5. Axions

5.1 Motivation

Recap: The Lagrangian for electromagnetism is

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \text{interactions} \]

with

\[ F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \]

\[ \hat{F}^{\mu
\nu} = \begin{pmatrix} 0 & -E^\mu \\ E^\nu & 0 \end{pmatrix} \]

\[ \Rightarrow -\frac{1}{4} F^{\mu\nu} \hat{F}_{\mu\nu} = \frac{1}{2} (E^2 - B^2) \]

We can define

\[ \hat{F}_{\mu\nu} = \frac{1}{2} e^{\nu \sigma \kappa \xi} \hat{F}_{\mu \sigma} = \begin{pmatrix} 0 & -B^\mu \\ B^\nu & 0 \end{pmatrix} \]

(dual field strength)

\[ \Rightarrow -\frac{i}{4} F^{\mu\nu} \hat{F}_{\mu\nu} = E \cdot B \]

Symmetries: C: -1 -1 (charge conjugation)

P: -1 +1 (parity)

CP: +1 -1

\[ \Rightarrow -\frac{i}{4} F^{\mu\nu} \hat{F}_{\mu\nu} \text{ breaks CP symmetry} \]

(time-reversal symmetry)
However, one can show that

\[ F_{\mu \nu} \sim \tilde{F}_{\mu \nu} = \frac{1}{2} \partial_{\mu} \tilde{A}_{\nu} - \partial_{\nu} \tilde{A}_{\mu} \]

\Rightarrow total derivative (surface term)

\Rightarrow no contribution to equations of motion

QED conserves CP symmetry

What about QCD?

5.1.1 Strong CP problem

\[ L_{QCD} = - \frac{1}{4} G_{\mu \nu}^{a} G^{a}_{\mu \nu} \]

\[ G^{a}_{\mu \nu} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} \]

\[ + g f^{abc} A_{\mu}^{b} A_{\nu}^{c} \]

gauge coupling \rightarrow group theory coefficients

Again we can include an extra term

\[ L_{QCD} \rightarrow L_{QCD} + \frac{\mathcal{L}}{32 \pi^2} \partial_{\mu} G^{a}_{\mu \nu} \tilde{G}^{a}_{\mu \nu} \]

In contrast to QED, this term contributes to physics via non-perturbative effects.

Most important: Neutron electric dipole moment

\[ d_{n} \sim 5 \times 10^{-16} \text{ e cm} \]
Experiment:

\[ d_n \leq 5 \cdot 10^{-26} \text{ cm} \Rightarrow \Theta \lesssim 10^{-10} \]

Why is \( \Theta \) so small?

Simple solution: Impose CP as a symmetry of QCD

\[ \Rightarrow \Theta = 0 \]

\[ \Rightarrow \text{Does not work, because weak interactions violate CP} \]

\[ \Leftrightarrow \text{complex phase } \alpha \text{ in quark mass matrix} \]

\[ \Leftrightarrow \text{induces CP violation in strong interactions via loop diagrams} \]

\[ \Rightarrow \Theta \Rightarrow \overline{\Theta} = 0 + \alpha \]

We need cancellation between \( \Theta \) and \( \alpha \) to obtain

\[ \overline{\Theta} < 10^{-10} \]

\[ \Rightarrow \text{Strong CP problem} \]
5.1.2 Axion solution

Consider a new field $a$ that couples to gluons

$$a \quad \Rightarrow \quad \mathcal{L} = \frac{1}{32 \pi^2} \frac{\alpha}{p^a} \xi \xi^a \xi^\mu \xi_{\mu}$$

- Spin: 0
- CP: -1
- Pseudo-scalar

$\Rightarrow \quad \overline{\xi} \rightarrow \overline{\xi} + \frac{\alpha}{p_a}$

After QCD phase transition, gluons become confined in pions

$\Rightarrow \quad$ Generates a potential for $a$

$$V_{eff} = V_0 + \frac{1}{8} \left( \overline{\xi} + \frac{\alpha}{p_a} \right)^2 m_{\pi}^2 f_{\pi}^2$$

$135 \text{ MeV} \quad 93 \text{ MeV}$

$\Rightarrow \quad$ Potential is minimised for non-zero value of $a$

$\Rightarrow \quad a$ obtains a vacuum expectation value

$$\langle a \rangle = - \frac{\overline{\xi}}{p_a}$$
$\Rightarrow$ Cancels dangerous contribution to neutron EDM

(Peccei-Quinn mechanism)

But $a$ is a quantum field

$\Rightarrow$ oscillations around $<\alpha>$

Write $\alpha = <\alpha > + \tilde{\alpha}$

$\Rightarrow V_{\text{eff}} \approx V_0 + \frac{1}{8} \frac{m_\pi^2 f_\pi^2}{f_\alpha^2} \tilde{\alpha}^2$

Central prediction:

new particle $\tilde{a}$ with mass

$\tilde{m}_a = \frac{m_\pi f_\pi}{2 f_\alpha}$

(Weinberg-Wilczek axion)

What is $f_\alpha$?

Original idea: $p_\alpha \sim V_{\text{EW}} \sim 246 \text{ GeV}$

$\Rightarrow \tilde{m}_a \sim 15 \text{ keV}$

Ruled out experimentally

But if $p_\alpha \gg V_{\text{EW}}$, axions are much lighter and much more weakly interacting

$\tilde{m}_a \approx 0.6 \text{ eV} \left( \frac{10^7 \text{ GeV}}{p_\alpha} \right)^2$ "invisible axions"
5.2 Axion cosmology

Axions are perfect cold DM candidates in spite of their small mass.

Reason: Non-thermal production

5.1.1 Misalignment mechanism

Treat the axion as a classical field $\phi(x,t)$:

$$L = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m_a^2 \phi^2$$

Assume $\phi$ takes the same initial value everywhere:

$$\phi(x_{1,0}) = \phi_0$$

Equation of motion in static universe:

$$\ddot{\phi} = -\frac{\partial V}{\partial \phi} = -m_a^2 \phi$$

$$\Rightarrow \phi(x_{1,t}) = \phi_0 \cos(m_a t)$$

(Harmonic oscillator)

In expanding universe, kinetic energy is dissipated (red shifted):

$$\ddot{\phi} = -\frac{\partial V}{\partial \phi} - 3H \dot{\phi}$$

(Hubble friction)
Damped harmonic oscillator

For \( 3H > m_a \): Overdamped

\[
\Rightarrow \phi \approx 0 \Rightarrow \phi \approx \phi_0
\]

For \( 3H < m_a \): Oscillations possible

\[
\phi(t, x) = \phi_a(t) \cos(\omega t)
\]

with

\[
3H \phi_a = -2 \dot{\phi}_a
\]

\[
\Rightarrow -\frac{3}{2} \frac{d}{dt} \frac{\phi_a}{a} = \frac{d}{dt} \frac{\phi_a}{a}
\]

\[
\Rightarrow \phi_a(t) \sim a(t)^{-3/2}
\]

Oscillations with frequency \( \omega \) and amplitude \( \phi_a(t) \)

Energy stored in field proportional to \( \phi_a(t)^2 \sim a^{-3} \)

\( \Rightarrow \quad \text{Just like non-relativistic matter!} \)

QFT: Deep analogy between coherently oscillating field and a collection of particles at rest
Expansion of Universe produces a very large number of non-relativistic axions


L. perfect cold DM

Calculate relic abundance

\[ \Omega h^2 = \frac{\Sigma\Omega_a}{\Omega_{cDM}} = \phi_0 \left( \frac{a(t)}{a(t_{today})} \right)^3 \]

where \( t_1 \) is given by \( 3H(t_1) = m \)

During radiation domination

\[ H \sim \frac{1}{a^2} \Rightarrow a \sim m^{-1/2} \]

\[ \Rightarrow \phi_0 \left( t_{today} \right) \sim m^{-3/4} \]

\[ \Rightarrow \Omega h^2 \sim m^{1/2} \phi_0^2 \]

Restoring missing factors

\[ \Omega h^2 = 0.1 \sqrt{\frac{m_a}{1 \text{ meV}}} \left( \frac{\phi_0}{10^{-13} \text{ GeV}} \right)^2 \]

For the QCD axion we can express \( \phi_0 \) and \( m_a \) in terms of \( \phi_a \).
\[ \phi_0 = \frac{f_0}{f_a} \]

\[ \text{Initial misalignment angle } \quad \phi_0 \in [0, 2\pi] \]

\[ m = \frac{m_{\chi}^2}{f_a} \quad (+ \text{finite } T \text{ effects}) \]

\[ \Rightarrow \Omega h^2 \approx 0.3 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \phi_0^2 \]

\[ \Rightarrow \text{All of DM for } m_a \approx 40 \text{ meV} \quad \phi_0 \approx 1 \]

Note: smaller \( m_a \) \( \leftrightarrow \) larger \( f_a \) \( \leftrightarrow \) larger \( \Omega h^2 \)

However, can reduce \( \Omega h^2 \) by tuning \( \phi_0 \ll 1 \)

\[ \Rightarrow \text{Axions can be all of DM for } m_a \ll 40 \text{ meV} \]

But there is an upper bound on \( \phi_0 \)

\[ \Rightarrow \text{Upper bound on axion mass } \sim 1 \text{ meV} \]
5.2.2 Topological defects

So far: Assumed homogeneous value for axion field in early Universe

\[ \text{Justified if } f_a > f_1 \]

Scale of inflation

If \( f_a < f_1 \), a can take different values in different regions of space

At high \( T \):

\[ \text{where } \rightarrow: \quad \varphi_a = 0 \]
\[ \rightarrow \uparrow: \quad \varphi_a = \frac{\pi}{2} \]
\[ \text{etc.} \]

At small \( T \):

Field moves to low-energy configuration (like ferromagnet)

\[ \text{Homogeneous value} \]
Sometimes this is not possible

E.g.

\[ \uparrow \rightarrow \rightarrow \rightarrow \downarrow \]

"Cosmological string"

or the axion approaches different values in different regions of space:

\[ \uparrow \uparrow \uparrow \rightarrow \]

"Domain wall"

\[ \uparrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \]

\[ \Rightarrow \text{Axion field becomes trapped in high-energy configuration} \]

\[ \Rightarrow \text{Large contribution to } \Omega_a h^2 \]

Topological defects are usually unstable & decay to axions

\[ \Omega_a \text{ (strings & walls)} \sim 0.4 \left( \frac{f_a}{10^{11} \text{ GeV}} \right)^{\frac{7}{6}} \]