Problem set 10

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Problem 22: Axion electrodynamics

The Lagrangian of the electromagnetic field, $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, yields the equations of motion

$$\partial_\mu F^{\mu\nu} = 0 \tag{1}$$

in the absence of charged matter. The field strength tensor can be expressed in terms of E- and B-fields as

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\mathbf{E} \\ \mathbf{E}^T & -\epsilon_{ijk}B_k \end{pmatrix} . \tag{2}$$

a) By considering $\nu = 0$ and $\nu = (1, 2, 3)$, show that these equations of motion are equivalent to two of the Maxwell equations in vacuum.

In the presence of an axion, the equations of motion become

$$\partial_\mu F^{\mu\nu} + g_{a\gamma}(\partial_\mu a)\tilde{F}^{\mu\nu} = 0 \tag{3}$$

where $\tilde{F}^{\mu\nu}$ is the dual field strength tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\mathbf{B} \\ \mathbf{B}^T & \epsilon_{ijk}E_k \end{pmatrix} . \tag{4}$$

b) Derive the resulting modified Maxwell equations of axion electrodynamics.

Consider an axion travelling along the x-axis, $a = a_0 \cos(\omega t - kx)$, in a constant magentic field pointing in the z-direction, $\mathbf{B} = B_0\,\mathbf{e}_z$.

c) Show that the modified Maxwell equations are not satisfied unless the axion creates an E-field.

d) In what direction does this E-field point? How about the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B}$?

e) *Optional:* Derive the equations of motion stated above using the Euler-Lagrange equations.

Problem 23: Axion-photon conversion

While the conversion of an axion into a photon conserves energy (i.e. $E_\gamma = E_a$), it does not need to conserve momentum, because the transverse magnetic field can provide the necessary momentum transfer $\mathbf{q} = \mathbf{p}_\gamma - \mathbf{p}_a \neq 0$. However, axion-photon conversion is efficient only as long as $q\, l < 1$, where $q = |\mathbf{q}|$ and l is the length of the magnetized volume.

a) Derive an expression for q in the limit $m_a \ll E_a$.

Let us now assume now that the axion does not travel in vacuum, but in a gas with electron density n_e. The photon then obtains the effective plasma mass

$$m_\gamma^2 = \frac{4\pi\alpha n_e}{m_e}\ . \tag{5}$$

b) Find the resulting expression for the momentum transfer q. How does this change the probability for axion-photon conversion?

c) How can this effect be used to increase the range of axion masses that can be explored experimentally?