Problem set 5

Tutorial: 16 May 2018, 12:15

Problem 10: Cascade annihilations

Consider a model in which a pair of non-relativistic DM particles annihilate into a pair of scalar particles of mass $m_\phi < m_\chi$, each of which subsequently decays into a pair of photons: $\chi\chi \to \phi\phi$, $\phi \to \gamma\gamma$.

a) What is the spectrum of the photons in the rest frame of the decaying scalar particle?

b) Assume that the scalar particles produced in the DM annihilation are emitted along the z-axis and that the two photons produced in the decay are emitted at an angle θ relative to the z-axis (in the rest frame of the decaying particle). What are the energies of the two photons in the centre-of-mass frame of the DM pair?

c) Assuming that the decays are isotropic in the rest frame of the decaying particle (this is always true for the decays of scalar particles), find the centre-of-mass-frame photon spectrum $\mathrm{d}N/\mathrm{d}x$, where $x = E_\gamma/m_\chi$, as a function of m_ϕ/m_χ.

d) Show that in the limit $m_\phi \to m_\chi$, one obtains a mono-energetic photon spectrum, such that all four photons have energy $m_\chi/2$. Why is this result to be expected?

Hint: You can make use of the fact that the number N of photons produced is Lorentz-invariant, and hence

$$\frac{\mathrm{d}N}{\mathrm{d}E_\gamma^{\mathrm{cm}}} = \int \mathrm{d}\cos\theta \frac{\mathrm{d}N}{\mathrm{d}\cos\theta\,\mathrm{d}E_\gamma^{\mathrm{dec}}} \frac{\mathrm{d}E_\gamma^{\mathrm{dec}}}{\mathrm{d}E_\gamma^{\mathrm{cm}}} , \tag{1}$$

where $\mathrm{d}E_\gamma^{\mathrm{cm}}$ and $\mathrm{d}E_\gamma^{\mathrm{dec}}$ denote the photon energy in the centre-of-mass frame and the rest frame of the decaying particle, respectively.

Problem 11: CMB constraints on DM annihilations

a) Using the present-day value of the DM abundance, $\Omega_{\rm DM}\, h^2 = 0.12$, and the critical density, $\rho_c = 1.05 \cdot 10^{-5}\, h^2\, \mathrm{GeV\, cm^{-3}}$, calculate the number density n_χ of DM particles as a function of redshift $z = 1/a - 1$.

b) Assuming that $\langle \sigma v \rangle$ is given by the thermal cross section, calculate the DM annihilation rate $\Gamma_{\rm ann}$ at recombination ($z = 1100$). Convince yourself that this rate is substantially smaller than the Hubble rate during recombination.

Even though DM annihilations during recombination only lead to a negligible change in the comoving number density of DM particles, they may have an important effect on the Cosmic Microwave Background if the partices produced in the annihilations can ionize neutral atoms. This is the case for example if DM particles annihilate dominantly into $\gamma\gamma$ or e^+e^-.

c) Show that the power injected into the photon-baryon fluid per unit volume can be written as

$$\frac{\mathrm{d}e}{\mathrm{d}t} = f(z) \frac{\langle \sigma v \rangle}{m_\chi} \, . \tag{2}$$

d) For the case of annihilations into e^+e^-, observations from Planck imply

$$\frac{\mathrm{d}e}{\mathrm{d}t}(z = 1100) \lesssim 3 \cdot 10^{-21}\ \mathrm{GeV\, cm^{-3}\, s^{-1}} \tag{3}$$

Determine the resulting bound on the DM mass.

In practice, the calculation of CMB constraints is complicated by the fact that recombination is sensitive to a range of redshifts and that energy is not deposited immediately after being released. Moreover, additional efficiency factors must be applied for different final states.