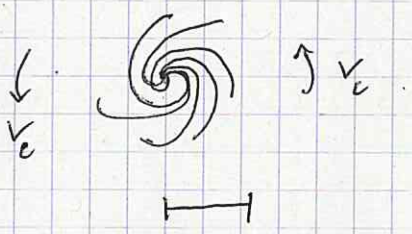
1. Evidence for Dark Matter

1.1 Galactic rotation curves

A typical spiral galaxy:



$R \sim 10\ \text{kpc}\ (\approx 3 \cdot 10^4\ ly)$

Expected rotational velocity:

$$F_N = m \cdot a \Rightarrow \frac{G \cdot M(r)}{r^2} = \frac{v_c^2}{r}$$

where $M(r) = 4\pi \int r^2 \rho(r)\, dr$
(total mass within radius r)

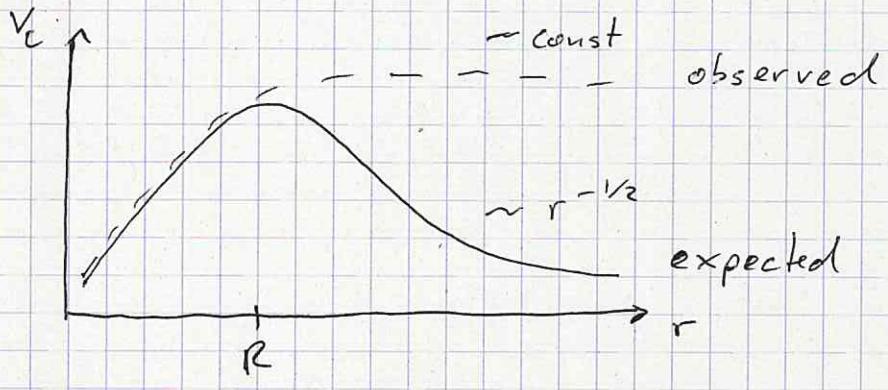
$$\Rightarrow \quad v_c(r) = \sqrt{\frac{G\, M(r)}{r}}$$

Can be measured using Doppler shift of star light.

Idea: Extend measurements beyond visible disk ($r > R$) using neutral hydrogen gas (21 cm line)

Expectation: For $r > R$ $M(r) \approx M(R)$

$$\Rightarrow v_c(r) = \sqrt{\frac{G \cdot M(R)}{r}} \sim r^{-1/2}$$



Observation: $v_c(r) \rightarrow$ const for $r > R$

$\Rightarrow \quad M(r) \sim r$

$\Rightarrow \quad \rho(r) \sim \frac{1}{r^2}$ slide

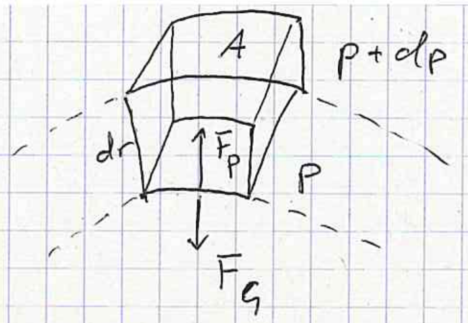
What kind of system has such a density profile?

Consider an isothermal gas.

Equation of state: $p = \frac{k_B T}{m} \rho$

If the system is self-gravitating, gradient in pressure = gradient in gravitational acceleration

$\rightarrow$ Hydrostatic equilibrium



$$F_p = (p + dp) \cdot A - p \cdot A = F_G$$

$$= - \frac{dm \cdot M(r) \cdot G}{r^2}$$

$$dm = \rho \cdot A \cdot dr$$

$$\Rightarrow \quad \frac{dp}{dr} = - \frac{\rho \cdot M(r) \cdot G}{r^2}$$

$$\Rightarrow \quad \frac{r^2}{\rho} \frac{k_B T}{G m} \frac{d\rho}{dr} = M(r)$$

$$\Rightarrow \quad \frac{k_B T}{G m} \frac{d}{dr} \left(\frac{r^2}{\rho} \frac{d\rho}{dr} \right) = 4\pi r^2 \rho(r)$$

Ansatz: $\rho(r) = C \cdot r^{-b}$

$$\Rightarrow \quad - \frac{b \, k_B T}{G m} = 4\pi C \, r^{2-b}$$

$$\Rightarrow \quad b = 2 \; ; \quad C = \frac{k_B T}{2\pi G m}$$

$$\Rightarrow \quad \rho(r) = \frac{k_B T}{2\pi G m} \frac{1}{r^2}$$

$\Rightarrow$ Flat rotation curves !

$\Rightarrow$ Stellar disk embedded in isothermal sphere of invisible particles

$\Rightarrow$ dark matter halo

Typical numbers:

Galaxy: $R \sim 100\ \text{kpc}$

$M(R) \sim 10^{12}\ M_\odot$

Dwarf galaxy: $R \sim 1\ \text{kpc}$

$M(R) \sim 10^{7}\ M_\odot$

What does this tell us about DM?

$\hookrightarrow$ Assume DM is elementary particle

De Broglie wave length must be smaller than R:

$$\lambda_{DB} = \frac{h}{m_{DM}\, v} = \frac{h}{m_{DM}} \sqrt{\frac{R}{M(R)\, G}} < R$$

$$\Rightarrow m_{DM} > \frac{h}{\sqrt{R\, M(R)\, G}}$$

$$\simeq \begin{cases} 5 \cdot 10^{-25}\ \text{eV} & \text{(galaxy)} \\ 10^{-21}\ \text{eV} & \text{(dwarf)} \end{cases}$$

↳ Assume DM is a fermion

Pauli exclusion: $f(\vec{x}, \vec{p}) \leq h^{-3}$

$$\Rightarrow M(R) = m_{DM} \int f(\vec{x}, \vec{p})\, d^3x\, d^3p$$

$$\lesssim m_{DM}\, R^3 \,(m_{DM}\, v_{DM})^3\, h^{-3}$$

$$= m_{DM}^4 \cdot R^3 \left(\frac{G\, M(R)}{R} \right)^{3/2} h^{-3}$$

$\Rightarrow$

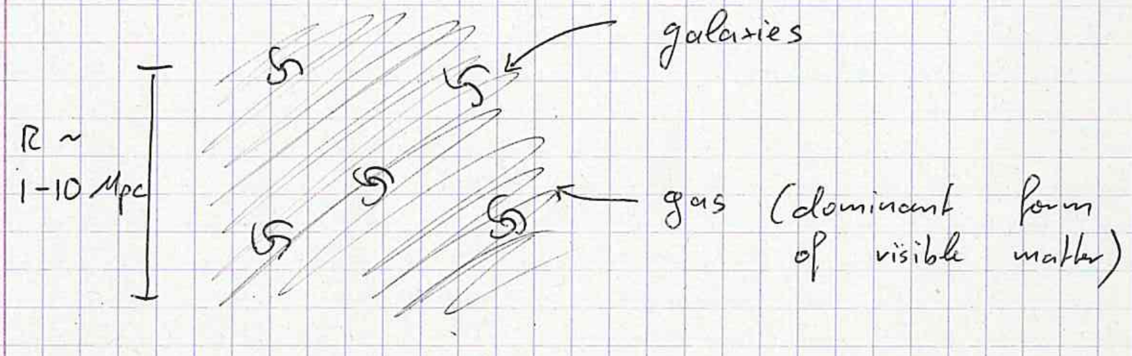
$$m_{DM} \gtrsim \left(G^3\, M(R)\, R^3\, h^{-6} \right)^{-1/8}$$

$$= \begin{cases} 20\ \text{eV} & \text{(galaxy)} \\ 500\ \text{eV} & \text{(dwarf)} \end{cases}$$

$\Rightarrow$ Neutrinos cannot be (all of) dark matter, because $m_\nu \lesssim 2\,\text{eV}$

↳ Tremaine-Gunn bound

1.2 Galaxy clusters



1.2.1 Virial theorem:

$$T = \frac{1}{2} |U|$$

$$T = \frac{1}{2} M(R) \langle v^2 \rangle$$

Can only measure $v_{\parallel}$ (redshift)

Isotropy: $\langle v^2 \rangle = 3 \langle v_{\parallel}^2 \rangle$

$$\Rightarrow T = \frac{3}{2} M(R) \langle v_{\parallel}^2 \rangle$$

$$|U| = \frac{G \, M(R)^2}{R}$$

$$\Rightarrow M(R) = \frac{3R \langle v_{\parallel}^2 \rangle}{G}$$

Coma cluster: $M_{vis} \sim 2 \cdot 10^{14} M_{\odot}$

$$\sqrt{\langle v_{\parallel}^2 \rangle} \sim 1000 \text{ km/s}$$

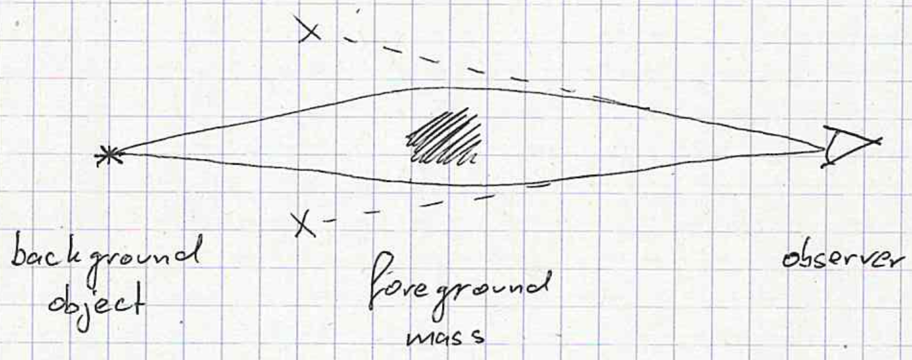
$$R \sim 2.2 \text{ Mpc}$$

$$\Rightarrow M(R) \sim 2 \cdot 10^{15} M_{\odot}$$

$\Rightarrow$ First evidence for DM (Zwicky, 1933)

1.2.2 Gravitational lensing

Basic idea: Gravity bends light (like a lense)



=> Look for distortion of background images (weak lensing) or multiple copies of the same object (strong lensing)

=> Measure gravitational potential of galaxy cluster in the foreground

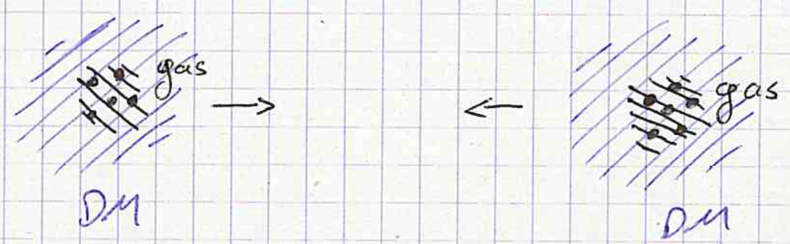
Observation: Potential deeper than expected from M_{vis}

1.2.3 The Bullet Cluster

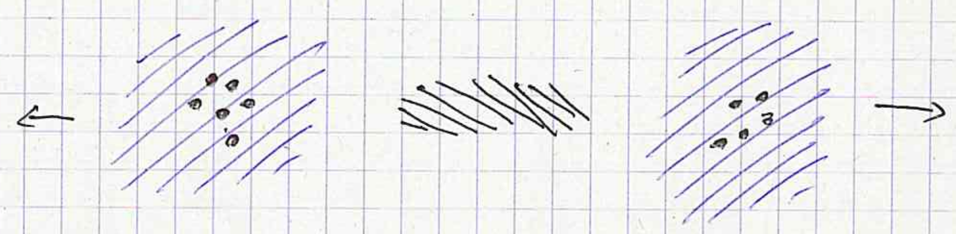
↳ Most famous evidence for DM

↳ Collision of two galaxy clusters

Before collision



After collision



=> Gas has electromagnetic interactions

=> Gas slows down

=> Gas and galaxies no longer coincident

Lensing observations: Peak of total mass remains coincident with galaxies

=> DM

Particle physics implications

- DM particles behave differently from baryons and electrons

  ↳ No dissipation
  ↳ No elastic scattering
  } DM is collisionless

Let's be more precise

σ: DM self-interaction cross section

N: number of unscattered particles from cluster A

$$\Rightarrow \dot{N} = -\sigma \cdot \underbrace{\frac{\rho}{m_{DM}} \cdot v}_{\text{flux of DM particles from cluster B}} \cdot N$$

$$\Rightarrow \frac{dN}{N} = -\sigma \frac{\rho}{m_{DM}} dx$$

$$\Rightarrow N = N_0 \exp\left(-\frac{\sigma}{m_{DM}} \int \rho \, dx\right)$$

Bullet Cluster: $\int \rho \, dx \approx 0.3 \frac{g}{cm^2}$

$$\frac{\Delta N}{N_0} = 1 - \frac{N}{N_0} \lesssim 0.3$$

$$\Rightarrow \frac{\sigma}{m_{DM}} \lesssim 1.2 \frac{cm^2}{g} \quad \left(= 2.4 \frac{barn}{GeV} \right)$$

For comparison:

Neutron-neutron scattering: $\frac{\sigma_n}{m_n} \sim 10 \frac{barn}{GeV}$

$\Rightarrow$ Bound on DM self-interactions is rather weak!