1. Evidence for Dark Matter

1.1 Galactoic rotation curves

A typical spiral galaxy:

\[ R \sim 10 \text{kpc} \quad (= 3 \times 10^4 \text{ Ly}) \]

Expected rotational velocity:

\[ F_r = m \cdot a \quad \Rightarrow \quad G \cdot M(r) = \frac{v^2}{r} \]

where \( M(r) = 4\pi \int r^2 \rho(r) \, dr \) (total mass within radius \( r \))

\[ \Rightarrow \quad v_c(r) = \sqrt{\frac{G \cdot M(r)}{r}} \]

Can be measured using Doppler shift of star light.

Idea: Extend measurements beyond visible disk using neutral hydrogen gas (21 cm line)
Expectation: For $r > R$, $M(r) \approx M(R)$

$$\Rightarrow v_c(r) = \sqrt{\frac{G \cdot M(R)}{r}} \sim r^{-1/2}$$

Observation: $v_c(r) \rightarrow \text{const}$ for $r > R$

$$\Rightarrow M(r) \sim r$$

$$\Rightarrow S(r) \sim \frac{1}{r^2} \quad \text{(isothermal sphere)}$$

Explanation: Visible disk embedded in 3D halo of dark matter

1.2 Can we trust Newton's law?

Assume that for very weak acceleration ($a \ll a_o$)

$$F_n = m \cdot \frac{a}{a_o} \quad \text{(modified Newtonian dynamics)}$$
\[ \frac{G \cdot M}{r^2} = \left( \frac{v^2}{r^2} \right) \]

\[ V_o(r) = (c_o \cdot G \cdot M)^{1/4} \rightarrow \text{const} \]

Explains galactic rotation curves!

We need more evidence for dark matter.

1.3 Gravitational lensing

Basic idea: Gravity bends light (like a lens)

\[ \text{Background object} \rightarrow \text{Observer} \]

\[ \text{Foreground mass} \]

\[ \Rightarrow \text{Look for distortion of background objects to measure gravitational potential (e.g. of galaxy clusters)} \]

Observation: Potential deeper than expected from stars and gas.

In some systems, the region of highest luminosity is different from the region of highest density!
14. How much dark matter is there?

Need observations of the early Universe

\( \rightarrow \) Cosmic Microwave Background (CMB)

Basic idea: - At temperatures \( T > 1 \text{eV} \)
the early Universe is ionized

\( \rightarrow \) Intransparent to radiation

- Once neutral atoms form (recombination), photons propagate freely

The CMB is almost exactly a black-body spectrum with \( T \approx 2.7 \text{K} \).
Tiny fluctuations across the sky:

\[ \frac{\delta T}{T} \sim 10^{-5} \]

Write

\[ \frac{\delta T}{T}(\theta, \phi) = \sum_c \sum_m a_{cm} Y_m^c(\theta, \phi) \]

spherical harmonic.

and define

\[ C_\ell = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \]

(temperature correlation)

Observation: Peak at \( \ell \sim 250 \) (\( \theta \sim 1^\circ \))

Reason: Oscillations in the plasma from competition between gravitational attraction and photon pressure.

Both dark matter (DM) and ordinary matter (baryons) contribute to gravitational potential.

Only ordinary matter feels photon pressure.

\[ R_n = R_{\text{on}} + R_B \]

\[ \frac{R_{\text{on}}}{R_n} \approx 0.85 \]

\[ \frac{R_{\text{on}}}{R_B} \approx 5.5 \]
2. Explanations for Dark Matter

2.1 Properties of Dark Matter (DM)

By definition:
- DM is stable
  \( \Omega_m \gg \Omega_{\text{b}} \) (universe)
- DM is electrically neutral
  \( \rho_m < \epsilon \)

- From CMB:
  \( \rho_b \ll \rho_m \)

Same conclusion obtained from
Big Bang Nucleosynthesis (abundance of elements)

\( \Rightarrow \) DM is non-baryonic

- From galactic rotation curves:
  DM does not collapse into a disk. Halos are 3D.

\( \Rightarrow \) DM does not dissipate energy
  (it is collisionless)
From structure formation:

DM moves sufficiently slowly that it can become gravitationally bound.

\[ \Rightarrow \text{DM is cold} \]

2.2 Conventional explanations

- Could DM just be ordinary matter that doesn't emit light?
  - Diffuse neutral gas
  - Planets
  - Faint stars

No, because DM must be non-baryonic.

- Could DM just be ordinary neutrinos?

No, because \( m_\nu \leq 0.2 \text{ eV} \) so would be relativistic during recombination (but DM)
Moreover, Pauli exclusion means that there would be an upper bound on halo masses:

\[ M_{\text{halo}} = m_\nu \int \int \rho(x, p) \, d^3x \, d^3p \leq \]

\[ \leq m_\nu \, R_{\text{halo}}^3 \left( m_\nu \, V_{\text{DM}} \right)^3 \]

with \( V_{\text{DM}} \sim \frac{\sqrt{G \cdot M_{\text{halo}}}}{R_{\text{halo}}} \)

\[ \Rightarrow m_\nu \geq \left( c^3 \cdot M_{\text{halo}} \cdot R_{\text{halo}}^3 \right)^{-1/8} \]

For \( M_{\text{halo}} \sim 10^{12} M_\odot \)

\( R_{\text{halo}} \sim 10 \text{ kpc} \)

\[ \Rightarrow m_\nu \gtrsim 10 \text{ eV} \]

(Stronger bound from dwarf galaxies: \( m_\nu \gtrsim 0.7 \text{ keV} \))
Could DM be black holes?

CMB: Black holes must have formed (long) before recombination.

Primordial Black Holes (PBH)

Can look for them via gravitational lensing:

\[ \text{Micro-lensing causes variation in brightness.} \]

Absence of such variations:

PBH cannot be all of DM for

\[ 10^{-3} M_\odot \leq M_{\text{PBH}} \leq 10 M_\odot \]

Larger masses may be viable (gravitational wave signals?)

- Most exciting possibility:
  New, yet undiscovered particle!
2.3 Structure formation with particle dark matter ($N$-body simulations)

- Assume simple initial conditions
- Evolve under influence of gravity
- Determine size, shape, and number of dark matter halos
- Challenge: Simulate star formation, supernova explosions, ...

Output (example):

$$n(M_h) \propto \frac{\text{Number of DM halos}}{\text{with mass } M > M_h}$$

$\Rightarrow$ Compare to observations

$\Rightarrow$ Upper bound on the mass of warm dark matter

N.b.: Individual particles macroscopic ($M_{\text{part}} \gg M_{\text{cm}}$)

$\Rightarrow$ Particle physics captured by initial conditions
N-body simulations make new prediction.

E.g. profile of DM halos is not

\[ g(r) \sim \frac{1}{r^2} \]

but

\[ g(r) \sim \frac{1}{r \left( r_s + r \right)^2} \]

\[ r_s : \text{scale radius} \]

\[ \log g \]

\[ \log r \]

\[ r_s \]

\[ (\text{cusp}) \]

Compare to observations:

- Good fit for large \( r \)
- In central region possible discrepancy:
  
  \[ v_c(r) \text{ smaller than predicted} \]

  \[ \Rightarrow g(r) \rightarrow \text{const} ? \]

  Cusp - core problem ?

  \[ \Leftrightarrow \text{Baryonic effects not included ?} \]

  \[ \Leftrightarrow \text{Dark matter self-interactions ?} \]
3. Dark matter in the early Universe

3.1 A brief introduction to cosmology

General relativity:

Geometry of Universe depends on the matter and energy content of points in space-time.

Homogeneous + isotropic Universe:

\[ ds^2 = -dt^2 + a(t)^2 \left( dx^2 + dy^2 + dz^2 \right) \]

The geometry is characterized by a metric, which defines distance between points in space-time.

Scale factor: describes how space expands (or shrinks)

\( a(t) \) captures evolution of Universe

Hubble expansion rate:

\[ H(t) = \frac{1}{a} \frac{da}{dt} \]
From Einstein equation derive relation between $H$ and $\mathcal{S}$ (energy density):

$$H^2 = \frac{8\pi G \mathcal{S}}{3} = \frac{8\pi}{3 M_p^2} \mathcal{S}$$

(Friedmann equation) $\mathcal{M}_p \sim 1.2 \times 10^{19}$ GeV

What is $\mathcal{S}^2$?

1) Universe filled with non-rel. particles:

$$\mathcal{S}_{\text{nr}}(t) = m \cdot n_{\text{nr}}(t) = m \cdot \frac{N}{V(t)} \sim a(t)^{-3}$$

2) Universe filled with rel. particles (or radiation)

$$\mathcal{S}_{\text{r}}(t) = E_r(t) \cdot n_{\text{r}}(t) \sim a(t)^{-4}$$

Redshift: $E_r(t) \sim a(t)^{-1}$

Case 1) $\frac{\dot{a}^2}{a^2} \sim a^{-3}$

$$\Rightarrow \dot{a} \sim a^{-1/2}$$

$$\Rightarrow a^{3/2} \sim t$$

$$\Rightarrow a(t) \sim t^{2/3}$$

Case 2) $\frac{\dot{a}^2}{a^2} \sim a^{-4}$ $\Rightarrow a(t) \sim t^{1/2}$
Let's focus on case $z$:

$$H = \frac{1}{a} \frac{da}{dt} \sim t^{-1/2} - t^{-1/2} \sim t^{-1}$$

Time not a convenient variable

More useful: temperature $T$

Assume the relativistic particles interact

$$\Rightarrow \text{Thermal distribution}$$

$$\rho(E) \sim e^{-E/T} \quad (k_B = 1)$$

Stefan-Boltzmann Law: $\mathcal{S}_r \sim T^{-4}$

$$\mathcal{S} = \frac{11^2}{30} \mathcal{G}(T) T^4$$

$^1$ degree of freedom

$$\Rightarrow T \sim a(t)^{-1} \sim t^{-1/2}$$

(Universe cools down)

$$\Rightarrow H = 1.66 \sqrt{\mathcal{G}} T^2 / M_p$$
As $T$ decreases:

- Relativistic particles lose energy
- Particles become non-relativistic
- Matter-radiation equality

$\Rightarrow$ Transition to case 1

Some useful notation:

Rather than $\eta_f$, one often quotes

$$\Omega_f = \frac{\eta_f \cdot \eta_f}{s_c}$$

where

$$s_c = 3H^2M_p^2/(8\pi)$$

For a flat Universe

$$\sum \Omega_f = 1$$

One also finds

$$\Omega_f h^2 = \Omega_f \left(\frac{H(\text{today})}{100 \text{ km} \text{ s}^{-1}}\right)^2 \approx 0.7$$
3.2 Thermal Freeze-out

Consider a Universe filled with relativistic particles $f$ with temperature $T$. Consider a heavy DM species $x$ with $m_x \gg T$.

**Annihilation process:**

$$\text{Rate: } \Gamma_{\text{ann}} = \langle \sigma_{\text{ann}} v \rangle \, n_f$$

**Production process:**

$$\Gamma_{\text{prod}} = \langle \sigma_{\text{prod}} v \rangle \, n_f$$

$\langle \rangle$: Thermal average

In thermal equilibrium

$$\Gamma_{\text{ann}} = \Gamma_{\text{prod}}$$
Since \( m_x < T < m_x \), annihilation is always possible.

But production is only possible if \( E_f \approx m_x \)

\[ \Rightarrow \langle \sigma v \rangle \sim e^{-m_x/T} \]

\[ \Rightarrow n_x \text{ must be exponentially suppressed:} \]

\[ n_x \sim e^{-m_x/T} \]

\[ \text{If thermal equilibrium continues} \]

\[ n_x \to 0 \text{ as } T \to 0 \]

To obtain sizeable abundance of DM, it must decouple from TE.

This happens if

\[ \Gamma_{\text{ann}} < H \]

\[ \text{interaction rate} \sim T^2 \]

\[ \sim e^{-m_x/T} \]

\[ \text{The exact expression is} \]

\[ n_x = g_x \left( \frac{m_x T}{2 \pi} \right)^{3/2} e^{-m_x/T} \]
The temperature for which

\[ \Gamma_{\text{ann}} = H \]

is called freeze-out temperature \( T_f \):

\[ \langle \sigma v \rangle = g_\chi \left( \frac{m_x T_f^2}{2 \pi} \right)^{3/2} e^{-m_x/T_f} \approx 1.66 \sqrt{g_*} \frac{T_f}{M_P} \]

Define \( x_f = m_x / T_f \), drop constants:

\[ \langle \sigma v \rangle \approx m_x^3 x_f^{-3/2} e^{-x_f} \sim \frac{m_x^2}{x_f^2} \frac{T_f^2}{M_P} \]

\[ \Rightarrow \langle \sigma v \rangle \sim m_x M_P \sqrt{x_f} \sim e^{x_f} \]

\[ \Rightarrow \frac{1}{2} \log x_f + \log \left( \langle \sigma v \rangle m_x M_P \right) + \text{const} = x_f \]

Typically \( \langle \sigma v \rangle \sim \frac{1}{m_x} \) and \( m_x \ll M_P \)

\[ \Rightarrow \log \left( \langle \sigma v \rangle m_x M_P \right) \gg \frac{1}{2} \log x_f + \text{const} \]

\[ \Rightarrow x_f \approx \log \left( \langle \sigma v \rangle m_x M_P \right) \]

\[ T_f = \frac{m_x}{\log \left( \langle \sigma v \rangle m_x M_P \right)} \ll m_x \]

*Ehmkay*:

\[ \langle \sigma v \rangle m_x M_P \sim \frac{M_P}{m_x} \sim \frac{10^{19}}{10^8} \text{GeV} \]

\[ \Rightarrow x_f = \log 10^{16} \approx 30 \]
We conclude
\[ n_x(T_f) \sim \frac{T_f^2}{\langle \sigma v \rangle M_p} \]

After freeze-out, DM particles stop interacting

\[ \Rightarrow n_x(T) \sim a(T)^{-3} \sim T^{-3} \]

\[ \Rightarrow n_x(\text{today}) = \left( \frac{T_{\text{today}}}{T_f} \right)^3 n_x(T_f) \]

\[ \sim \frac{1}{\langle \sigma v \rangle M_p T_f} \]

\[ \Omega_x = \frac{m_x n_x(\text{today})}{\rho_c(\text{today})} \sim \frac{X_f}{\langle \sigma v \rangle M_p} \]

Restoring missing particles:

\[ \Omega_x h^2 \approx \frac{10^3 \text{ GeV}^{-1} X_f}{\langle \sigma v \rangle M_p} \frac{1}{g_x} \]

Typically \( g_x \approx 80 \) and \( X_f \approx 3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \)

\[ \Rightarrow \Omega_x h^2 \approx 3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \]

18
The WIMP miracle

Typical annihilation process:

For \( m_x \gg m_z, m_y \), dimensional analysis gives:

\[ \langle \sigma v \rangle \sim \frac{1}{m_x^2} \]

Constant of proportionality depends on coupling strength.

For weak interactions:

\[ \langle \sigma v \rangle \sim \frac{A x_z^2}{m_x^2} \quad (x_z \sim 10^{-2}) \]

Assume \( m_x \) is close to electroweak scale: \( m_x \sim v \sim 250 \text{ GeV} \)

\[ \Rightarrow \langle \sigma v \rangle \sim 10^{-26} \text{ cm}^3/\text{s} \]

\[ \Rightarrow \Omega_x h^2 \sim 0.1 \]

Observed value: \( \Omega_x h^2 = 0.1199 \)

Remarkable coincidence!
Why is it interesting to consider $m_X \sim 250 \text{ GeV}$?

May play a role in electroweak symmetry breaking!

Hierarchy problem:

Why is $m_h \ll M_p$?

New symmetry?

$\Rightarrow$ Expect new states at the electroweak scale!

Lightest new state can be stable!

Prominent example:

Supersymmetric neutralino

E.g. $\tilde{h}$

higgsino $\tilde{\chi}_1^0$ $\tilde{\chi}_2^0$

$\tilde{h}, \tilde{h}^2 \approx 0.1 \left( \frac{m_h}{1 \text{ TeV}} \right)^2$
3.3 Misalignment mechanism

Consider scalar field \( \phi(x, t) \) with potential \( V(\phi) = \frac{1}{2} m^2 \phi^2 \)

\[
L = T - V
= \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2
\]

Assume \( \phi \) is the same everywhere:

\( \phi(x, t) = \phi_0 \)

Equation of motion in static universe:

\[
\ddot{\phi} = -\frac{\partial V}{\partial \phi} = -m^2 \phi
\]

\( \Rightarrow \phi(x, t) = \phi_0 \cos(m \tau) \)

\( \Rightarrow \) Oscillation between large \( T \) and large \( V \)

In expanding universe, kinetic energy is dissipated (redshifted):

\[
\ddot{\phi} = -\frac{2V}{\partial \phi} - 3 \dot{H} \phi \quad (\text{Hubble friction})
\]

\( \Rightarrow \) Damped harmonic oscillator
For $\Delta H > m$: Over-damped

$\Rightarrow \phi = 0 \Rightarrow \phi = \phi_0$

For $\Delta H < m$: Oscillations possible

$\phi(t, x) = \phi_0(t) \cos(\omega t)$

with

$\Delta H \phi_0 \sim -2 \phi_0$

$\Rightarrow -\frac{3}{2} \frac{d\alpha}{\alpha} \sim \frac{d\phi_0}{\phi_0}$

$\Rightarrow \phi_0(t) \sim \alpha(t)^{-3/2}$

Oscillation with frequency $\omega$ and amplitude $\phi_0(t)$

Energy stored in field proportional to $\phi_0(t)^2 \propto \alpha^{-3}$

Just like non-relativistic matter!

Coherently oscillating field

$\equiv$ Collection of particles at rest
* Analogy to quantum mechanics:

Consider ground state \( |0\rangle \) of harmonic oscillator

\[
\hat{a} |0\rangle = 0
\]

\( \hat{A} \) Annihilation operator \( \hat{a} = \sqrt{\frac{\hbar \omega}{2}} \left( \hat{x} - \frac{i}{\hbar \omega} \hat{p} \right) \)

Construct coherent oscillation with amplitude \( x_0 \):

\[
|X_0\rangle = \hat{T}(x_0) |0\rangle
\]

\( \hat{T} \) translation operator \( [\hat{T}(x_0), \hat{x}] = x_0 \)

\[
\hat{a} |X_0\rangle = \hat{a} \hat{T}(x_0) |0\rangle = \hat{T}(x_0) \left( \hat{a} + \sqrt{\frac{\hbar \omega}{2}} x_0 \right) |0\rangle = \sqrt{\frac{\hbar \omega}{2}} x_0 |X_0\rangle
\]

\( \Rightarrow |X_0\rangle \) is eigenstate of \( \hat{a} \), but not of the number operator

\[
\hat{N} = \hat{a} + \hat{a}^\dagger
\]

Expectation value of \( \hat{N} \):

\[
\langle \hat{N} \rangle = \langle X_0 | \hat{N} | X_0 \rangle = \langle X_0 | \hat{a} + \hat{a}^\dagger | X_0 \rangle = \frac{\hbar \omega}{2} x_0^2
\]

\( \Rightarrow \) Number of excited quanta prop. to \( x_0^2 \)
Calculate relic abundance:

$$\Omega h^2 \sim m^2 \phi_0 (\text{today})^2$$

$$\phi_0 (\text{today}) = \phi_0 \cdot \left(\frac{a(t_1)}{a(\text{today})}\right)^{3/2}$$

where $$3H(t_1) = m$$

Radiation domination:

$$a(t_1) \sim t_1^{1/2}$$

$$H(t_1) \sim t_1^{-1}$$

$$\Rightarrow \phi_0 (\text{today}) \sim m^{-3/4}$$

$$\Rightarrow \Omega h^2 \sim m^{1/2} \phi_0^2$$

Restoring missing factors:

$$\Omega h^2 = 0.1 \sqrt{\frac{m}{1 \text{ MeV}}} \left(\frac{\phi_0}{10^{-13} \text{ GeV}}\right)^2$$

Example: The QCD axion

$$\phi_0 = \frac{p_a}{f}$$ solves strong CP problem

$$m = \frac{m_*^2}{f}$$ ( + finite temperature corrections )

$$\Rightarrow \Omega h^2 \approx 0.3 \left(\frac{p_a}{10^{12} \text{ GeV}}\right)^{7/6}$$
4. Phenomenology of WIMPs

4.1 Case study: Scalar singlet dark matter (SSDM)

Arguably simplest WIMP model

Consider scalar boson $s$ that is completely neutral (no strong, weak, EM charge)

$\Rightarrow$ No couplings to SM fermions or gauge bosons.

$\Rightarrow$ But: Can couple to Higgs boson $h$:

\[ s \xrightarrow{\lambda_{hs} s^2} h \]

*QFT*: Couplings arise from

\[ \mathcal{L} = \frac{1}{2} \lambda_{hs} s^2 H^+ H \]

After spontaneous symmetry breaking

\[ H \rightarrow \frac{1}{\sqrt{2}} (h + v) \]

Higgs boson $H$
\[ \mathcal{L} = \frac{1}{2} \lambda_{h s} s^2 v^2 + \lambda_{h s} s v^2 h + \frac{1}{2} \lambda_{h s} s^2 h \]

- mass term
- trilinear term
- quartic term

Note: No vertices involving single scalar boson

\[ s \rightarrow \ell \nu (\text{forbidden}) \]

Otherwise, \( s \) is unstable

\[ \Rightarrow \text{Two parameters:} \]

- \( m_s \): Singlet mass
- \( \lambda_{h s} \): Higgs coupling

4.1.1

**Relic density**

\[ \Delta \chi^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} = 0.12 \]

DM annihilation cross section
For $m_s \leq m_W$: \[ \sigma \approx \sigma (ss \rightarrow b \bar{b}) \]

For $m_s \geq m_h$: \[ \sigma \approx \sigma (ss \rightarrow W^+W^-) + \sigma (ss \rightarrow ZZ) + \sigma (ss \rightarrow hh) \]

For given $m_s$: Unique $\lambda_{ks}$ yields $\rho h^2 = 0.12$

$\lambda_{ks}$ versus $m_h$ in the plot. $m_s = m_h/2 \Rightarrow$ resonant enhancement
4.1.2 Searching for SSDM

- Indirect detection:
  
  DM annihilations stop after freeze-out except in regions of high density (e.g., Galactic centre)

  → Look for anomalous emission

  Example: Gamma rays

  \[ e^+ e^- \rightarrow \gamma \gamma \]

  \[ \gamma \rightarrow \text{mono-energetic gamma-rays line} \]
  (weak signal, very low BG)

  → Strong constraints (Fermi-LAT)

  Can also look for continuum

  \[ e^+ e^- \rightarrow \text{Bremsstrahlung} \]

  \[ \gamma \rightarrow \text{for other final states} \] (positrons, anti-protons)
- Direct detection

Higgs couples to nuclei proportionally to mass

\[ \sigma \sim \lambda_{hs}^2 m_n^2 \]

\[ N \rightarrow h N \rightarrow s s \]

\[ \Rightarrow \text{Strong bounds from Xe-based experiments} \]

- Collider experiments

For \( m_3 < m_h/2 \), Higgs can decay into DM:

\[ h \rightarrow s s \text{ invisible at colliders} \]

How to observe?

- Look for Higgs produced together with something else
Invisible Higgs decay

$\Rightarrow$ Apparent violation of momentum conservation
("missing transverse momentum")

$\Rightarrow$ Can constrain invisible decays

LHC run 2: $\text{BR}(h \rightarrow \text{inv}) \approx 25\%$

4.1.3 Summary

Two viable regions

1) $m_s \approx m_H/2$ (difficult to probe)

2) $m_s \gtrsim 300$ GeV (probed by XENON1T)
4.2 Model-independent approaches

Idea: At sufficiently low energies, the details model-specific details do not matter.

Example: Weak interactions

\[ \text{\begin{align*}
&\text{\includegraphics[width=0.5\textwidth]{diagram.png}} \\
&\downarrow \text{low energies} \\
&\text{\includegraphics[width=0.5\textwidth]{diagram2.png}} \\
&G_F = \frac{\sqrt{2}}{8} \frac{g^2}{m_{W}^2} = 1.17 \cdot 10^{-5} \text{ GeV}^{-2}
\end{align*}} \]

\( \Rightarrow \text{Effective Field Theory (EFT)} \)

\( \Rightarrow \text{beta decay} \)

\( \Rightarrow \text{neutrino scattering} \)

\( \Rightarrow \ldots \)
In analogy:

\[ \frac{1}{L^2} \]

E.g., consider interactions with quarks:

\[ x \rightarrow \text{DM annihilation} \]
- relic density
- indirect detection

\[ x \rightarrow \text{DM scattering} \]
- direct detection

\[ \frac{1}{3} p_{1,2} \]

\[ \rightarrow \text{DM production} \]
- LHC searches
- (need initial state radiation)
4.3 Possible exceptions (incomplete)

Relic density:
- Resonant enhancement
- Co-annihilations \((X, X \rightarrow SM SM)\)

Direct detection:
- Momentum dependent interactions
  \(\sigma \sim \left(\frac{q}{m_X}\right)^n \approx 10^{-3n}\) \((n = 2-4)\)
- Inelastic scattering
Indirect detection:
- Velocity-suppressed (p-wave) annihilation
  \( \sigma V \sim v^2 = 10^{-6} \)

LHC searches:
- EFT approach not valid
  (mediator \( \ll 1 \) TeV)

Or it's just not a WIMP:
- Axion
- Gravitino (super-WIMP)
- Asymmetric dark matter
- FIMPs (freeze-in mechanism)
- Sterile neutrinos
- ...

For experiments to decide!