Problem set 7

Tutorial: 6 June 2018, 12:15

Problem 15: Invisible Higgs decays

After electroweak symmetry breaking, the scalar singlet DM model contains a term in the Lagrangian that describes the interactions of two DM particles with a Higgs boson:

$$\mathcal{L} \supset \frac{v_0}{2} \lambda_{hs} \, s^2 \, h \; , \tag{1}$$

where $v_0 = 246\,\text{GeV}$ is the Higgs vacuum expectation value. For sufficiently light DM particles, $m_s < m_h/2$, this interaction induces the decay $h \to ss$. The corresponding matrix element is simply given by

$$\mathcal{M} = v_0 \lambda_{hs} \; . \tag{2}$$

To calculate the partial decay width in the rest frame of the Higgs boson, where $P_h = (m_h, 0, 0, 0)$, one needs to calculate the available phase space, which is obtained by integrating over the 3-momenta of the particles in the final state while imposing energy and momentum conservation:

$$\Gamma(h \to \chi\chi) = \frac{1}{4\,m_h} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3 (2\,E_1)} \frac{\mathrm{d}^3 p_2}{(2\pi)^3 (2\,E_2)} |\mathcal{M}|^2 (2\pi)^4 \, \delta^{(4)}(P_1 + P_2 - P_h) \; , \tag{3}$$

where $P_i = (E_i, \mathbf{p}_i)$ are the 4-momenta of the two DM particles and a symmetry factor 1/2 has been included to account for the two identical particles in the final state.

a) Calculate $\Gamma(h \to \chi\chi)$ as a function of λ_{hs} and m_s. Convince yourself that your result has the expected behaviour in the limit $m_s \to m_h/2$.

b) Calculate the invisible branching ratio of the Higgs boson (using $\Gamma(h \to \text{SM}) = 4\,\text{MeV}$) in the limit $m_s \to 0$ as a function of λ_{hs}.

c) Imposing the experimental constraint $\text{BR}(h \to \text{inv}) < 25\%$, derive the experimental upper bound on λ_{hs}.

Problem 16: Missing energy searches

Consider a new particle ϕ with mass m_ϕ that can be produced in from a quark anti-quark pair. The corresponding cross section can be written as

$$\sigma(q\bar{q} \to \phi) = k\delta(s - m_\phi^2) \ , \tag{4}$$

where $\sqrt{s}$ denotes the centre-of-mass energy and k is a dimensionless pre-factor that depends on the details of the model.

a) Obtain an expression for $\sigma(pp \to \phi)$ by convoluting the parton-level cross section with the parton distribution functions $f_q(x_1)$ and $f_{\bar{q}}(x_2)$. What is the distribution of longitudinal momenta $|p_z|$ of the produced particles?

b) Assume that ϕ decays isotropically (in its rest frame) into one visible and one invisible particle (both of which have a mass that is negligible compared to m_ϕ). Derive the distribution of $\not{E}_\mathrm{T}$. Why is it not necessary to know the parton distribution functions for this purpose?

c) An analogous process can happen in the Standard Model: $q\bar{q}' \to W^\pm \to \ell^\pm\nu$. Propose an analysis cut on $\not{E}_\mathrm{T}$ that would fully suppress this background. What fraction of signal events pass this cut (as a function of m_ϕ)?

Problem 17: Unitarity bound revisited

Let us revisit the unitarity bound on the mass of WIMPs in the context of the scalar singlet DM model.

a) Draw all diagrams that contribute to the scattering process $h\,s \to h\,s$.

b) Argue that in the limit of large centre-of-mass energy, $\sqrt{s} \to \infty$, only one diagram gives a relevant contribution, while the matrix elements for all other diagrams go to zero. What is the matrix element for this diagram?

Now we can make use of the requirement of partial wave unitarity, which implies

$$\left|\mathrm{Re}\left(\frac{1}{32\pi}\int \mathcal{M}\,\mathrm{d}\cos\theta\right)\right| < \frac{1}{2} \ . \tag{5}$$

c) Find the corresponding upper bound on λ_{hs}.

In the limit $m_s \gg m_h$, the total DM annihilation cross section is approximately given by

$$\langle\sigma v\rangle = \frac{\lambda_{hs}^2}{16\pi\, m_s^2} \ . \tag{6}$$

d) Determine the largest value of m_s for which the scalar singlet DM model can reproduce the observed DM abundance without violating the unitarity requirement.