Problem set 2

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Problem 3: Self-interacting dark matter

Isothermal DM halos are predicted to have a density profile $\rho(r) \propto 1/r^2$. This appears to be in contradiction with observations indicating that DM halos have cores, i.e. $\rho(r) = \text{const}$ for $r < r_\text{c}$. A possible solution for this discrepancy is the idea of self-interacting dark matter, which states that DM particles would start forming a core as soon as the self-interaction rate R_SIDM times the age of the system t_age is larger than unity:

$$R_\text{SIDM}\, t_\text{age} > 1 \tag{1}$$

a) Show that $R_\text{SIDM} = \sigma_\text{SIDM}\, \rho\, v/m_\text{DM}$, where σ_SIDM is the self-interaction cross section, ρ is the DM density and v is the typical DM velocity.

b) Consider the Draco dwarf satellite, which has a velocity dispersion of about $\sigma_v \approx 20\,\text{km/s}$. Determine the density profile in the absence of self-interactions, assuming an isothermal halo.

c) Assume that the age of Draco is about $t_\text{age} \approx 10\,\text{Gyr}$. Determine the range of r for which eq. (1) is satisfied, as a function of $\sigma_\text{SIDM}/m_\text{DM}$.

d) It has been claimed that Draco has a core size of about $r_c \approx 1\,\text{kpc}$. What would that imply for $\sigma_\text{SIDM}/m_\text{DM}$?

e) Compare your result to the bound on $\sigma_\text{SIDM}/m_\text{DM}$ from the Bullet Cluster.

Problem 4: Big-Bang Nucleosynthesis

The number density of particles in thermal equilibrium is proportional to $\exp(-m/T)$, where m is the particle mass and T is the temperature, which is related to the age of the Universe via

$$t \approx \frac{M_\text{Pl}}{1.66\sqrt{g_*}\,T^2} \tag{2}$$

in terms of the Planck mass M_Pl and the number of relativistic degrees of freedom g_*.

a) Calculate the ratio of the neutron number density to the proton number density in terms of their mass difference $\Delta m = 1.3\,\text{MeV}$.

b) For temperatures below T_n the conversion between protons and neutrons becomes inefficient, so that neutron and proton number densities now evolve independently. The only process changing the ratio are then neutron decays with a lifetime of $\tau \approx 880\,\text{s}$. Calculate the ratio of neutron to proton number density for $T < T_n$, assuming $g_* = 3.36$. The change of the proton number density due to neutron decays can be neglected for simplicity.

c) Assuming that all neutrons that survive until $T_{\rm BBN} \approx 80\,{\rm keV}$ end up in helium, calculate T_n from the observed helium mass fraction $Y_p \approx 0.25$.

The decoupling temperature T_n can also be calculated in terms of the baryon-to-photon ratio η. This makes it possible to determine $\Omega_{\rm b}$ from measurements of Y_p.

Problem 5: Warm dark matter

To obtain a simple estimate for the free-streaming length of warm dark matter, we can assume that dark matter particles erase all structures as long as they are relativistic and start forming structures as soon as they become non-relativistic. The transition between the two regimes happens approximately for $T \approx m_{\rm DM}/3$.

a) Using the expression for the age of the Universe as a function of temperature from eq. (2), calculate the distance a dark matter particle travels before becoming non-relativistic as a function of the dark matter mass. This corresponds to the size of the smallest structures that can form in the early Universe, called $\lambda_{\rm fs}$.

b) Making use of the Stefan-Boltzmann law $\rho = \pi^2\, g_*\, T^4/30$, calculate the total amount of energy contained in a sphere with radius $\lambda_{\rm fs}$. The result gives an estimate for the mass $M_{\rm min}$ of the smallest DM halos that can form in the early Universe.

c) Calculate the value of $M_{\rm min}$ for $m_{\rm DM} = 1\,{\rm keV}$. Compare your result to the mass of the Milky Way ($M_{\rm MW} \approx 10^{12}\, M_\odot$).

If dark matter is warm, halos with mass $M < M_{\rm min}$ can only form via the fragmentation of larger structures. This so-called *top-down structure formation* is however strongly constrained by data.