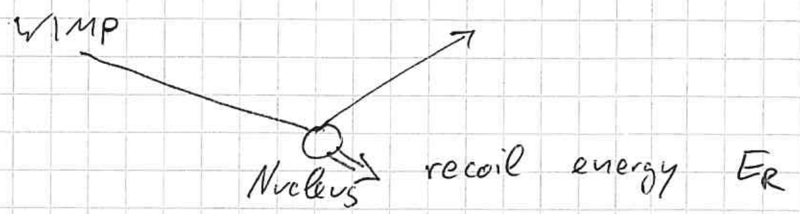
3.2 Direct detection

Basic idea: WIMPs can scatter off SM particles

Example:



DM is everywhere (e.g. in the Milky Way halo)
$\Rightarrow$ Scattering can happen on Earth

First estimates:

$$\rho_{DM} \sim 0.3 \ \frac{\text{GeV}}{\text{cm}^3}$$

$$v_{DM} \sim 200 \ \frac{\text{km}}{\text{s}}$$

$$\Rightarrow \cancel{\text{flux}} \ j_{DM} = \frac{\rho_{DM} \, v_{DM}}{m_\chi} \sim 10^4 \text{ WIMPs per cm}^2 \text{ s}$$

Scattering cross section

$$\sigma_\chi \sim \frac{\alpha_w^2}{m_\chi^2} \sim 10^{-36} \text{ cm}^2 \quad (= 1 \text{ pb})$$

$\Rightarrow$ Scattering ~~probability~~ rate per nucleus

$$R = j_{DM} \cdot \sigma_\chi \sim 10^{-32} \text{ s}^{-1}$$

Typical energy transfer

$$E_R \sim \frac{1}{2} \mu \, v_{DM}^2 \sim (1-100) \text{ keV}$$

reduced mass: $\mu = \dfrac{m_\chi \, m_N}{m_\chi + m_N}$

Impossible to detect?

For a detector with N_T target nuclei, one has

$$R = N_T \cdot j_{DM} \cdot \sigma_\chi$$

Ton-scale detector: $N_T \sim 10^{28}$

$\Rightarrow$ $R \sim$ 1 event per hour

$\Rightarrow$ Observable with low-background and low-threshold detectors

3.2.1 Differential event rates

- Not all DM particles have the same velocity

$$v_{DM} \rightarrow \int v \, f(\vec{v}) \, d^3v$$

velocity distribution

- ~~Only~~ Energy & momentum conservation:

Need $v > v_{min} = \sqrt{\frac{m_N E_R}{2\mu^2}}$

to produce recoil energy E_R (problem 12)

$$\Rightarrow \frac{dR}{dE_R} = \frac{\rho_{DM}}{m_\chi} N_T \int_{v_{min}}^{\infty} v \, f(\vec{v}) \, \frac{d\sigma}{dE_R} \, d^3v$$

implicit v-dependence

Convenient to normalize rate to total detector mass $m_D = N_T \cdot m_N$

$$\frac{dR}{dE_R} = \frac{\rho_{DM}}{m_N \, m_\chi} \int_{v_{min}}^{\infty} v \, f(\vec{v}) \, \frac{d\sigma}{dE_R} \, d^3 v$$

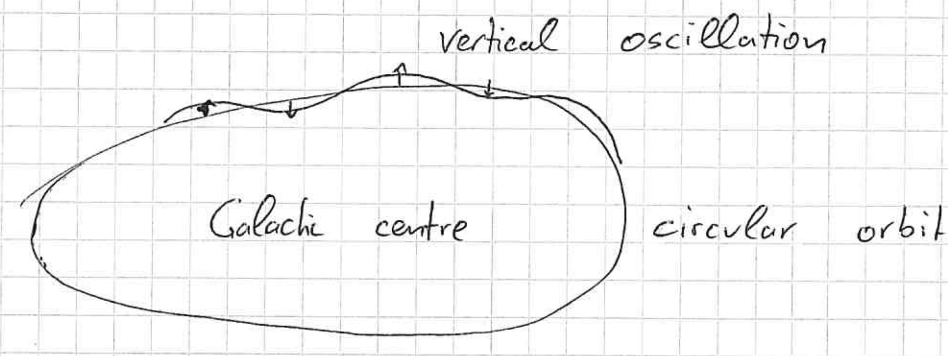
3.2.2 Input from astrophysics

ρ_{DM} : local DM density

→ can be inferred from galactic rotation curves

→ alternative: measure vertical motion of stars in solar neighbourhood

Idea:



Oscillation period depends on ρ_{DM}

Result: $\rho_{DM} \sim (0.2 - 0.5) \, \frac{GeV}{cm^3}$

Common choice: $\rho_{DM} = 0.3 \, \frac{GeV}{cm^3} \; (\equiv \rho_0)$

$f(v)$: DM velocity distribution

→ currently impossible to measure

→ need to rely on N-body simulations

Result: $f(v)$ is approximately of MB form with velocity dispersion proportional to rotational velocity

$$f(v) \sim \exp\left(-\frac{v^2}{v_0^2}\right)$$

$v_0 \sim 220$ km/s

Comment 1: An important correction arises from the finite escape velocity

$$v_{esc} = \sqrt{\frac{2G\, M(r_{sun})}{r_{sun}}} \sim 600 \text{ km/s}$$

$$\Rightarrow f(v) = 0 \quad \text{for} \quad v > v_{esc}$$

Comment 2: Need to transform into laboratory frame

$$f_{lab}(\vec{v}) = f(\vec{v} + \vec{v}_E)$$

with

$$\vec{v}_E = \vec{v}_{sun} + \Delta\vec{v}$$

velocity of Earth rel. to Sun

⇒ Problem 13

3.2.3 Input from particle physics

↳ Encoded in $d\sigma/dE_R$

Important simplification: $v_{DM} \ll c$

$\Rightarrow$ Scattering is non-relativistic

$\Rightarrow$ Matrix element can only depend on

$\vec{S_\chi}$: DM spin (if $S_\chi > 0$)
$\vec{S_N}$: Nucleus spin (if $S_N > 0$)
$\vec{v}$: relative velocity
$\vec{q}$: momentum transfer

↳ must enter as $\frac{\vec{q}}{m_\chi}$ or $\frac{\vec{q}}{m_N}$

Since $\vec{v}, \frac{\vec{q}}{m_\chi}, \frac{\vec{q}}{m_N} \ll 1$, it is usually sufficient to include only $\vec{S_\chi}$ and $\vec{S_N}$.

$\Rightarrow$ 2 possibilities:

a) $\mathcal{M} = \text{const}$ (spin-independent)

b) $\mathcal{M} = f(\vec{S_\chi}, \vec{S_N})$
$= f(\vec{S_\chi} \cdot \vec{S_N})$ (spin-dependent)

Note: $\mathcal{M}$ is a scalar quantity
$\Rightarrow$ cannot depend only on $\vec{S_\chi}$ or only on $\vec{S_N}$

Important complication:

Need to map DM-quark interactions to DM-nucleus interactions

Example: $\mathcal{L}^{EFT} \supset c_u \bar{\chi}\gamma^\mu\chi \, \bar{u}\gamma_\mu u + c_d \bar{\chi}\gamma^\mu\chi \, \bar{d}\gamma_\mu d$

(Vector couplings)

Vector current is conserved: $\partial_\mu \bar{u}\gamma^\mu u = 0$

$$\Rightarrow \mathcal{L}^{EFT} \longrightarrow \mathcal{L}^{EFT}_{nucleon} = c_p \bar{\chi}\gamma^\mu\chi \, \bar{p}\gamma_\mu p + c_n \bar{\chi}\gamma^\mu\chi \, \bar{n}\gamma_\mu n$$

with $c_p = 2c_u + c_d$, $\quad c_n = 2c_d + c_u$

$$\mathcal{L}^{EFT}_{nucleon} \rightarrow \mathcal{L}^{EFT}_{nucleus} = c_N \bar{\chi}\gamma^\mu\chi \, \bar{N}\gamma_\mu N$$

with $c_N = Z c_p + (A - Z) c_n$

E.g. for $c_u = c_d = c \Rightarrow c_N = 3Ac$

for $c_u = \frac{2}{3}c$, $c_d = -\frac{1}{3}c \Rightarrow c_N = Zc$

$\Rightarrow$ Calculate scattering cross section from $\mathcal{L}^{EFT}_{nucleus}$

$$\sigma_N = \frac{c_N^2 \mu^2}{\pi}$$

↳ spin-independent

Scattering is also isotropic

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\sigma_N}{2}$$

$$\Rightarrow \quad \frac{d\sigma}{dE_R} = \frac{d\sigma}{d\cos\vartheta} \frac{m_N}{\mu^2 v^2}$$

$$= \frac{m_N}{2\mu^2 v^2} \sigma_N$$

$$= \frac{m_N}{2v^2} \frac{C_N^2}{\pi}$$

Convenient to write in terms of DM-proton cross section

$$\sigma_p = \frac{C_p^2 \, \mu_{\chi p}^2}{\pi}$$

$$\Rightarrow \quad \frac{d\sigma}{dE_R} = \frac{m_N}{2\mu_{\chi p}^2 v^2} \left(\frac{C_N}{C_p}\right)^2 \sigma_p$$

$$\Rightarrow \quad \frac{dR}{dE_R} = \frac{\rho_{DM}}{2 m_\chi \mu_{\chi p}^2} \underbrace{\int_{v_{min}} d^3v \frac{f(\vec{v})}{v}}_{\text{"velocity integral"}} \left(\frac{C_N}{C_p}\right)^2 \sigma_p$$

For $\quad C_u = C_d : \quad \left(\frac{C_N}{C_p}\right)^2 = A^2$

$\Rightarrow$ Scattering enhanced for heavy targets

Reason: All quarks interfere constructively

$\Rightarrow$ Coherent enhancement

But: Nucleus not a point-like object

$\Rightarrow$ Loss of coherence for $q \gtrsim r_N$

$\Rightarrow \left(\frac{c_N}{c_p}\right)^2 \longrightarrow F^2(q^2)$ (form factor)

$$\lim_{q\to 0} F^2(q^2) = \left(Z + (A-Z)\frac{c_n}{c_p}\right)^2$$

Another example:

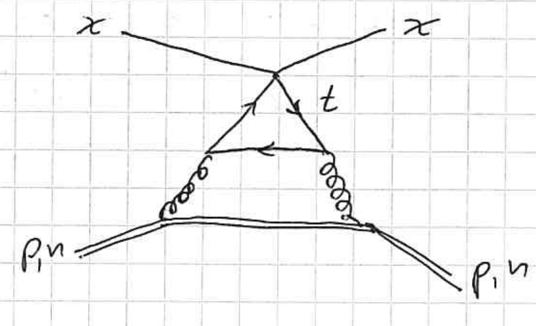
$$\mathcal{L}^{EFT} \supset \sum_q \frac{1}{\Lambda^3} m_q \bar{q} q \bar{\chi}\chi$$

(scalar couplings)

$$\mathcal{L}^{EFT} \longrightarrow \mathcal{L}^{EFT}_{nucleon} = \frac{f_p}{\Lambda^3} m_p \bar{p} p \bar{\chi}\chi + \frac{f_n}{\Lambda^3} m_n \bar{n} n \bar{\chi}\chi$$

with $f_p \approx f_n \approx 0.3$

↳ Challenging calculation!
Contributions from diagrams like



But: matching nucleon $\rightarrow$ nucleus same as before

$$f_N = Z f_p + (A-Z) f_n \approx A \cdot f_p$$

$$\Rightarrow \frac{\sigma_N}{\sigma_p} = A^2$$

Finally: $\mathcal{L}^{EFT} \supset c_u \bar{\chi} \gamma^\mu \gamma^5 \chi \, \bar{u} \gamma_\mu \gamma^5 u$
$+ c_d \bar{\chi} \gamma^\mu \gamma^5 \chi \, \bar{d} \gamma_\mu \gamma^5 d$

(axialvector couplings)

=> Leads to spin-dependent scattering

$$\sigma_N = \underbrace{J(J+1)}_{=0 \text{ for } J=0} \frac{4 a_N^2 \mu^2}{\pi}$$

a_N needs to be calculated / measured for every isotope

$$\frac{d\sigma_N}{dE_R} = S(q^2) \frac{m_N \sigma_p^{SD}}{2 \mu_{n\chi}^2 v^2}$$

Spin-dependent form factor (no enhancement for heavy nuclei)

Summary:

$\frac{dR}{dE_R}$ is:
- proportional to A^2 (or Z^2) for spin-independent int.
- proportional to $J(J+1)$ for spin-dependent int.
- decreases for large E_R due to loss of coherence
- decreases for large v_{min} due to velocity integral

3.2.4 Experimental strategies

Three possibilities to detect $E_R \sim keV$:

1) Ionization (detect free charges)

2) Scintillation (detect light)

3) Heat (detect phonons)

To identify a DM signal, at least 2 channels should be detected.

Main strategies:	Channels	Threshold	Mas
a) Nobel gas detectors:	1 + 2	~10 keV	ton
b) Cryogenic semiconductors:	1 + 3	~1 keV	kg
c) Cryogenic scintillators:	2 + 3	~0.1 keV	g

Typical result: