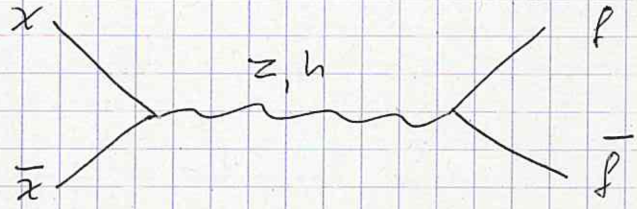
2.2 The WIMP miracle

Typical annihilation process:



For $m_\chi \gg m_Z, m_h, m_f$ dimensional analysis gives

$$\langle \sigma v \rangle \sim \frac{1}{m_\chi^2}$$

Constant of proportionality depends on coupling strength

For weak interactions

$$\langle \sigma v \rangle \sim \frac{\alpha_W^2}{m_\chi^2} \qquad (\alpha_W \sim 10^{-2})$$

Assume m_χ close to electroweak scale:

$$m_\chi \sim v_{EW} \sim 250 \text{ GeV}$$

$$\Rightarrow \langle \sigma v \rangle \sim 10^{-9} \text{ GeV}^{-2}$$

$$\sim 10^{-26} \text{ cm}^3/\text{s}$$

$$\Rightarrow \Omega_\chi h^2 \sim 0.1$$

Very close to observed value

$$\Omega_{DM} h^2 = 0.12$$

2.3 Supersymmetry

Why is $m_\chi \sim 250$ GeV interesting?

$\rightarrow$ Possible connection to EW symmetry breaking

Example: SUSY

2.3.1 Motivation

SM is renormalizable theory

↳ valid at all energies

But: Does not describe gravity

$\Rightarrow$ SM incomplete

$\Rightarrow$ There must be new scale's

$$\Lambda \gg v_{EW}$$

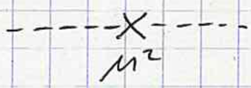
$$\text{e.g.} \quad \Lambda = M_{Pl}$$

Consider Higgs potential

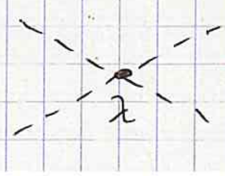
$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$\lambda \approx 0.1$

Tree level:

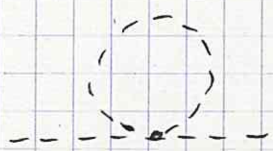


mass term



interaction term

Loop level: correction to mass term



$$\Delta\mu^2 \sim \frac{\lambda \Lambda^2}{16\pi^2}$$

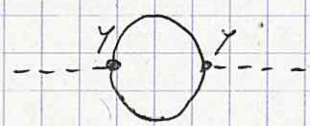
$$\Rightarrow \quad \underset{\substack{\text{renormalized}\\ \text{coupling}\\ (\approx 10^4\ \text{GeV}^2)}}{\mu_r^2} = \underset{\text{tree-level}}{\mu_t^2} + \underset{\approx 10^{35}\ \text{GeV}^2}{\Delta\mu^2}$$

$\Rightarrow$ Need very precise cancellation

$\Rightarrow$ Fine tuning!

Key observation:

Fermion loops have opposite sign



$$\sim \; - \frac{y^2 \Lambda^2}{16\pi^2}$$

Bosonic and fermionic loops cancel!

Need underlying relation between λ and y

$\Rightarrow$ Symmetry relating bosons & fermions

$\Rightarrow$ Supersymmetry

2.3.2 The MSSM

Idea: Each particle has a superpartner with $\Delta s = 1/2$

quark $\longrightarrow$ squark $(s=0)$
lepton $\longrightarrow$ slepton $(s=0)$
Higgs $\longrightarrow$ higgsino $(s=1/2)$
gauge boson $\longrightarrow$ gaugino $(s=1/2)$

masses not identical
SUSY is broken!

Simplest realization:
Minimal Supersymmetric Standard Model

SM + 2^{nd} Higgs doublet + superpartner

4 new neutral fermions:

- 2 higgsinos $\tilde{H}$ (one from each doublet)
- 1 bino $\tilde{B}$ (superpartner of hypercharge
- 1 wino $\tilde{W}$ (" of neutral $SU(2)$ ~~SU(2)~~ boson)

$\Rightarrow$ "Neutralinos"

↳ mix with each other

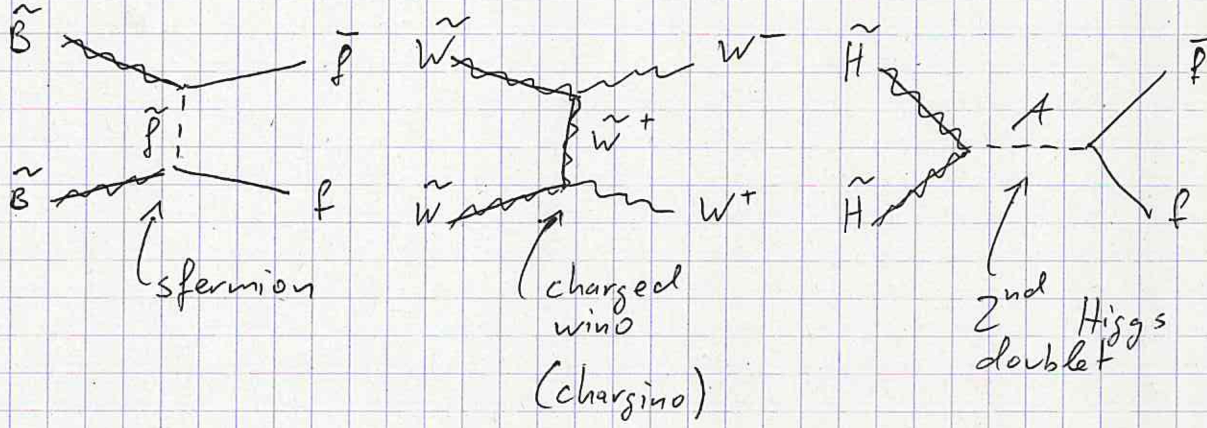
↳ mass eigenstate $\neq$ interaction eigenstate

2.3.3 Supersymmetric WIMPs

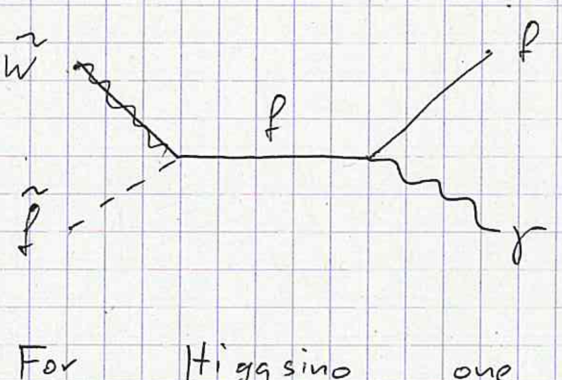
R-parity: Lightest neutralino can be stable

$\Rightarrow$ perfect WIMP candidate

Many possible annihilation channels, depending on neutralino mixing



Also, co-annihilation (involving only 1 DM particle)



For Higgsino one finds

$$\Omega_{\tilde{H}} h^2 \approx 0.1 \left(\frac{m_{\tilde{\chi}}}{1\,\mathrm{TeV}} \right)^2$$

$\Rightarrow$ 1.1 TeV Higgsino makes a great DM candidate!

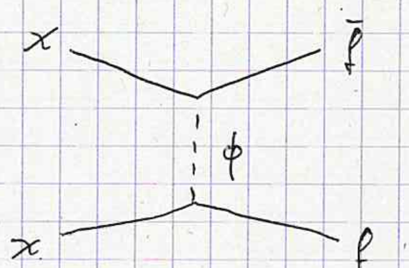
2.4 Simplified models and effective theories

SUSY is complicated

↳ Many new particles
↳ Many annihilation channels

Idea: Focus on specific aspects

Two examples:

i) 

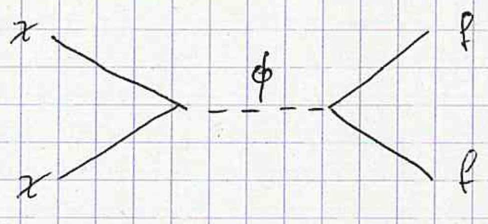
(t - channel simplified model)

ϕ: mediator

$$\mathcal{L} \supset g \phi \bar{\chi} f$$

charged under $SU(3) \times SU(2) \times U(1)$

3 parameters: g, m_χ, m_ϕ

2) 

(s - channel simplified model)

$$\mathcal{L} \supset g_f \phi \bar{f} f + g_\chi \phi \bar{\chi} \chi$$

4 parameters: g_f, g_χ, m_χ, m_ϕ

=> Study phenomenology in terms of these few parameters

=> Combine different simplified models to cover full parameter space

Simplifying further:

Propagator: 1) $$\frac{1}{s - m_\phi^2} = \frac{1}{(p_1 - p_3)^2 - m_\phi^2}$$

2) $$\frac{1}{t - m_\phi^2} = \frac{1}{(p_1 + p_2)^2 - m_\phi^2}$$

Freeze-out: $v \ll 1$ & $m_\chi \gg m_f$

$$\Rightarrow \quad p_1 \approx p_2 \approx \begin{pmatrix} m_\chi \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p_3 \approx \begin{pmatrix} m_\chi \\ 0 \\ m_\chi \sin\vartheta \\ m_\chi \cos\vartheta \end{pmatrix}$$

$$\Rightarrow \quad s \approx 4 m_\chi^2$$

$$t \approx - m_\chi^2$$

If $m_\phi \gg m_\chi$, we can approximate

$$\frac{1}{s - m_\phi^2} \approx \frac{1}{t - m_\phi^2} \approx -\frac{1}{m_\phi^2}$$

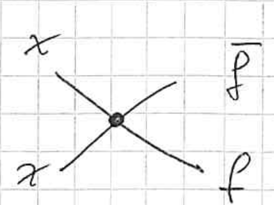
$\Rightarrow$ Replace propagator by constant factor

↳ "Integrate out heavy degrees of freedom"

Define new Lagrangian with only light degrees of freedom:

1) $\mathcal{L}^{EFT} \supset c_1 \, (\bar{\chi} f)(\bar{f} \chi)$

2) $\mathcal{L}^{EFT} \supset c_2 \, (\bar{\chi} \chi)(\bar{f} f)$



$\underbrace{\qquad\qquad\qquad}_{\text{4-fermion interactions}}$

↳ Analogous to Fermi theory of weak interactions

What are c_1 and c_2?

Mass dimension: fermion: $\frac{3}{2}$

Lagrangian: 4

c_1 & c_2: -2

$\Rightarrow$ Determine coefficients such that effective theory and full theory agree:

$$c_1 = \frac{g^2}{m_\phi^2} \qquad c_2 = \frac{g_\chi \cdot g_f}{m_\phi^2}$$

Literature: $c_1 \rightarrow \frac{1}{\Lambda_1^2}$ $\qquad c_2 \rightarrow \frac{1}{\Lambda_2^2}$

Advantage: Only 2 parameters (m_χ, Λ)

Disadvantage: Non-renormalizable theory

↳ Only valid at low energies

↳ Loop calculations difficult

Example:

For case 2 one finds

where $N_c = \begin{cases} 3 & \text{quark} \\ 1 & \text{lepton} \end{cases}$

$$\sigma v = \frac{N_c v^2 m_\chi^2}{8\pi \Lambda^4} \left(1 - \frac{m_f^2}{m_\chi^2}\right)^{3/2}$$

Note: $\sigma v \sim v^2$ (p-wave annihilation)

$\langle \sigma v \rangle \sim \frac{1}{x_f}$ (see tutorial)

For simplicity, focus on $\bar{\chi}\chi \to \bar{b}b$
(assuming $m_b \ll m_\chi$)

$$\langle \sigma v \rangle \approx \frac{9 \, m_\chi^2}{4\pi \, x_f \, \Lambda^4}$$

$$\Omega_\chi h^2 \approx 10^{-8} \, \text{GeV}^{-2} \, \frac{\Lambda^4}{m_\chi^2}$$

$\Rightarrow$ Observed abundance for

$$\Lambda \approx 50 \cdot \sqrt{\frac{m_\chi}{1 \, \text{GeV}}}$$

E.g. $m_\chi = 100$ GeV, $\Lambda = 500$ GeV

Note: For $m_\chi = 1$ TeV, $\Lambda \sim 1.8$ TeV

$\Rightarrow s \approx 4 m_\chi^2 > \Lambda^2$

$\Rightarrow$ Effective theory no longer valid!