5. Axions

5.1 Motivation

Recap: The Lagrangian for electromagnetism is

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \text{interactions}$$

with

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E} \\ \vec{E}^T & \varepsilon_{ijk} B_k \end{pmatrix}$$

$$\Rightarrow -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}(E^2 - B^2)$$

We can define

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = \begin{pmatrix} 0 & -\vec{B} \\ \vec{B}^T & \varepsilon_{ijk} E_k \end{pmatrix}$$

(dual field strength)

$$\Rightarrow -\frac{1}{4} F^{\mu\nu} \tilde{F}_{\mu\nu} = \vec{E} \cdot \vec{B}$$

Symmetries:			
C:	-1	-1	(charge conjugation)
P:	-1	$+1$	(parity)
CP:	$+1$	-1	

$\Rightarrow -\frac{1}{4} F^{\mu\nu} \tilde{F}_{\mu\nu}$ breaks CP symmetry

(time-reversal symmetry)

However, one can show that

$$F^{\mu\nu} \tilde{F}_{\mu\nu} = \frac{1}{2} \partial_\mu \varepsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}$$

=> total derivative (surface term)

=> no contribution to equations of motion

QED conserves CP symmetry

What about QCD ?

5.1.1 Strong CP problem

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^{a\,\mu\nu} G^a_{\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

gauge coupling   group theory coefficients

Again we can include an extra term

$$\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{QCD} + \frac{\vartheta}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$$

In contrast to QED, this term contributes to physics via non-perturbative effects.

Most important: Neutron electric dipole moment

$$d_n \sim 5 \cdot 10^{-16}\, \vartheta \text{ cm}$$

Experiment:

$$d_n \lesssim 5 \cdot 10^{-26} \text{ cm} \quad \Rightarrow \quad \vartheta \lesssim 10^{-10}$$

Why is ϑ so small?

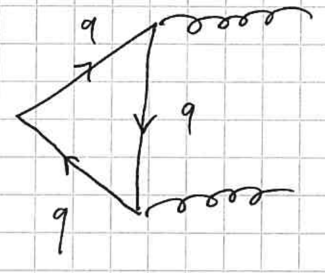
Simple solution: Impose CP as a symmetry of QCD

$$\Rightarrow \vartheta = 0$$

$\Rightarrow$ Does not work, because weak interactions violate CP

↳ complex phase α in quark mass matrix

↳ induces CP violation in strong interactions via loop diagrams



"axial anomaly"

$$\Rightarrow \vartheta \rightarrow \bar{\vartheta} = \vartheta + \alpha$$

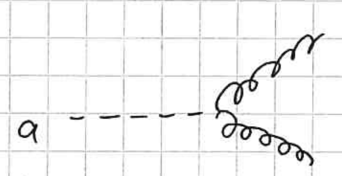
We need cancellation between ϑ and α to obtain

$$\bar{\vartheta} < 10^{-10}$$

$\Rightarrow$ Strong CP problem

5.1.2 Axion solution

Consider a new field a that couples to gluons



$$\Rightarrow \quad \mathcal{L} = \frac{1}{32\pi^2} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

spin: 0

CP: −1

⇒ pseudoscalar

f_a: unknown scale of new physics

$$\Rightarrow \quad \bar{\vartheta} \longrightarrow \bar{\vartheta} + \frac{a}{f_a}$$

After QCD phase transition, gluons become confined in pions

⇒ Generates a potential for a

$$V_{eff} \approx V_0 + \frac{1}{8}\left(\bar{\vartheta} + \frac{a}{f_a}\right)^2 m_\pi^2 f_\pi^2$$

m_π: 135 MeV, f_π: 93 MeV

⇒ Potential is minimised for non-zero value of a

⇒ a obtains a vacuum expectation value

$$\langle a \rangle = -\bar{\vartheta} f_a$$

$\Rightarrow$ Cancels dangerous contribution to neutron EDM

(Peccei - Quinn mechanism)

But a is a quantum field
$\Rightarrow$ oscillations around $\langle a \rangle$

Write $a = \langle a \rangle + \tilde{a}$

$$\Rightarrow V_{eff} \approx V_0 + \frac{1}{8} \frac{m_\pi^2 f_\pi^2}{f_a^2} \tilde{a}^2$$

Central prediction:

new particle $\tilde{a}$ with mass

$$m_a = \frac{m_\pi f_\pi}{2 f_a}$$

(Weinberg - Wilczek axion)

What is f_a ?

Original idea: $f_a \sim v_{EW} \sim 246$ GeV

$\Rightarrow m_a \sim 15$ keV

$\Rightarrow$ Ruled out experimentally

But if $f_a \gg v_{EW}$, axions are much lighter and much more weakly interacting

$$m_a \approx 0.6\,\text{eV} \left(\frac{10^7\ \text{GeV}}{f_a} \right)$$ "invisible axions"

5.2 Axion cosmology

Axions are perfect cold DM candidates in spite of their small mass

Reason: Non-thermal production

5.1.1 Misalignment mechanism

Treat the axion as a classical field $\phi(x,t)$:

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2} m_a^2 \phi^2$$

Assume ϕ takes the same initial value everywhere

$$\phi(x, t_0) = \phi_0$$

Equation of motion in static universe:

$$\ddot{\phi} = -\frac{\partial V}{\partial \phi} = - m_a^2 \phi$$

$$\Rightarrow \phi(x,t) = \phi_0 \cos(m_a t)$$

(harmonic oscillator)

In expanding universe, kinetic energy is dissipated (redshifted)

$$\ddot{\phi} = -\frac{\partial V}{\partial \phi} - 3H\dot{\phi}$$

Hubble friction

$\Rightarrow$ Damped harmonic oscillator

For $3H > m_a$: Overdamped

$\Rightarrow \dot{\phi} \approx 0 \Rightarrow \phi \approx \phi_0$

For $3H < m_a$: Oscillations possible

$$\phi(t, x) = \phi_a(t) \cos(mt)$$

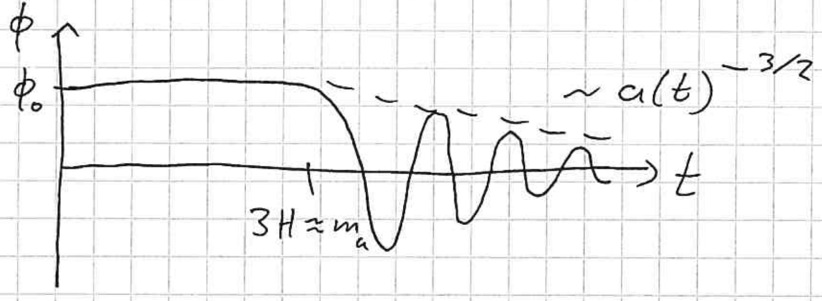
with

$$3H \phi_a \approx -2 \dot{\phi}_a$$

$$\Rightarrow -\frac{3}{2} \frac{da}{a} = \frac{d\phi_a}{\phi_a}$$

$$\Rightarrow \phi_a(t) \sim a(t)^{-3/2}$$

Oscillations with frequency m and amplitude $\phi_a(t)$



Energy stored in field proportional to $\phi_a(t)^2 \sim a^{-3}$

$\Rightarrow$ Just like non-relativistic matter!

QFT: Deep analogy between coherently oscillating field and a collection of particles at rest

$\Rightarrow$ Expansion of Universe produces a very large number of non-relativistic axions

↳ perfect cold DM

Calculate relic abundance

$$\Omega h^2 \sim m_a^2 \, \phi_a(\text{today})^2$$

$$\phi_a(\text{today}) = \phi_0 \left(\frac{a(t_1)}{a(\text{today})} \right)^{3/2}$$

where t_1 is given by $3H(t_1) = m$

During radiation domination

$$H \sim \frac{1}{a^2} \Rightarrow a \sim m^{-1/2}$$

$$\Rightarrow \phi_a(\text{today}) \sim m^{-3/4}$$

$$\Rightarrow \Omega h^2 \sim m^{1/2} \, \phi_0^2$$

Restoring missing factors

$$\Omega h^2 = 0.1 \sqrt{\frac{m_a}{1\,\mu eV}} \left(\frac{\phi_0}{10^{13}\,\text{GeV}} \right)^2$$

For the QCD axion we can express ϕ_0 and m_a in terms of f_a.

$$\phi_0 \hat{=} \vartheta_0 f_a$$

initial misalignment angle

$$\vartheta_0 \in [0, 2\pi]$$

$$m = \frac{m_\pi^2}{f_a} \quad (+ \text{ finite } T \text{ effects})$$

$$\Rightarrow \Omega h^2 \approx 0.3 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{7/6} \vartheta_0^2$$

$\Rightarrow$ All of DM for $m_a \sim 40\ \mu\mathrm{eV}$
$\vartheta_0 \sim 1$

Note: smaller m_a $\leftrightarrow$
larger f_a $\leftrightarrow$
larger Ωh^2

However, can reduce Ωh^2 by tuning $\vartheta_0 \ll 1$

$\Rightarrow$ Axions can be all of DM for $m_a \ll 40\ \mu\mathrm{eV}$

But there is an upper bound on ϑ_0

$\Rightarrow$ Upper bound on axion mass (~ 1 meV)

5.2.2 Topological defects

So far: Assumed homogeneous value for axion field in early Universe

↳ Justified if $f_a > \Lambda_I$ ← scale of inflation

If $f_a < \Lambda_I$, a can take different values in different regions of space

At high T:

↑ ↗ → ↑ ↘
↗ → ↘ ↗ ↘
↖ ↓ ↘ → ↗

where →: $\vartheta_0 = 0$

↑: $\vartheta_0 = \frac{\pi}{2}$

etc.

At small T:

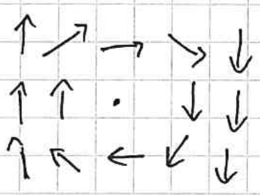
Field moves to low-energy configuration (like ferromagnet)

↗ ↗ ↗ ↗ ↗
↗ ↗ ↗ ↗ ↗
↗ ↗ ↗ ↗ ↗

(homogeneous value)

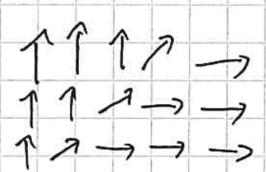
Sometimes this is not possible

E.g.



"Cosmological string"

or the axion approaches different values in different regions of space:



"Domain wall"

$\Rightarrow$ Axion field becomes trapped in high-energy configuration

$\hookrightarrow$ very large contribution to $\Omega_a h^2$

Topological defects are usually unstable & decay to axions

$$\Omega_a \text{ (strings \& walls)} \sim 0.4 \left(\frac{f_a}{10^{11}\,\text{GeV}} \right)^{7/6}$$