Problem set 12

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Problem 25: Pseudo-Dirac neutrinos

Let us again consider the neutrino mass matrix

$$\overline{\nu^c} M \nu = \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_\mathrm{D} \\ m_\mathrm{D} & m_\mathrm{M} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \,, \tag{1}$$

but this time rather than assuming $m_\mathrm{M} \gg m_\mathrm{D}$ (as for the see-saw mechanism) we assume that $m_\mathrm{M} \ll m_\mathrm{D}$.

a) Show that in this case the eigenvalues of the mass matrix are given by $m_\pm = m_\mathrm{D}(1 \pm \epsilon)$ with $\epsilon = m_\mathrm{M}/(2\,m_\mathrm{D}) \ll 1$ and that the corresponding mass eigenstates can be written as $\psi_+ = \sin\theta\,\nu_L + \cos\theta\,\nu_R^c$ and $\psi_- = i(\cos\theta\,\nu_L - \sin\theta\,\nu_R^c)$.[1]

b) Write the neutrino mass matrix in terms of the vector $\psi = (\psi_+, \psi_-^c)^\mathrm{T}$.

c) Show that for $\epsilon \to 0$ one obtains $\theta \to \pi/4$, i.e. maximal mixing between active and sterile neutrino.

Although Pseudo-Dirac neutrinos are not viable DM candidates, the same idea has been successfully applied to models of DM, for example in the context of inelastic DM (see problem 12).

Problem 26: Sterile neutrino decay

In addition to the observable decay $N \to \gamma\nu$, sterile neutrinos can also decay fully invisibly into SM neutrinos.

a) Why are the decays $N \to \nu\nu$ and $N \to \nu\bar{\nu}$ forbidden?

b) Starting from the Feynman diagram for neutrino-neutrino scattering, construct the Feynman diagram for the decay $N \to \nu\nu\bar{\nu}$.

c) Argue that the decay width for this process must be proportional to $\theta^2\,G_\mathrm{F}^2$. Use dimensional analysis to derive an order-of-magnitude estimate for Γ.

d) Identify the region in m_N–θ parameter space where sterile neutrinos would decay too quickly to act as DM in the present Universe.

A more detailed calculation gives a somewhat weaker bound due to an extra factor $1/(96\pi^3)$ in the decay width arising from the phase space integration for the three-particle final state.

[1] Note that the second mass eigenstate has been multiplied by a factor i to ensure positivity of the mass eigenvalues.

Problem 27: Sterile neutrinos as SuperWIMPs

In addition to the usual production via mixing, sterile neutrinos can also be produced in the decays of heavy particles. As an example, we consider a scalar singlet S with mass m_S that obtains a thermal abundance via the freeze-out mechanism and then decays into sterile neutrinos: $S \to N\bar{N}$.

a) Assuming that the annihilation cross section of the scalar singlet into SM particles is simply given by $\langle \sigma v \rangle = \lambda_{hs}^2/(16\pi m_S^2)$ (see problem 17), calculate the abundance of sterile neutrinos as a function of m_S, λ_{hs} and m_N.

b) What value of λ_{hs} is needed in order to obtain 10% of sterile neutrinos from decays, assuming $m_N = 10\,\text{keV}$ and $m_S = 1\,\text{TeV}$?

c) If the scalar singlets only start decaying after freeze-out, how do the momenta of sterile neutrinos produced via decays compare to the momenta of sterile neutrinos produced via mixing?

d) Does the additional contribution from decays increase or decrease the free-streaming length λ_{fs}?