6. Sterile neutrinos

6.1 Motivation

Weak interactions violate parity

↳ different charges for left- and right-handed fermions

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

are $SU(2)_L$ doublets

d_R, u_R, e_R are $SU(2)_L$ singlets

Mass terms couple left- and right-handed fermions:

$$m_u \bar{u} u = m_u (\bar{u}_R u_L + \bar{u}_L u_R)$$

=> $SU(2)_L$ gauge symmetry forbids mass terms

=> fermions are massless before EW symmetry breaking

=> mass generation via Higgs mechanism

$$\mathcal{L} \supset y_d \bar{Q}_L d_R H \qquad \text{etc.}$$

SM does not contain ν_R

$\Rightarrow$ neutrinos remain massless

↳ Inconsistent with experiments!

Neutrinos oscillate, e.g.

$$\nu_e \rightarrow \nu_\mu, \nu_\tau$$

for solar neutrinos

$\Rightarrow$ requires $\delta m^2 \neq 0$

$\Rightarrow$ requires $m_\nu \neq 0$

Simplest solution: include ν_R

$$\Rightarrow \quad \mathcal{L} \supset y_\nu \bar{\ell}_L \nu_R \tilde{H} + h.c.$$

↑ $i\sigma_2 H^*$

$$\xrightarrow{EWSB} m_D \bar{\nu}\nu \quad \text{(Dirac mass term)}$$

with $m_D = \frac{1}{\sqrt{2}} y_\nu v$

↑ EW ver

$$\nu = \begin{pmatrix} \nu_R \\ \nu_R \end{pmatrix}$$

Remaining puzzle: Why is m_D so small?

Require $y_\nu \lesssim 10^{-12}$

↳ fine-tuning

Key observation: neutrinos have no colour or electric charge

$\Rightarrow$ ν_R is a SM singlet

$\Rightarrow$ Possible to write down Majorana mass term

$$\mathcal{L} \supset \frac{1}{2} m_M \overline{\nu_R} \nu_R^{\,c} + \text{h.c.}$$

where

$$\nu_R^{\,c} = \underset{\substack{\uparrow \\ \text{charge conjugation matrix}}}{C} \, \overline{\nu_R}^{\,T}$$

While consistent with gauge symmetry, this term violates lepton number by two units

Prediction: Neutrinoless double-beta decay

↳ so far unobserved

If both $m_D \neq 0$ and $m_M \neq 0$ we can write total mass term as

$$\mathcal{L} \supset \frac{1}{2} \overline{\nu^c} \mathcal{M} \nu + \text{h.c.}$$

with $\nu = \begin{pmatrix} \nu_L \\ \nu_R^{\,c} \end{pmatrix}$ and $\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}$

For $m_M \gg m_D$ the eigenvalues of $\mathcal{M}$ are

$$m_1 = \frac{m_D^2}{m_M} \quad \text{and} \quad m_2 = m_M$$

The corresponding mass eigenstates are

$$\nu \approx \nu_L - \frac{m_D}{m_M} \nu_R^c \Rightarrow \text{very light, mostly } \nu_L$$

and

$$N \approx \frac{m_D}{m_M} \nu_L + \nu_R^c \Rightarrow \text{heavier, mostly } \nu_R^c$$

⇒ Explanation for small mass of active neutrinos

Prediction: sterile neutrino with small active component

$$\vartheta \approx \frac{m_D}{m_M}$$

→ See-saw mechanism

Realistic models: Need 3 sterile neutrinos

Simplest example: νMSM

$$m_M \sim \text{GeV}, \quad m_D \sim \text{keV}$$

$$\Rightarrow m_\nu \sim \text{meV}, \quad m_N \sim \text{GeV}$$

↳ explains neutrino oscillations

Note: To agree with experiments only 2 neutrinos must have $m_\nu \sim \text{meV}$

3^{rd} neutrino can have $m_\nu \ll \text{meV}$

→ Corresponding sterile neutrino can have $m_N \sim \text{keV}$

⇒ interesting DM candidate

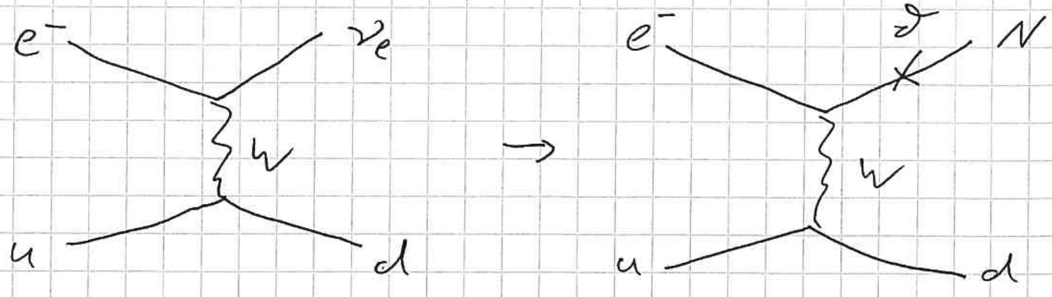
6.2 Sterile neutrino cosmology

2 parameters:
- Mass $m_N \sim \text{keV}$
- Active - sterile mixing angle $\vartheta \ll 1$

For small enough ϑ, sterile neutrinos never enter into thermal equilibrium with SM

→ Assume negligible initial abundance of sterile neutrinos

Any process producing neutrinos can also produce sterile neutrinos



$\langle \sigma v \rangle \sim G_F^2 \vartheta^2 T^2$ (for $T < m_W$)

$\Gamma_{prod} \sim n_{eq} \langle \sigma v \rangle \sim G_F^2 \vartheta^2 T^5$

$\Rightarrow$ Continuous production of sterile neutrinos through "energy leakage"

↳ freeze-in mechanism (see problem 18,

$$\frac{dY_N}{dT} = - \frac{\langle \sigma v \rangle n_{SM}^2}{H T s}$$

Detailed calculation more complicated!

↳ ϑ depends on temperature and density of thermal bath

$$\vartheta \longrightarrow \vartheta_M \approx \frac{\vartheta}{1 + (T/T_0)^6}$$

$T_0 \sim 100$ MeV

$\Rightarrow$ Sterile neutrinos produced dominantly around $T \sim T_0$

$$\Omega_N h^2 \sim 0.1 \left(\frac{\sin^2 \vartheta}{3 \cdot 10^{-9}}\right)\left(\frac{m_s}{3\ \mathrm{keV}}\right)^{1.8}$$

(Dodelson-Widrow mechanism)

Additional production modes possible, e.g. resonant oscillations or production via SuperWIMP mechanism (see § 4.1)

$\Rightarrow$ Even smaller ϑ can be interesting!

6.3 Constraints from structure formation

Since sterile neutrinos are fermions, their phase-space density is limited

$\Rightarrow$ Tremaine-Gunn bound

$$m_\nu \gtrsim 500\ \mathrm{eV} \qquad (\text{see } \S 1.1)$$

Moreover, keV DM particles are warm DM

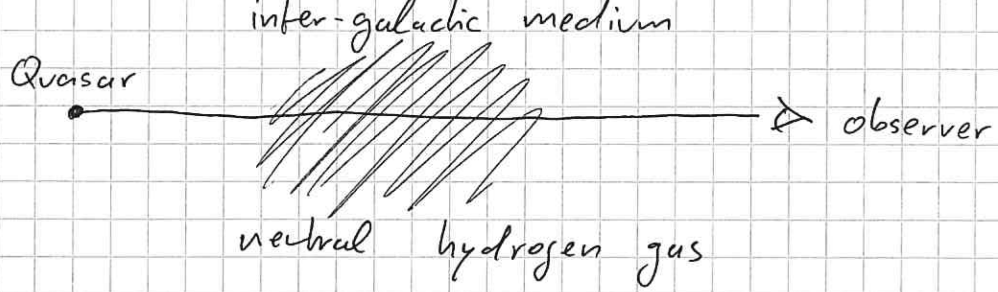
$\Rightarrow$ sizeable free-streaming length λ_{fs}:

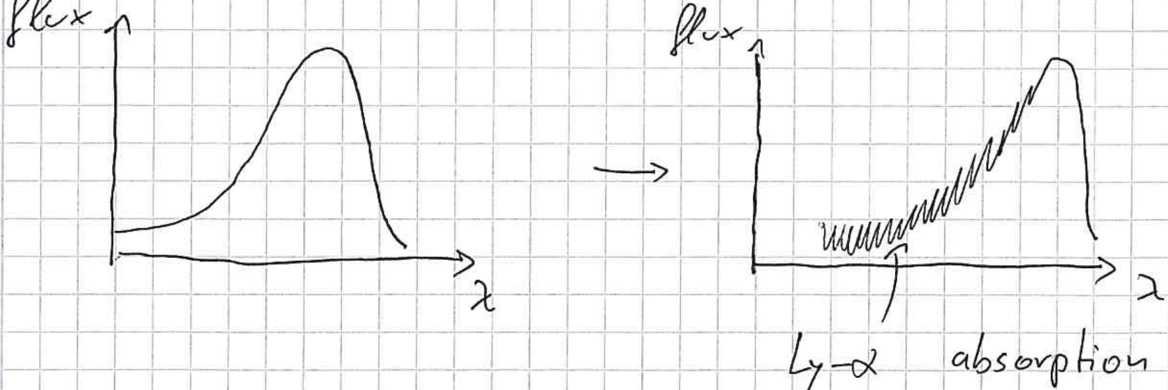
$$\lambda_{fs} \sim \mathrm{Mpc} \quad \text{for} \quad m_\nu \sim \mathrm{keV}$$

↳ Potential conflict with structure formation (e.g. halo mass function, see §1.5)

Sensitive probe: Lyman-α forest

Basic idea:





Position of Ly-α absorption redshifts

→ measure absorption at diffent frequencies

→ infer distribution (and temperature) of matter along the way

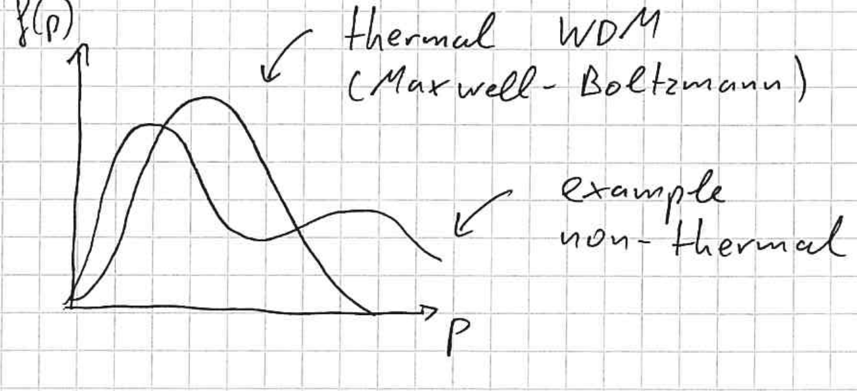
⇒ measure matter power spectrum

With increasing λ_{fs}, amount of small-scale structure is suppressed

⇒ fewer absorption features

⇒ constraints on warm DM

Problem: For non-thermal DM it is not straight-forward to define a DM temperature

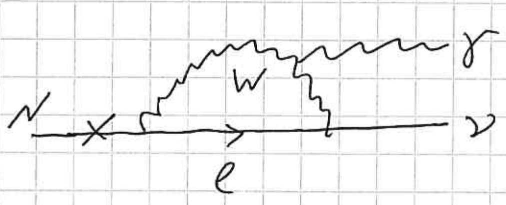


⇒ Need to define equivalent thermal mass

⇒ Ly-α constraints depend on production mechanism

6.4 Astrophysical constraints

Sterile neutrinos are not stable!



$$\tau_N \sim 10^{30}\,\mathrm{s} \left(\frac{\sin^2 2\vartheta}{10^{-8}}\right)^{-1} \left(\frac{m_N}{1\,\mathrm{keV}}\right)^{-5}$$

↳ Much larger than age of Universe

↳ Potentially observable:

$$E_\gamma \approx \frac{m_N}{2} \quad \Rightarrow \quad \text{mono-energetic x-ray line}$$

Search strategy similar to WIMP indirect detection, but instead of

$$\underset{\text{rate}\nearrow}{R} \sim \int \rho_{DM}^2 \, dl \qquad (\text{J factor})$$

for annihilations, one finds

$$R \sim \int \rho_{DM} \, dl = \underset{\substack{\uparrow \\ \text{column density}}}{\Sigma_{DM}}$$

for decays.

⇒ Rather than small halos with high density (e.g. dwarf spheroidals), more interesting to look at systems with large total mass

Promising target: Perseus cluster

$d \sim 70\ \mathrm{Mpc}$

$M_{halo} \sim 10^{15}\ M_\odot$

=> Strong constraints on sterile neutrinos

↳ Dodelson-Widrow production ruled out

=> But also: hint for a signal at $E_\gamma \sim 3.5\ \mathrm{keV}$

Best-fit interpretation:

$m_N \sim 7\ \mathrm{keV}, \quad \sin^2 2\vartheta \sim 10^{-10} - 10^{-9}$

→ Possible tension with Ly-α constraints and with x-ray constraints from the ~~GC~~ Galactic Centre

↳ New satellite missions planned to clarify picture