* A brief introduction to cosmology

General relativity:

- Geometry of Universe depends on matter and energy content
- Geometry is characterised by <u>metric</u>, which defines distances between points in spacetime

Homogeneous + isotropic (flat) Universe:

$$ds^2 = -dt^2 + \underbrace{a(t)^2}_{} [dx^2 + dy^2 + dz^2]$$

scale factor: describes how space expands (or shrinks)

$a(t)$ captures evolution of Universe

Hubble expansion rate:

$$H(t) = \frac{1}{a} \frac{da}{dt}$$

Useful definition

$$h = \left(\frac{H(\text{today})}{100 \text{ km/s } \text{Mpc}^{-1}} \right) \approx 0.7$$

Einstein equation relates H to the energy density ρ:

$$H^2 = \frac{8\pi G \rho}{3} = \frac{8\pi}{3 M_p^2} \rho$$

(Friedmann equation)

$M_p \approx 1.2 \cdot 10^{19}$ GeV

What is ρ?

1) Universe filled with non-rel. particles

$$\rho_{nr}(t) = m \cdot n_{nr}(t) = \frac{m \cdot N}{V(t)} \sim a(t)^{-3}$$

2) Universe filled with rel. particles or radiation

$$\rho_r(t) = E_r(t) \cdot n_r(t) \sim a(t)^{-4}$$

redshift: $E_r(t) \sim \lambda_r(t)^{-1} \sim a(t)^{-1}$

Case 1) $\dfrac{\dot{a}^2}{a^2} \sim a^{-3}$

$$\Rightarrow \dot{a} \sim a^{-1/2}$$

$$\Rightarrow a^{3/2} \sim t$$

$$\Rightarrow a(t) \sim t^{2/3}$$

*2

Case 2) $\frac{\dot{a}^2}{a^2} \sim a^{-4}$

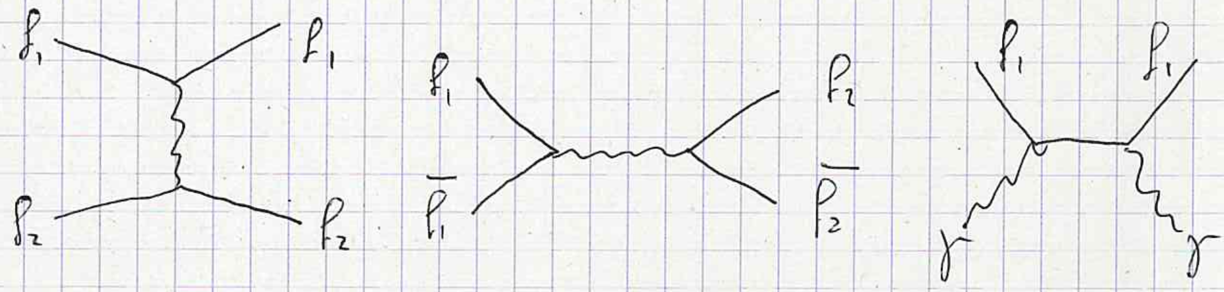
$$\Rightarrow a(t) \sim t^{1/2}$$

$$\Rightarrow H = \frac{1}{a} \frac{da}{dt} \sim t^{-1/2} t^{-1/2} \sim t^{-1}$$

Time not a convenient variable

More useful: temperature T

Assume relativistic particles interact



$\Rightarrow$ Thermal distribution

$$p(E) \sim e^{-E/T} \qquad (k_B = 1)$$

Stefan - Boltzmann law: $\rho_r \sim T^4$

$$\rho = \frac{\pi^2}{30} g_*(T)\, T^4 \sim a(t)^{-4}$$

degrees of freedom

$$\Rightarrow T \sim a(t)^{-1} \sim t^{-1/2}$$

(Universe cools down)

$$\Rightarrow H(T) = 1.66 \sqrt{g_*} \frac{T^2}{M_P}$$

*3

As T decreases:

- Relativistic particles lose energy
- Particles become non-relativistic
- Matter - radiation equality

↳ transition to case 1)

Useful notation:

Critical energy density of flat Universe

$$\rho_c = \frac{3 H^2 M_p^2}{8\pi}$$

Rather than n_f, define

$$\Omega_f = \frac{m_f \cdot n_f}{\rho_c}$$

For a flat Universe

$$\sum_f \Omega_f = 1$$

Note that

$$\Omega_f \, h^2$$

is independent of H

↳ easier to measure!

1.3 Big Bang Nucleosynthesis

↳ Earliest time probed by observations

Basic idea:

1) $T \gg 1\,\mathrm{MeV}$ $(t \ll 1\,s)$:

- p/n equilibrium: $p + e \longleftrightarrow n + \nu_e$
- no heavier elements

2) $T \lesssim 1\,\mathrm{MeV}$

- neutrinos decouple
- neutrons start decaying: $n \to p + e^- + \bar{\nu}_e$

3) $T \lesssim 100\,\mathrm{keV}$

- remaining neutrons form nuclei

$$p + n \longrightarrow D + \gamma$$
$$D + p/n \longrightarrow {}^3He/T$$
$${}^3He/T + D \longrightarrow {}^4He + p/n$$

- almost all neutrons end up in ${}^4He$:

$$\rho_{^4He} = 2 \cdot \rho_n \qquad (m_n \approx m_p)$$

$$\Rightarrow Y_p = \frac{\rho_{^4He}}{\rho_{^4He} + \rho_H} = \frac{2\rho_n}{\rho_n + \rho_p}$$

$$= \frac{2n_n}{n_n + n_p}$$

Observations:

In regions with low star formation (e.g. dwarf galaxies)

$$Y_p\,(T = 50\ \mathrm{keV}) \approx Y_p\,(\text{today})$$

$$= 0.249 \pm 0.009$$

$$D/H = (2.8 \pm 0.2) \cdot 10^{-5}$$

Predictions:

- Masses, ~~...~~ binding energies and lifetimes measured on Earth
- Only free parameter:

baryon-to-photon ratio $\eta = \frac{n_b}{n_\gamma}$

BBN: $\eta \approx 6 \cdot 10^{-10}$

Today: $n_\gamma^0 = \frac{2\zeta(3)}{\pi^2} T^3$ ← 2.73 K

$$\approx 400\ \mathrm{cm}^{-3}$$

$$\Rightarrow \rho_b = m_p \cdot \eta \cdot n_\gamma = 2.3 \cdot 10^{-7}\ \frac{\mathrm{GeV}}{\mathrm{cm}^3}$$

$$\Rightarrow \Omega_b h^2 = 0.0227 \pm 0.0006$$

(or $\Omega_b \approx 5\%$)

$\Rightarrow$ Baryons only small fraction of Universe's energy density

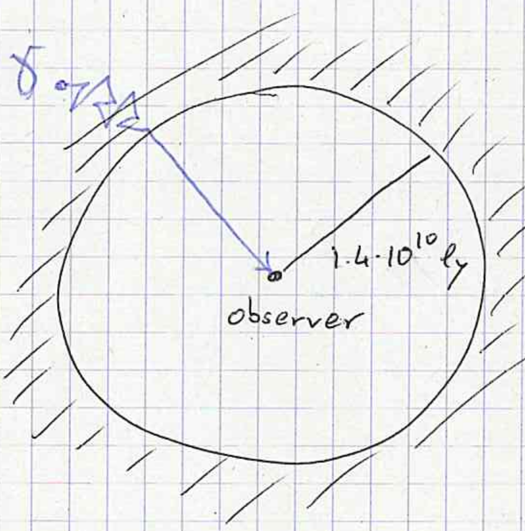
1.4 Cosmic Microwave Background

↳ More detailed discussion in "The perturbed Universe"

Basic idea: $t \sim 3.8 \cdot 10^5$ yr

- For $T > 1\,\text{eV}$, Universe is ionized
  → intransparent to radiation

- $T < 1\,\text{eV}$, neutral atoms form (recombination)
  → photons propagate freely



Universe ionized

→ cosmic background radiation

→ almost exactly black-body spectrum with $T \approx 2.7\,K$

Tiny fluctuations across the sky

$$\delta T / T \sim 10^{-5}$$

Write $$\frac{\delta T}{T}(\vartheta, \phi) = \sum_{\ell} \sum_{m} a_{\ell m} Y_{\ell m}(\vartheta, \phi)$$

spherical harmonics

and define

$$C_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$$

(temperature correlations)

=> Lots of structure! Origin?

1.4.1 Acoustic oscillations

Consider local overdensity

→ potential well

(1) Baryons fall into well

=> density ↑

=> temperature ↑

=> photon pressure ↑

(2) Photon pressure stops infall

=> turn-around

(3) Baryons get pushed away

=> density ↓

=> temperature ↓

=> photon pressure ↓

(4) Gravity wins over photon pressure

=> "acoustic" oscillations

("sound" speed $c_s \approx c/\sqrt{3}$)

Sound horizon $r_s = c_s \cdot t_{CMB}$

$\sim 10^5$ lyr

If size of well $\approx r_s$

recombination happens at (2).

=> Maximal temperature fluctuations

=> Peak

If size of well $\approx r_s/2$

recombination happens at (4)

=> Maximum rarefication

=> 2^{nd} peak

Etc.

What can we learn?

More baryons => stronger compression

=> enhanced odd peaks

=> Measure Ω_b

What creates potential wells?

Dark matter!

↳ ~~No photon pressure~~

↳ ~~No oscillations~~

1.4.2 Matter power spectrum

CMB is sensitive to the assumed density fluctuations

$$\delta(\vec{x}) = \frac{\delta\rho(\vec{x})}{\rho}$$

Calculate correlation function

$$\delta(\vec{k}) = \int d^3x \; e^{-i\vec{k}\vec{x}} \; \delta(\vec{x})$$

$$\langle \delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3 \; \delta(\vec{k} - \vec{k}') \; P(k)$$

matter power spectrum

Idea: Fluctuations generated by inflation (right after Big Bang)

$\Rightarrow$ Fluctuations at all scales

$\Rightarrow$ Power law (exponent model-dependent)

$\Rightarrow$ Grow under influence of gravity

Perturbations in DM grow faster than in baryons

↳ No photon pressure

↳ No oscillations

$\Rightarrow$ Measure Ω_{DM}

$\Rightarrow$ CMB measures both Ω_b and Ω_{DM}

$$\Omega_b h^2 = 0.02205 \pm 0.0003$$
$$\Omega_{DM} h^2 = 0.1199 \pm 0.003$$

(or $\Omega_b \approx 5\%$, $\Omega_{DM} \approx 26\%$)

$\Rightarrow$ consistent with BBN

$\Rightarrow$ 5 times more DM than baryons!

CMB not only sensitive to amount but also to properties of DM

Idea: Relativistic particles escape from overdensities (free-streaming)

$\Rightarrow$ Fluctuations are washed out

Free-streaming length $\lambda_{fs} \sim \frac{1}{m_{DM}}$

Only structures with $k < \frac{1}{\lambda_{fs}}$ survive

$\Rightarrow$ Lower bound on DM mass

$\Rightarrow$ DM cannot be too hot!