptagons !
Intermediate Top Resonances

- Putting simply $\Gamma_t \neq 0$ violates gauge invariance
- Gauge-invariant treatment $\rightarrow$ complex-mass scheme
- $\Gamma_t$ incorporated into top mass via:

  \[ \mu_t^2 = m_t^2 - i m_t \Gamma_t \]

  A. Denner, S. Dittmaier, M. Roth, D. Wackeroth ’99
  A. Denner, S. Dittmaier, M. Roth, L. H. Wieders ’05

- All matrix elements evaluated using complex masses
- $\mu_t^2$ identified with the position of pole of top-quark propagator
- Top-mass counter-term $\delta \mu_t$ related to top-quark self-energy at: $p_t^2 = \mu_t^2$

Another non trivial aspect: evaluation of one-loop scalar integrals in presence of complex masses!
- Scalar integrals with complex masses $\rightarrow$ supported e.g. by ONELOOP

  A. van Hameren ’11
HELAC-NLO

- **HELAC-1LOOP** → Virtual corrections in ’t Hooft-Veltman version of dimensional regularization
- **CUTTOOLS** → Ossola-Papadopoulos-Pittau (OPP) reduction technique
- **ONELOOP** → Evaluation of scalar integrals with complex masses

- **HELAC-DIPOLES** → The singularities from soft or collinear parton emissions isolated via subtraction methods for NLO QCD:
  - Catani-Seymour dipole subtraction
  - Nagy-Soper subtraction scheme
  - Both for massive and massless cases
  - Restriction on the phase space of the subtraction → $\alpha_{\text{max}}$

- Reweighting & unweighting techniques, helicity, and color sampling methods for optimization

- **KALEU** → Phase-space integration
  - Multi-channel Monte Carlo techniques
  - Adaptive weight optimization
  - Dedicated additional channels for each subtraction term for both subtractions

[https://helac-phegas.web.cern.ch/helac-phegas/](https://helac-phegas.web.cern.ch/helac-phegas/)
Results for ttj

\[
pp \rightarrow e^+ \nu e \mu^- \bar{\nu}_\mu b\bar{b} j + X
\]

- Different lepton generations \(\rightarrow\) Effects at the level of 0.5% \(\rightarrow\) checked @ LO
- Diagrams for gg initial state @ LO: 508 for e^+\mu^- \(\rightarrow\) 1240 for e^+e^-
- SM Parameters in \(G_\mu\) scheme:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_F)</td>
<td>(1.16637 \times 10^{-5} \text{ GeV}^{-2})</td>
</tr>
<tr>
<td>(m_t)</td>
<td>173.3 GeV</td>
</tr>
<tr>
<td>(m_W)</td>
<td>80.399 GeV</td>
</tr>
<tr>
<td>(\Gamma_W)</td>
<td>2.09974 GeV</td>
</tr>
<tr>
<td>(m_Z)</td>
<td>91.1876 GeV</td>
</tr>
<tr>
<td>(\Gamma_Z)</td>
<td>2.50966 GeV</td>
</tr>
<tr>
<td>(\Gamma_t^{\text{LO}})</td>
<td>1.48132 GeV</td>
</tr>
<tr>
<td>(\Gamma_t^{\text{NLO}})</td>
<td>1.3542 GeV</td>
</tr>
</tbody>
</table>

- MSTW2008 set of PDF & \(\mu_R = \mu_F = \mu_0 = m_t\)
- All light quarks including \(b\)-quarks and leptons are massless
- Suppressed contribution from \(b\) quarks in the initial state neglected
- Amounts to 0.8% @ LO
Top Width

- Finite W width contributions included in matrix elements & in $\Gamma_t$
- Top width for unstable W bosons, neglecting bottom quark mass @ LO & NLO

$$\Gamma_{t}^{\text{LO}} = \frac{G_\mu m_t^5}{16\sqrt{2}\pi^2 M_W^2} \int_0^1 \frac{dy \gamma_W}{(1-y/\bar{y})^2 + \gamma_W^2} F_0(y)$$

$$\gamma_W = \frac{\Gamma_W}{M_W}, \quad \bar{y} = (M_W/m_t)^2 \quad F_0(y) = 2(1-y)^2(1+2y)$$

$$\Gamma_{t}^{\text{NLO}} = \frac{G_\mu m_t^5}{16\sqrt{2}\pi^2 M_W^2} \int_0^1 \frac{dy \gamma_W}{(1-y/\bar{y})^2 + \gamma_W^2} \left[ F_0(y) - \frac{2\alpha_s}{3\pi} F_1(y) \right]$$

$$F_1(y) = 2(1-y)^2(1+2y) \left[ \pi^2 + 2\text{Li}_2(y) - 2\text{Li}_2(1-y) \right]$$
$$+ 4y(1-y-2y^2) \ln(y) + 2(1-y)^2(5+4y) \ln(1-y)$$
$$- (1-y)(5+9y-6y^2).$$

- In the limit $\gamma_W \rightarrow 0$

$$\frac{\gamma_W}{(1-y/\bar{y})^2 + \gamma_W^2} \rightarrow \pi \bar{y} \delta(y - \bar{y}).$$
**Cuts**

\[ pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu \bar{b}b j + X \]

- **Jets:**
  - Final-state quarks and gluons with pseudo-rapidity \(|y| < 5\) converted into infrared-safe jets using *anti-k\(_T\)* jet algorithm with \(R=0.5\)

- **Requirement:**
  - exactly 2 b-jets, at least one light-jet, 2 charged leptons, and missing \(p_T\)

- **Final states:**
  - have to fulfill the following kinematical requirements (fairly inclusive cuts)

\[
\begin{align*}
p_{T\ell} & > 30 \text{ GeV}, & p_{Tj} & > 40 \text{ GeV}, \\
p_{T}^{\text{miss}} & > 40 \text{ GeV}, & \Delta R_{jj} & > 0.5, \\
\Delta R_{\ell\ell} & > 0.4, & \Delta R_{\ell j} & > 0.4, \\
|y_\ell| & < 2.5, & |y_j| & < 2.5,
\end{align*}
\]
**Scale Dependence**

- **Total cross section @ LHC with 8 TeV (MSTW2008 PDF)**

  \[ pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X \]

  \[ \sigma_{\text{HELAC-NLO}}^{\text{LO}} = 183.1^{+112.2(61\%)}_{-64.2(35\%)} \text{ fb}, \]

  \[ \sigma_{\text{HELAC-NLO}}^{\text{NLO}} = 159.7^{-33.1(21\%)}_{-7.9(5\%)} \text{ fb}. \]

- **NLO corrections:** -13%

- **Theoretical uncertainties:**
  - 61% (48%) @ LO
  - 21% (13%) @ NLO

\[ \mu_R = \mu_F = \mu_0 = m_t \]

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*G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek ’16*
Hardest Light-Jet

- Upper panel: distributions and scale dependence bands: \( \{0.5m_t, m_t, 2m_t\} \)
- Lower panel: differential K-factor

- NLO do not rescale shape of LO
- Distortions up to 50% with \( \mu_0 = m_t \)
- Properly described only via NLO
- Negative NLO in \( p_T \) tails → LO higher than NLO

- The dynamic scale should depend on hardest jet \( p_T \)
- Asymptotic freedom → \( \alpha_s \) in tails
- Dependence on \( \alpha_s \) @ LO >> @ NLO
- Would drive positive NLO/LO ratio in this region

\[ pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b\bar{b} j + X \]

G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek ’16
Hardest Light-Jet

- Upper panel: distributions and scale dependence bands
- Lower panel: differential K-factor

- Negative, moderate but ... quite stable NLO corrections
- Dimensionless nature of $y_j$
- Receives contributions from various scales $\rightarrow$ also from these sensitive to threshold for $ttj$ production
- For $\mu_0 = m_t$ effects of phase-space regions close to $ttj$ threshold dominate

$$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu \bar{b}bj + X$$

G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek '16
Lepton and b-Jet

- Upper panel: distribution and scale dependence bands
- Lower panel: differential K-factor
- $be^+$ pair that returns the smallest invariant mass

$M_{be^+} = \sqrt{m_t^2 - m_W^2} \approx 153 \text{ GeV}$

- If both top and W decay on-shell $\rightarrow$ end-point given by sharp cut
- Additional radiation & off-shell effects introduce smearing
- Highly sensitive to the details of the description of the process

$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b\bar{b}j + X$

G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek ’16
Summary

- Complete description for $ttj$ process with “resonant” and “non-resonant” contributions at NLO QCD

- First calculation of $2 \rightarrow 5$ process with HELAC-NLO

- Further studies are needed:
  - Look for judicious choice of a dynamical scale
  - PDF uncertainties
  - Bottom-mass effects
  - Off-shell effects for differential distributions (comparison to NWA)

- Phenomenological applications $\rightarrow m_t$ extraction

- Shape-based $m_t$ measurement relies on precise modeling of differential distributions

- $m_t$ extraction $< 1$ GeV $\rightarrow$ Predictions should go beyond simple approximation of factorizing top production & decays
Outlook

- Alternative method for $m_t$
- $m_t$ from normalized differential cross section for $ttj$

\[ \mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{tt+1{-}\text{jet}}} \frac{d\sigma_{tt+1{-}\text{jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s), \]
\[ \rho_s = \frac{2m_0}{\sqrt{s_{tt+1{-}\text{jet}}}}, \]

S. Alioli, et al. ‘13

- $\mathcal{R}$ has been calculated using $ttj$ @ NLO + POWHEG matched with PYTHIA → top-quark decays via PS only with spin correlations @ LO

- Theoretical uncertainties & PDF uncertainties should affect $m_t$ extraction < 1 GeV

- **For now:** ATLAS @ 7 TeV: $m_t = 173.7 \pm 2.2$ GeV

- Worth looking at with complete off-shell effects included…
Backup slides...
**b-Jet**

- Upper panels: distributions and scale dependence bands
- Lower panels: differential $K$-factors

$$pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j + X$$

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**G. Bevilacqua, H. B. Hartanto, M. Kraus, M. Worek ’16**
Leptons

- Upper panels: distributions and the scale dependence bands
- Lower panels: differential $K$-factors

$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} j + X$

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**NWA for ttj**

- Inclusive NLO $\sigma(ttj)$ in NWA convolution of production $\sigma(tt+nj)$ & $\Gamma(tt+nj)$ $n \leq 2$

$$d\sigma_{\text{incl}} = \Gamma_{t,tot}^{-2}(d\sigma_{t\bar{t}+0j} + d\sigma_{t\bar{t}+1j} + d\sigma_{t\bar{t}+2j} + \cdots) \otimes (d\Gamma_{t\bar{t}+0j} + d\Gamma_{t\bar{t}+1j} + d\Gamma_{t\bar{t}+2j} + \cdots).$$

- Expanded version with terms up to $\alpha_s^4$ only

$$d\sigma_{\text{NLO}}^{ttj} = \Gamma_{t,tot}^{-2}(d\sigma_{LO,t+1j}^{tt}d\Gamma_{LO,t}^{tt} + d\sigma_{LO}^{tt}d\Gamma_{LO,t+1j}^{tt} + (d\sigma_{\text{virt},t+1j}^{tt} + d\sigma_{\text{real},t+2j}^{tt})d\Gamma_{LO,t}^{tt} + d\sigma_{LO}^{tt}(d\Gamma_{\text{virt},t+1j}^{tt} + d\Gamma_{\text{real},t+2j}^{tt})$$

- Mixed contribution

K. Melnikov, M. Schulze '12

(a) jet emission in production

(b) jet emission in decay

(c) mixed contribution
NWA for $ttj$

LHC @ 7 TeV with inclusive cuts

$\sigma_{LO} = 316.9 \text{(Pr)} + 33.4 \text{(Dec)} = 350.3 \text{ fb}$,

$\sigma_{NLO} = 323 \text{(Pr)} + 40.5 \text{(Dec)} - 75.5 \text{(Mix)} = 288 \text{ fb}$.

14%  26%

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Full NWA versus NWA with LO decays