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The Ratio R of Hadronic and Electronic Z Widths and the Strong Coupling Constant α_s

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Abstract

We review the relation between the ratio of hadronic and electronic Z widths, $R = \Gamma(\mathrm{Z} \to \mathrm{q\bar{q}})/\Gamma(\mathrm{Z} \to \mathrm{e^+e^-})$, and the strong coupling constant at the Z mass, α_s. The theoretical uncertainty of α_s derived from R is estimated to be

$$\Delta\alpha_s = \pm 0.002\,(\mathrm{electroweak}) \pm 0.002\,(\mathrm{QCD})\,{}^{+0.004}_{-0.003}\,(m_{\mathrm{top}}, m_{\mathrm{Higgs}}) \ .$$

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Introduction

One of the most important quantities measured by the LEP experiments is R, defined as the ratio of the hadronic and the electronic partial widths of the Z boson. Perhaps the main reason for considering this ratio is that the QCD correction on R can be calculated without specific knowledge about the hadronization mechanism and therefore R is supposed to allow a determination of the strong coupling constant $\alpha_s(m_Z)$ with very small theoretical uncertainties. The main goal of this paper is to give an estimate of the actual size of these uncertainties. In addition, we try to cast the relation between R and α_s into a simple form, which includes all the recently calculated electroweak and QCD corrections, including the mass effects. We make use of three independent analytical programs, BHM [1], TOPAZ0 [2] and ZFITTER [3], to calculate and cross check our results. Since there are different ways of implementing radiative corrections, *i.e.* different renormalization schemes or different implementations of higher order corrections, we have carefully analyzed the corresponding theoretical uncertainties. Relations for the α_s dependence of R have first been given in ref. [4] and more recently in ref. [5], while in ref. [6] the relation between α_s and the masses of the top and Higgs particles is given in tabular form for a fixed value of R. We point out that these previous publications do not include all the electroweak and strong corrections as they are known today.

Definitions

R is defined as the ratio of the hadronic and electronic partial Z widths,

$$R \;=\; \Gamma(\mathrm{Z} \to \mathrm{q\bar{q}})/\Gamma(\mathrm{Z} \to \mathrm{e^+e^-}) \;=\; \Gamma_\mathrm{q}/\Gamma_\mathrm{e}\ . \tag{1}$$

Γ_q denotes the partial Z width into hadrons resulting from a primarily produced $\mathrm{q\bar{q}}$ pair, including all five allowed flavours. Note that its value is slightly smaller than

$$R' = \Gamma_\mathrm{q}/\Gamma_\mathrm{l} = 3 \cdot \frac{\Gamma_\mathrm{q}}{\Gamma_\mathrm{e} + \Gamma_\mu + \Gamma_\tau} = 1.0007 \cdot R \tag{2}$$

due to the masses of the μ and in particular of the τ lepton [7, 8].
Of interest is also the ratio

$$r_\mathrm{b} \;=\; \Gamma(\mathrm{Z} \to \mathrm{b\bar{b}})/\Gamma(\mathrm{Z} \to \mathrm{q\bar{q}}) \;=\; \Gamma_\mathrm{b}/\Gamma_\mathrm{q}\ , \tag{3}$$

where Γ_b denotes the partial Z width into hadrons resulting from a primarily produced $\mathrm{b\bar{b}}$ pair. The decay width to $\mathrm{b\bar{b}}$ differs from the decay width into $\mathrm{d\bar{d}}$ because of non-universal weak corrections given by the $\mathrm{Z} \to \mathrm{b\bar{b}}$ vertex, which involves the b-t mass splitting, and because of the non-negligible b-quark mass effects. The peculiar top mass dependence of r_b, which can be measured precisely at LEP, allows in principle to constrain m_top by disentangling it from m_top-dependent loop corrections entering $\Delta\rho$. In other words, r_b is a genuine indicator of large m_top effects.

Dependence of R on α_s

A convenient way to summarize the effects of all the pertinent radiative corrections is to cast the relation between R and $\alpha_s \equiv \alpha_s(m_Z)$ into the form

$$R = R^0 \cdot (1 + \delta_\mathrm{QCD})\ , \tag{4}$$

where R^0 is the ratio of the hadronic and the electronic partial widths when final state QCD corrections are absent ($\alpha_s = 0$), and

$$\delta_{\mathrm{QCD}} = a_1 \cdot \frac{\alpha_{\mathrm{s}}}{\pi} + a_2 \cdot \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2 + a_3 \cdot \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^3 + \ldots \tag{5}$$

is the perturbative final state QCD correction; non-perturbative effects are negligible, as will be discussed below.

The above factorization is not exact for, at least, two reasons. First of all the QCD corrections affect also the vector boson self energies and the $\mathrm{Z} \to \mathrm{b}\bar{\mathrm{b}}$ vertex, giving rise to $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ or $\mathcal{O}(\alpha_{\mathrm{s}} G_{\mathrm{F}} m_{\mathrm{top}}^2)$ terms in R^0. Secondly, the total hadronic decay rate is the sum of the vector current induced rate Γ^{V} and of the axial decay rate Γ^{A}, which receive different QCD corrections, $f_{\mathrm{QCD}}^{\mathrm{V,q}}$ and $f_{\mathrm{QCD}}^{\mathrm{A,q}}$, respectively:

$$\Gamma(\mathrm{Z} \to \mathrm{q}\bar{\mathrm{q}}) = \Gamma^{\mathrm{V}} + \Gamma^{\mathrm{A}} = \frac{G_{\mathrm{F}} m_{\mathrm{Z}}^3}{2\sqrt{2}\,\pi} \sum_q \left[v_{\mathrm{q}}^2 \, f_{QCD}^{\mathrm{V,q}} + a_{\mathrm{q}}^2 \, f_{\mathrm{QCD}}^{\mathrm{A,q}} \right] . \tag{6}$$

Our strategy will be to use the programs BHM, TOPAZ0 and ZFITTER to calculate R and in the end the QCD coefficients a_i are derived from a fit to the exact result. These programs contain all the one loop electroweak corrections and all the leading higher order electroweak contributions presently known, including the $\mathcal{O}(G_{\mathrm{F}}^2 m_{\mathrm{top}}^4)$ terms [9] and the $\mathcal{O}(\alpha_{\mathrm{s}} G_{\mathrm{F}} m_{\mathrm{top}}^2)$ terms [10] for ρ and $\sin^2\theta$ and the $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ corrections for the vector boson self energies. The QCD corrections are calculated for massless quarks up to third order [11], and quark mass effects, including top-bottom mass splitting, are calculated in ref. [12, 13, 14, 15, 16]. As we have mentioned before, there are differences in the actual implementation of these corrections among the three programs, and therefore our first goal has been to produce results under similar conditions.

Here we simply list the most relevant choices that were made for the comparison, which correspond to the options used in ref. [8] for instance, some of which are different from the choices for our final result:

– QED and QCD corrections were factorized with respect to weak flavour-dependent corrections, *i.e.* for a given quantity A we compute $A = (A_{\mathrm{IB}} + \Delta_{\mathrm{W}})(1+\delta_{\mathrm{QED}})(1+\delta_{\mathrm{QCD}})$, where the subscript IB stands for improved Born (namely absorbing universal weak corrections), whereas W stands for weak flavour-dependent corrections.
– for the light quark contribution to the running of α_{em} we used the result of ref. [17] since, so far, no updated evaluation has been published.
– The c-quark mass is not running, but the b-quark mass corrections are taken into account according to eq. (30) of ref. [18].

A comparison of the R values calculated by the three programs under these conditions is shown in fig. 1. For the same input parameters $\alpha_s = 0.120, m_{\mathrm{b}} = 4.7$ GeV, $m_{\mathrm{top}} = 150$ GeV and $m_{\mathrm{Higgs}} = 300$ GeV the three programs BHM, TOPAZ0 and ZFITTER differ by only $\Delta\alpha_{\mathrm{s}} \sim \pi\Delta R/R \approx \pm 0.0005$. This is remarkably good agreement and strengthens confidence in the technical precision of the calculations.

Having understood the three programs for a common setup, we decided to include some new results for our final formulae:

– Motivated by the recent calculation of the $\mathcal{O}(\alpha_{\mathrm{s}} G_{\mathrm{F}} m_{\mathrm{top}}^2)$ corrections [10], QED and QCD corrections were not factorized, *i.e.* $A = A_{\mathrm{IB}}(1+\delta_{\mathrm{QED}}+\delta_{\mathrm{QCD}})+\Delta_{\mathrm{W}}$. This choice decreases the value of α_{s} derived from R by 0.001.
– As noted in ref. [12] there is an additional difference between vector (Γ^{V}) and axial (Γ^{A}) contributions even for massless final state quarks. This difference which starts at $\mathcal{O}(\alpha_{\mathrm{s}}^2)$ is a consequence of the large t-b mass splitting. However, it is not yet clear how to implement

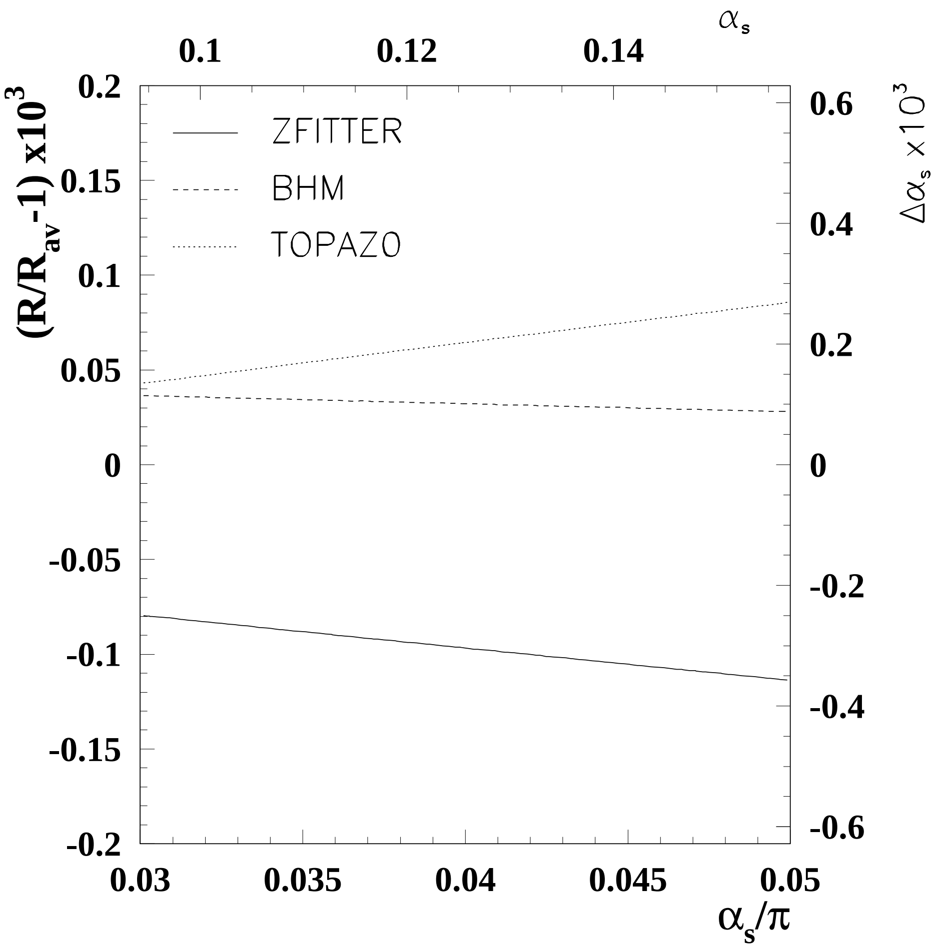

Figure 1: Relative difference between the value of R from each the three programs and the average, as a function of α_s, for $m_{top} = 150$ GeV, $m_{Higgs} = 300$ GeV and $m_b = 4.7$ GeV. The vertical scale on the right gives the change in α_s that corresponds to the relative change in R shown on the left scale.

unambiguously these singlet corrections, but the individual changes compensate in the total rate and therefore the determination of α_s is not affected.

- The c-quark mass corrections were taken into account, leading to a reduction of α_s by 0.0005.
- We included some of the recently computed QCD corrections of ref. [18, 19, 20, 21], namely:
  1. the next-to-next-to-leading running b-quark mass in the complete mass corrections of $\mathcal{O}(\alpha_s^2 m_b^2/m_Z^2)$ to the axial Z-boson decay rate;
  2. the improved QCD corrections of $\mathcal{O}(\alpha_s^2)$ to the singlet part $\Gamma^{A,s}(Z \to b\bar{b})$;
  3. the power suppressed quark mass corrections to the Z decay rate which affect at $\mathcal{O}(\alpha_s^2)$ the non-singlet term;
  4. the $\mathcal{O}(\alpha_s^3)$ corrections to the Z decay rate into hadrons, *i.e.* the $\mathcal{O}(\alpha_s^3)$ for $\Gamma^{A,s}$.

  Only the correction listed last is of some relevance, decreasing α_s by about 0.0005, while the rest together only produce a decrease of the order of 0.0001.

It is worth noticing that all these corrections have the same sign and therefore lower α_s from R with respect to the evaluation given in ref. [8].

The sizes of the above effects were calculated independently from TOPAZ0 and BHM, and good agreement was obtained. Figure 2 shows the final result, obtained as the average of the predictions from the three programs[1)]. The curve represents the result of a third order parameterization in α_s/π, which agrees with the full prediction to better than $\Delta R \approx 0.0005$ or, equivalently, $\Delta\alpha_s = 0.0001$, in the range of 0.10 to 0.15 in α_s. The parameterization is given by

$$R = 19.943 \cdot \left[1 + 1.060 \cdot \frac{\alpha_s}{\pi} + 0.90 \cdot \left(\frac{\alpha_s}{\pi}\right)^2 - 15 \cdot \left(\frac{\alpha_s}{\pi}\right)^3 \right] . \tag{7}$$

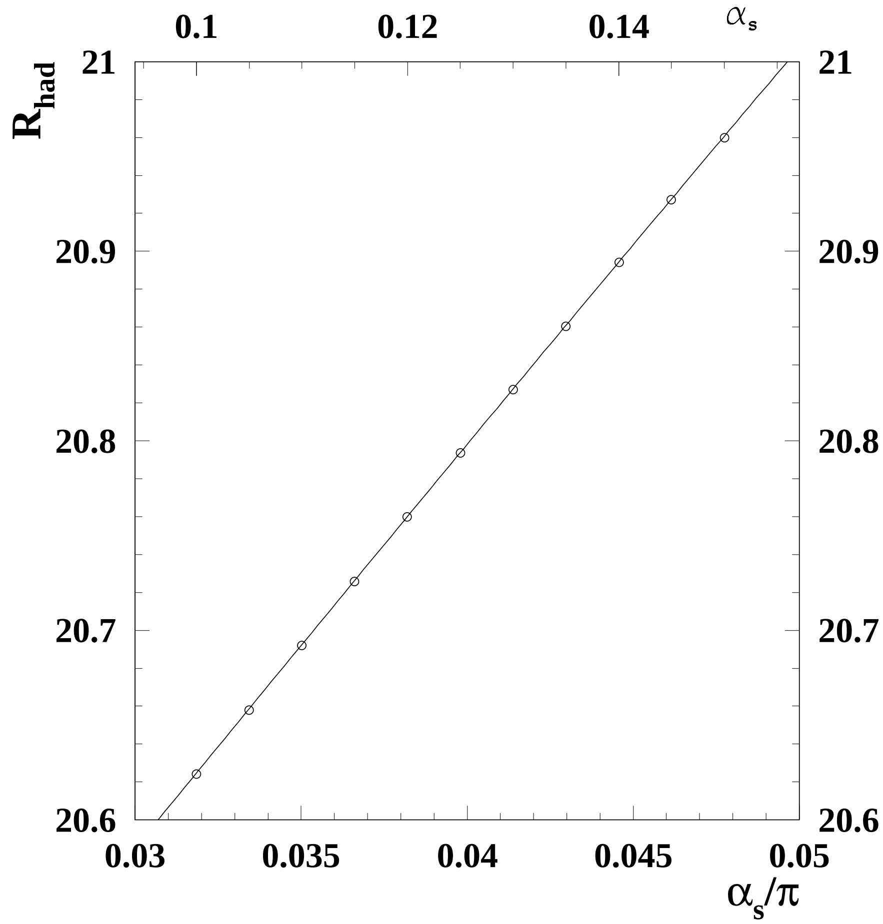

Figure 2: R as a function of α_s/π from the average of the three programs, including some recent calculations mentioned in the text. The solid line was obtained from the effective formula, where R^0 and the coefficient a_1 were fitted, whereas a_2 and a_3 were fixed to 0.90 and -15, respectively. The circles represent the result from the full calculation.

It is worth remarking here that a formula representing the complete calculation should have a different and more complicated structure than given here. This is due to the fact that we have decided not to factorize final state corrections in the actual complete calculation. In order

1) For this analysis the average was actually performed before taking into account any of the effects in the second list, and then the average of the corrections from BHM and TOPAZ0 was added. We have been informed that an updated version of ZFITTER will be released soon [22], showing that the new R is systematically bigger by an absolute amount of $0.007 \div 0.010$, which supports our findings.

to enforce the simpler structure we have fitted the two most important coefficients, R^0 and a_1, to the complete results, thus obtaining a numerically precise approximation of the full calculation.

Similarly, we obtain a formula for r_b, again for the canonical values $m_\mathrm{top} = 150$ GeV and $m_\mathrm{Higgs} = 300$ GeV. The different programs lie within ±0.0001 around the parameterization given by

$$r_\mathrm{b} = 0.21598 \cdot \left[1 + 0.106 \cdot \frac{\alpha_\mathrm{s}}{\pi} - 0.09 \cdot \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^2 - 23 \cdot \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^3\right] . \tag{8}$$

While for R the QCD correction amounts to about 4%, it is only about 0.2% for r_b due to a cancellation of all but the b mass dependent terms.

Dependence of R on m_top and m_Higgs

The values that we have quoted refer to fixed Higgs boson and top quark masses, but R_0 depends, via loop corrections, on these masses. In the range of ‘allowed’ values for these parameters, $60\,\mathrm{GeV} \leq m_\mathrm{Higgs} \leq 1000\,\mathrm{GeV}$ [23] and $100\,\mathrm{GeV} \leq m_\mathrm{top} \leq 200\,\mathrm{GeV}$ [24, 7], these dependencies can be parameterized to a good approximation by

$$R^0 \sim \left(1 - 2.4 \cdot 10^{-4}\, \ln\left(\frac{m_\mathrm{Higgs}}{m_\mathrm{Z}}\right)^2\right) \cdot \left(1 - 2.5 \cdot 10^{-4}\left(\frac{m_\mathrm{top}}{m_\mathrm{Z}}\right)^2\right) . \tag{9}$$

for BHM and ZFITTER and by

$$R^0 \sim \left(1 - 2.2 \cdot 10^{-4}\, \ln\left(\frac{m_\mathrm{Higgs}}{m_\mathrm{Z}}\right)^2\right) \cdot \left(1 - 4.7 \cdot 10^{-4}\left(\frac{m_\mathrm{top}}{m_\mathrm{Z}}\right)^2\right) . \tag{10}$$

for TOPAZ0.

While all programs agree well on the Higgs mass dependence, TOPAZ0 predicts a top mass dependence almost twice as big as the dependence obtained from ZFITTER or BHM. This difference has no sizable effect on the actual determination of α_s, and the numerical studies that we have performed show that its basic origin is due to the different choices made in the programs concerning the calculation of the purely electroweak corrections, namely:

- Absorb part of the Higgs corrections into ρ resummed or not resummed. This introduces a dependence of the resummation on the renormalization procedure.
- Squares of quantities, *e.g.* $a = a_0 + g\,a_1 + \mathcal{O}(g^2)$ are performed by squaring a numerically or by suppressing orders g^2, *i.e.* $a^2 = a_0^2 + 2\,g\,a_0\,a_1$.

Concerning these points, the choice in BHM and ZFITTER has been the first alternative whereas it has been the second for TOPAZ0.

The coefficient a_2 in δ_QCD also varies with the top mass,

$$\Delta_{m_\mathrm{top}} a_2 = -0.002 \cdot (m_\mathrm{top}/\mathrm{GeV} - 150) . \tag{11}$$

while the top mass dependence of the third order coefficient a_3 is negligible [21].

Theoretical Uncertainties

An important result of our analysis is that the differences between BHM, TOPAZ0 and ZFITTER for a similar configuration are within 0.0005 in α_s over a range $0.10 < \alpha_\mathrm{s} < 0.15$. The following uncertainties contribute to the total theoretical error in α_s as determined from R:

- uncertainties in the electroweak calculations and from their implementations into the programs. From a comparison of three different programs (BHM, TOPAZ0, ZFITTER) we have found that the largest difference comes from the factorization *vs.* non-factorization of QED and QCD corrections. For any of the Z partial widths we can compute

$$\Gamma_{\rm f} = (\Gamma_{\rm IB} + \Delta\Gamma_{\rm W})(1+\delta_{\rm QED})(1+\delta_{\rm QCD}) \quad (12)$$

$$\Gamma_{\rm nf} = \Gamma_{\rm IB}(1+\delta_{\rm QED}+\delta_{\rm QCD}) + \Delta\Gamma_{\rm W} \quad (13)$$

where $\Gamma_{\rm IB}$ is the width computed with the improved Born approximation (universal corrections), and $\Delta\Gamma_{\rm W}$ includes the non-universal weak corrections. Due to the well known $m_{\rm top}$ dependence of the $\rm Z \rightarrow b\bar{b}$ vertex the difference between the two approaches is more pronounced for the $\rm b\bar{b}$ partial decay rate. Since the actual difference among the two possibilities is linked to the assumption on higher orders, even though we have chosen the second one as our baseline option, we have to quote the difference among both as a theoretical uncertainty. From our analysis it follows that the corresponding uncertainty for $\alpha_{\rm s}$ is $\Delta\alpha_{\rm s} \approx 0.001$.

- The uncertainty coming from the hadronic contribution to $\alpha_{\rm em}$ [17], $\Delta(\delta[\alpha_{\rm em}(\text{light})]) = 0.0009$, leads to an uncertainty on $\alpha_{\rm s}$ of $\Delta\alpha_{\rm s} \approx 0.001$. Therefore, if we had used the preliminary value $1/\alpha_{\rm em}(\text{light}) = 128.87$ [25] as our base option, this would have produced a shift of $\Delta\alpha_{\rm s} \approx 0.0005$.
- A variation of the physical b-quark mass in the range $m_{\rm b} = 4.7 \pm 0.2$ GeV gives $\Delta\alpha_{\rm s} \approx 0.0003$.
- Of all the new QCD effects that have recently appeared in the literature [19, 20, 21] only the $\mathcal{O}(\alpha_{\rm s}^3)$ corrections to the singlet part of the axial-vector width are of some relevance, giving $\Delta\alpha_{\rm s} \approx 0.0005$, while the rest amounts to small corrections, $\Delta\alpha_{\rm s} \approx 0.0001$. We might consider $\Delta\alpha_{\rm s} = 0.0005$ as a conservative estimation of the uncertainty due to still unknown mass corrections.
- Missing higher order in the massless QCD corrections, the effect of which can be estimated in different ways:
  (a) A variation of the renormalization scale μ between $m_{\rm Z}/4$ and $m_{\rm Z}$ [26] changes $\delta_{\rm QCD}$ by $8 \cdot 10^{-4}$ for $\alpha_{\rm s} = 0.12$ [27]. The corresponding change in $\alpha_{\rm s}$ is about 0.002.
  (b) A guess of the fourth order correction of $\mathcal{O}[100] \cdot (\alpha_{\rm s}/\pi)^4$. Estimates in the literature [28, 29] are of this order of magnitude. For $\alpha_{\rm s} = 0.12$ the changes in $\delta_{\rm QCD}$ and $\alpha_{\rm s}$ amount to approximately 0.0002 and 0.0005, respectively. (c) A variation of the renormalization scheme leads to $\Delta\alpha_{\rm s} < 0.001$ [30]. In all cases the uncertainty is not bigger than 0.002 in $\alpha_{\rm s}$.
- Non-perturbative corrections are expected to be $\Delta\delta_{\rm QCD} = \mathcal{O}[(\Lambda/m_{\rm Z})^2]$ [31]. With a value for the QCD scale parameter Λ of 0.3 GeV one obtains $\Delta\delta_{\rm QCD} = \mathcal{O}[1 \cdot 10^{-5}]$ and $\Delta\alpha_{\rm s} = \mathcal{O}[0.0003]$, which is negligible.
- A variation of the Higgs mass between 60 and 1000 GeV leads to $\Delta\alpha_{\rm s} = \pm 0.002$. Taking the larger top mass dependence from TOPAZ0, a change of the top mass from 100 to 200 GeV translates into $\Delta\alpha_{\rm s} =^{+0.003}_{-0.002}$. Adding these two sources in quadrature yields $\Delta\alpha_{\rm s} =^{+0.004}_{-0.003}$.

Therefore the most important contributions are due to the unknown masses of the top quark and of the Higgs boson and to the interplay between pure weak and QCD corrections. We can conservatively claim that the theoretical uncertainty in the extraction of $\alpha_{\rm s}$ from R is:

$$\Delta\alpha_{\rm s} = \pm 0.002\,(\text{electroweak}) \;\pm 0.002\,(\text{QCD})\; ^{+0.004}_{-0.003}\,(m_{\rm top}, m_{\rm Higgs}) \;. \quad (14)$$

$\alpha_{\rm s}$ from R

The most recent combined LEP value for R is 20.763 ± 0.049(exp.) [8]. By use of the effective formula, we find $\alpha_{\rm s} = 0.120 \pm 0.007$(exp.). Assuming that $m_{\rm top} = 150$ GeV and

$m_{\rm Higgs} = 300$ GeV are reasonable central values, no correction is needed. Adding the theoretical uncertainties as estimated above we get $\alpha_{\rm s} = 0.120 \pm 0.008$ as final result.

Summary and Conclusions

Using three independent analytical programs which incorporate all the recently calculated electroweak loop corrections and all the numerically relevant mass effects in the QCD correction, we have estimated the theoretical uncertainty of $\alpha_{\rm s}$ derived from R to be

$$\Delta\alpha_{\rm s} = \pm 0.002\,(\rm electroweak) \pm 0.002\,(\rm QCD)\;^{+0.004}_{-0.003}\,(m_{\rm top}, m_{\rm Higgs}) \;\; .$$

The relation between R and $\alpha_{\rm s}$ can be represented through an effective, but highly accurate formula

$$R = 19.943 \cdot \left[1 + 1.060 \cdot \frac{\alpha_{\rm s}}{\pi} + 0.9 \cdot \left(\frac{\alpha_{\rm s}}{\pi}\right)^2 - 15 \cdot \left(\frac{\alpha_{\rm s}}{\pi}\right)^3 \right]$$

for $m_{\rm Higgs} = 300$ GeV and $m_{\rm top} = 150$ GeV.

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