1.5 Structure formation

- Assume initial conditions (matter power spectrum $P(k)$)
- Evolve under influence of gravity
- Determine size, shape, number of DM halos

$\Rightarrow$ Numerical simulations (N-body)

E.g. Millennium XXL:

$\hookrightarrow$ $3 \cdot 10^{11}$ simulation "particles" with mass $7 \cdot 10^{9}\, M_\odot$ each

Challenge: Star formation, supernova explosions, ...

$\Rightarrow$ Hydrodynamical simulations

Output (example)

Halo mass function $n(M_h)$: Number of DM halos with mass $M > M_h$

$\Rightarrow$ Compare to observations

1.5.1 Warm dark matter

- Simulations cannot resolve individual DM particles
- Particle physics captured by initial conditions

Example: Free-streaming modifies $P(k)$ for large k

This changes $n(M_h)$ for small M_h

$\Rightarrow$ Suppression of small-scale structure

Observations:
- Lower bound on m_{DM}

  $m_{DM} \gtrsim 1\ \mathrm{keV}$

  also for bosons!

- Cold dark matter does not fit perfectly ("missing satellite problem")

1.5.2* Halo profiles

Simulations make new predictions

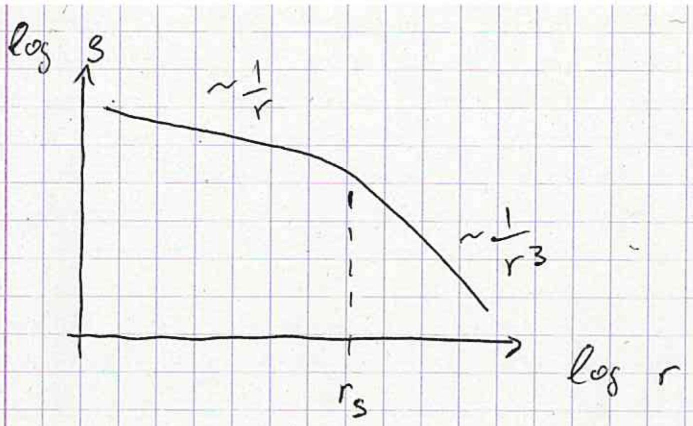
Example: DM halo profile is not

$$\rho(r) \sim \frac{1}{r^2}$$

but

$$\rho(r) \sim \frac{1}{r(r_s + r)^2}$$

r_s: scale radius



Observations: – Good fit for large r
– Discrepancy in central region ($v_c(r)$ smaller than predicted)

$$\Rightarrow \rho(r) \xrightarrow{r \to 0} \text{const} \; ?$$

$\Rightarrow$ Central core ?

("cusp-core problem")

But: Baryonic effects are important (mis-modelling ?)

Possible resolution: Self-interacting dark matter

1.6 Properties of Dark Matter

- DM is <u>stable</u> ($\tau_{DM} \gg t_{universe}$)
- DM is <u>electrically neutral</u> ($q_{DM} \ll e$)

by definition

- DM is <u>non-baryonic</u> ($\Omega_B \ll \Omega_{DM}$) — CMB + BBN
- DM is <u>dissipationless</u> and <u>collisionless</u> — Bullet Cluster + rotation curves
- DM is <u>cold</u> — Matter power spectrum + large-scale structure

$\Rightarrow$ No Standard Model particle has these properties

↳ Neutrinos would be hot DM ($m_\nu \lesssim 0.2$ eV)

$\Rightarrow$ DM cannot be diffuse gas, planets or faint stars

New yet undiscovered particle ?

2. Weakly-interacting massive particles (WIMPs)

Evidence for DM only from gravitational interactions!

What if DM has additional (weak) interactions?

$\Rightarrow$ DM enters into thermal equilibrium

$\Rightarrow$ Must be much heavier than neutrinos (bound on hot DM)

$$\Downarrow$$

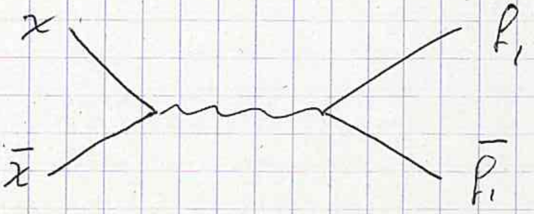
WIMP

2.1 Thermal freeze-out

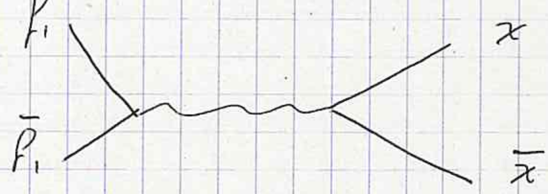
Consider Universe with temperature T filled with relativistic particles f.

Consider a heavy DM species χ with $m_\chi \gg T$.

Annihilation process:



Production process:



Rate per particle:

$$\Gamma_{ann} = \langle \sigma_{ann} \, v \rangle \, n_\chi$$

$$\Gamma_{prod} = \langle \sigma_{prod} \, v \rangle \, n_f$$

$\langle \rangle$: Thermal average

In thermal equilibrium

$$\Gamma_{ann} = \Gamma_{prod}$$

Since $m_f \ll T \ll m_\chi$, annihilation is always possible

Production is only possible if $E_f \sim m_\chi$

$$\Rightarrow \frac{\langle \sigma_{prod} \, v \rangle}{\langle \sigma_{ann} \, v \rangle} \sim e^{-m_\chi / T}$$

Equilibrium only maintained if

$$n_x \sim e^{-m_x/T}$$

In fact, DM number density given by

$$n_x = g_x \int \frac{d^3p}{(2\pi)^3} e^{-E/T}$$

$$= g_x \int \frac{4\pi p^2}{(2\pi)^3} e^{-m_x/T} e^{-p^2/2m_xT} dp$$

$$= g_x \left(\frac{m_x T}{2\pi} \right)^{3/2} e^{-m_x/T}$$

If DM stays in equilibrium

$$n_x \rightarrow 0 \quad \text{as} \quad T \rightarrow 0$$

To obtain sizeable abundance of DM, it must decouple from T. E.

This happens if

$$\underset{\substack{\text{annihilation} \\ \text{rate}}}{\Gamma_{ann}} \lesssim \underset{\substack{\text{expansion} \\ \text{rate}}}{H}$$

$\sim e^{-m_x/T}$ $\qquad \sim T^2$

The temperature for which

$$\Gamma_{ann} = H$$

is called freeze-out temperature T_f:

$$\langle \sigma v \rangle \, g_x \left(\frac{m_x T_f}{2\pi} \right)^{3/2} e^{-m_x/T_f} \approx 1.66 \sqrt{g_*} \frac{T_f^2}{M_{pl}}$$

Define $\quad x_f = m_x / T_f$

$$C = \frac{1.66 \sqrt{g_*}}{g_x} (2\pi)^{3/2}$$

$$\Rightarrow \langle \sigma v \rangle \, m_x^3 \, x_f^{-3/2} \, e^{-x_f} = \frac{C \, m_x^2}{x_f^2 \, M_p}$$

$$\Rightarrow \langle \sigma v \rangle \, m_x \, M_p \, \sqrt{x_f} \cdot \frac{1}{C} = e^{x_f}$$

$$\Rightarrow \underbrace{\log \frac{\sqrt{x_f}}{C}}_{\substack{\text{small} \\ \text{number}}} + \underbrace{\log \left(\langle \sigma v \rangle m_x \, M_p \right)}_{\text{very large number}} = x_f$$

very large number:

$$\langle \sigma v \rangle \sim \frac{1}{m_x^2}$$

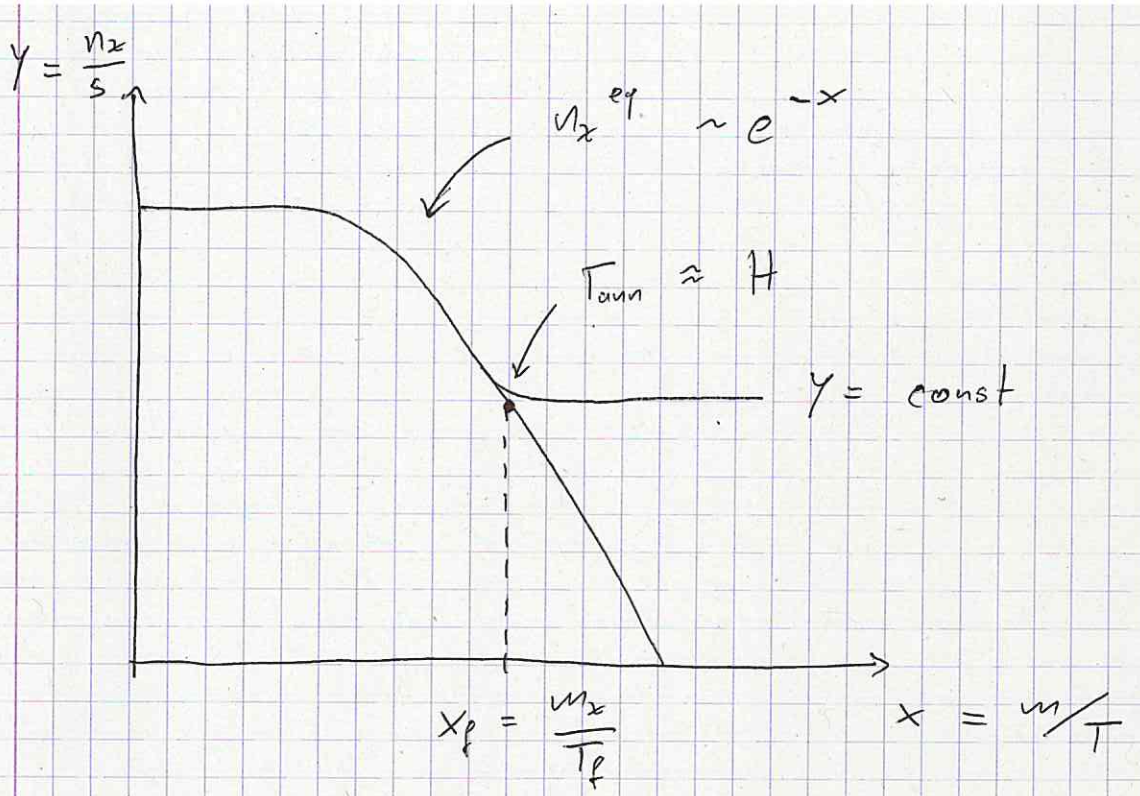
$$m_x \ll M_p$$

Example: $\quad m_x \approx 1 \text{ TeV}$

$$\Rightarrow x_f \approx \log \langle \sigma v \rangle m_x M_p$$

$$\approx \log 10^{16} \approx 30$$

$$T_f = \frac{m_x}{x_f} \ll m_x$$



From $\langle \sigma v \rangle n_\chi = H(T_f)$

$$\Rightarrow \quad n_\chi(T_f) \approx \frac{T_f^2}{\langle \sigma v \rangle M_P} 1.66 \sqrt{g_*}$$

After freeze-out DM particles stop interacting

$$\Rightarrow \quad n_\chi(T) \sim a(T)^{-3} \sim T^3$$

Convenient to define

$$Y_\chi = \frac{n_\chi(T)}{s(T)}$$

with

$$s(T) = \frac{2\pi^2}{45} g_* T^3$$

25 the entropy density

$$\Rightarrow \quad Y_x(\text{today}) = Y_x(T_f)$$

$$= \frac{3.8}{\sqrt{g_*}} \cdot \frac{1}{\langle \sigma v \rangle \, M_P \, T_f}$$

$$\Rightarrow \Omega_x h^2 = \frac{m_x \, Y_x(\text{today}) \, s(\text{today}) \, h^2}{\rho_c(\text{today})}$$

$$\approx \frac{10^9 \text{ GeV}^{-1} \quad x_f}{\sqrt{g_*} \, M_P \, \langle \sigma v \rangle}$$

Typically $g_* \approx 80$ and $x_f \approx 30$

$$\Rightarrow \Omega_x h^2 \approx \frac{3 \cdot 10^{-27} \text{ cm}^3/\text{s}}{\langle \sigma v \rangle}$$