M_h to N^3LO+N^3LL

Robert Harlander
RWTH Aachen University

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based on arXiv:1910.03595
in collaboration with

Jonas Klappert
Alexander Voigt

supported by DFG

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Ingredients

Fixed-order result of `FlexibleSUSY+Himalaya`

1- and 2-loop results

+ 3-loop $\alpha_t\alpha_s^2$ [no $\mathcal{O}$(v/MS) terms]

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Hybrid result of **FlexibleEFTHiggs**

1-loop results + resummation + $\mathcal{O}$(v/M$_S$) terms

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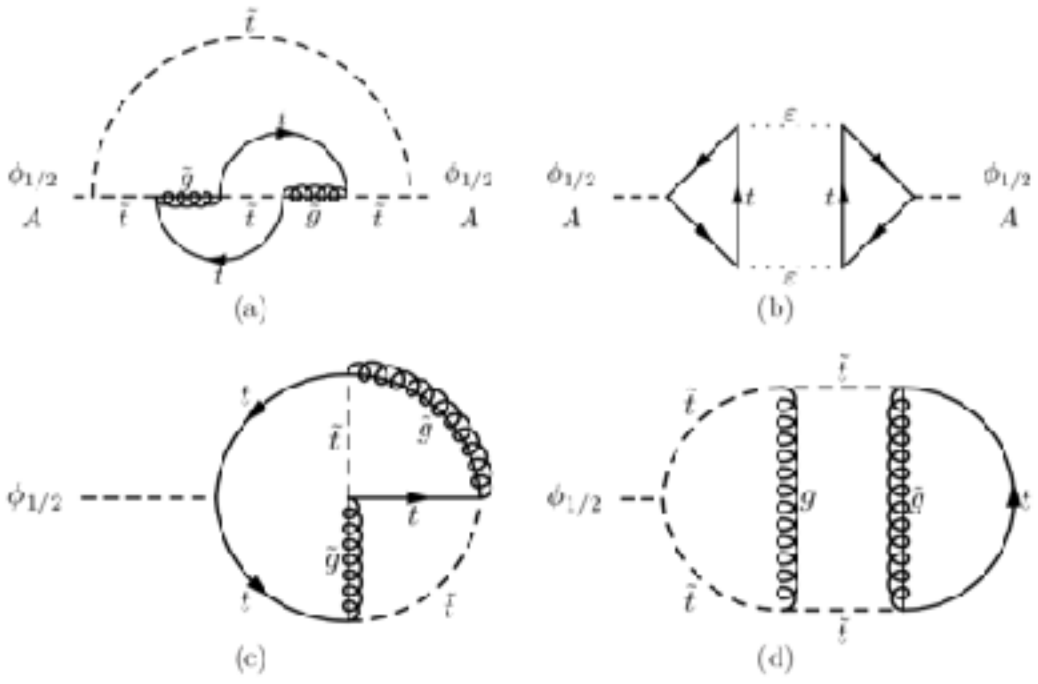
Kant, RH, Mihaila, Steinhauser '10



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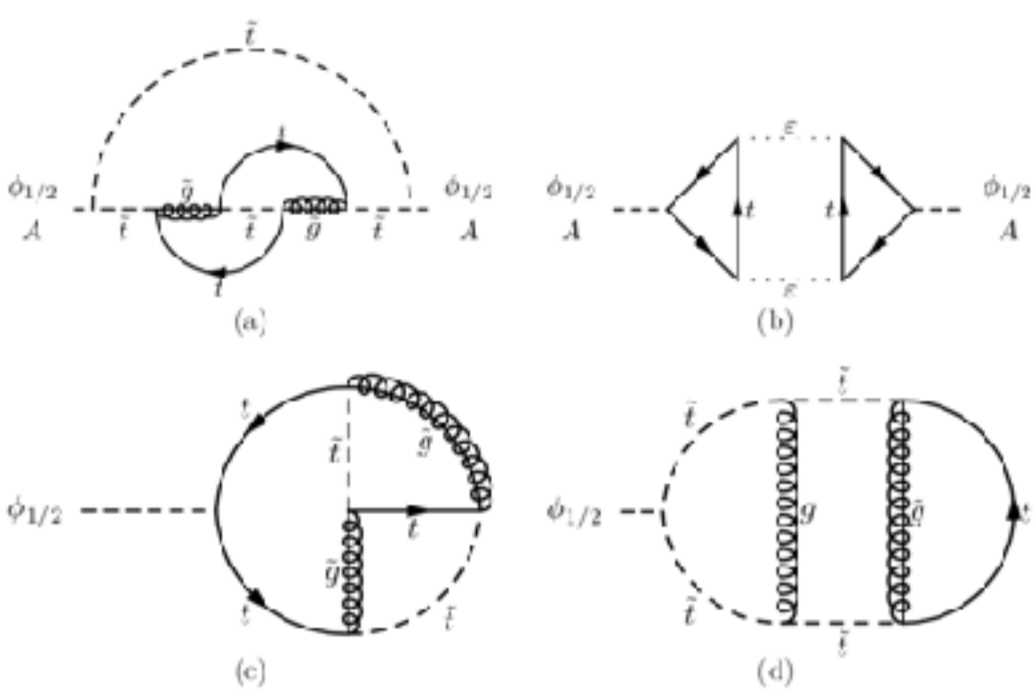

(h3)	$m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}\,,$
(h4)	$m_{\tilde{q}} \gg m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}}\,,$
(h5)	$m_{\tilde{q}} \gg m_{\tilde{t}_2} \gg m_{\tilde{t}_1} \approx m_{\tilde{g}}\,,$
(h6)	$m_{\tilde{q}} \gg m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}\,,$
(h6b)	$m_{\tilde{q}} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1}\,,$
(h9)	$m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \gg m_{\tilde{g}}\,,$

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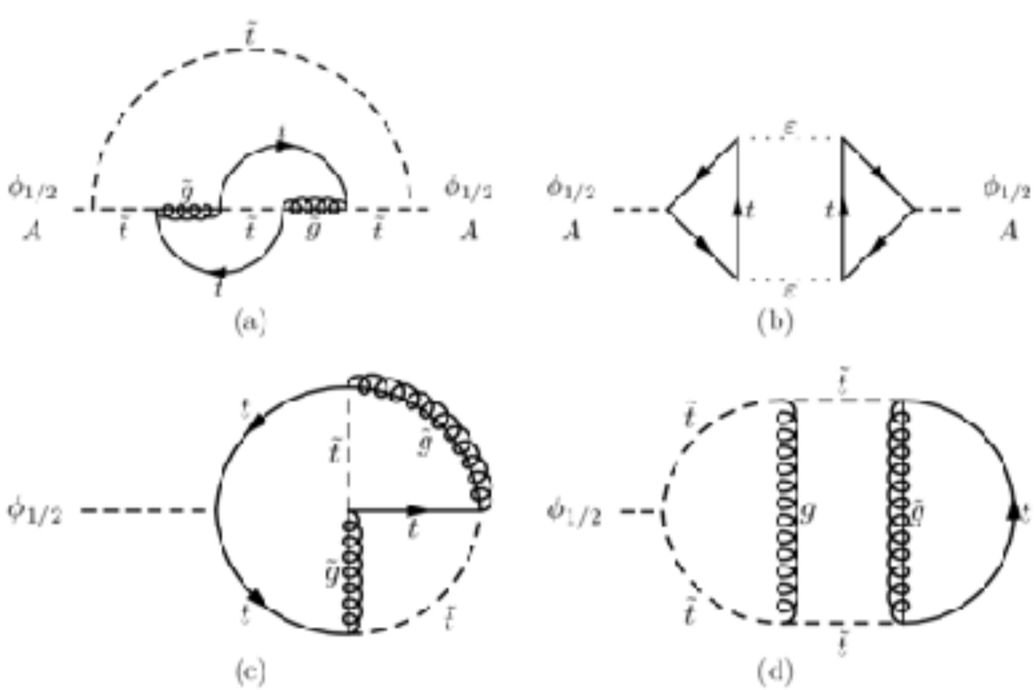

$$
\begin{array}{ll}
\text{(h3)} & m_{\tilde q} \approx m_{\tilde t_1} \approx m_{\tilde t_2} \approx m_{\tilde g}\,, \\
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\end{array}
$$

however, see Fazio, Reyes ‘19

Resummation

$$M_h^2 = \lambda(\mu_t)v^2 + m_t^2\alpha_t[\ldots]$$

	running	matching
LL	$\mu\frac{d}{d\mu}\bar{\lambda} = \quad \beta_{\alpha_t}$	$\bar{\lambda}(\mu_S) = (M_Z^2/v^2)\cos^2\beta$
NLL	$+\,\beta_{\alpha_t\alpha_s}$	$+\,\Delta\lambda_{\alpha_t}$
N2LL	$+\,\beta_{\alpha_t\alpha_s^2}$	$+\,\Delta\lambda_{\alpha_t\alpha_s}$
N3LL	$+\,\beta_{\alpha_t\alpha_s^3}$	$+\,\Delta\lambda_{\alpha_t\alpha_s^2}$

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(Arrows: NLO links β_{α_t} ↔ $\Delta\lambda_{\alpha_t}$; N²LO links $\beta_{\alpha_t \alpha_s}$ ↔ $\Delta\lambda_{\alpha_t \alpha_s}$.)

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Chetyrkin, Zoller '13
Bednyakov, Pikelner, Velizhanin '13

Resummation

$$M_h^2 = \lambda(\mu_t)v^2 + m_t^2\alpha_t[\ldots]$$

running / matching

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LL	$\mu\frac{d}{d\mu}\bar{\lambda} = \beta_{\alpha_t}$	NLO	$\bar{\lambda}(\mu_S) = (M_Z^2/v^2)\cos^2\beta$
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N2LL	$+\ \beta_{\alpha_t\alpha_s^2}$	N3LO	$+\ \Delta\lambda_{\alpha_t\alpha_s}$
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Chetyrkin, Zoller '13
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extract from FO
RH, Klappert, Ochoa, Voigt '18

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Hybrid result of **FlexibleEFTHiggs**

1-loop results + resummation + $\mathcal{O}$(v/MS) terms

Combination

FlexibleSUSY+
Himalaya

1-, 2-loop w/ $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/o resummation

Combination

FlexibleSUSY+ Himalaya	HSSUSY+ Himalaya
1-, 2-loop w/ $O(v/M_S)$	1-, 2-loop w/o $O(v/M_S)$
3-loop w/o $O(v/M_S)$	3-loop w/o $O(v/M_S)$
w/o resummation	w/ resummation

Combination

FlexibleSUSY+
Himalaya

1-, 2-loop w/ $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/o resummation

HSSUSY+
Himalaya

1-, 2-loop w/o $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/ resummation

FlexibleEFTHiggs

1-loop w/ $O(v/M_S)$
w/ resummation

Combination

FlexibleSUSY+
Himalaya

1-, 2-loop w/ $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/o resummation

HSSUSY+
Himalaya

1-, 2-loop w/o $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/ resummation

FlexibleEFTHiggs

1-loop w/ $O(v/M_S)$
w/ resummation

add $O(v/M_S)$

Combination

FlexibleSUSY+ Himalaya

1-, 2-loop w/ $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/o resummation

HSSUSY+ Himalaya

1-, 2-loop w/o $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/ resummation

FlexibleEFTHiggs

1-loop w/ $O(v/M_S)$
w/ resummation

add $O(v/M_S)$

1-loop: FlexibleEFTHiggs - HSSUSY$|_{\text{1-loop}}$

Combination

FlexibleSUSY+ Himalaya	HSSUSY+ Himalaya	FlexibleEFTHiggs
1-, 2-loop w/ $O(v/M_S)$ 3-loop w/o $O(v/M_S)$ w/o resummation	1-, 2-loop w/o $O(v/M_S)$ 3-loop w/o $O(v/M_S)$ w/ resummation	1-loop w/ $O(v/M_S)$ w/ resummation

add $O(v/M_S)$

1-loop: FlexibleEFTHiggs - HSSUSY$|_{\text{1-loop}}$

2-loop: FlexibleSUSY$|_{\text{2-loop}}$ - $M_h^2|_{\text{EFT}\to\text{FO,2-loop}}$

Combination

FlexibleSUSY+ Himalaya

1-, 2-loop w/ $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/o resummation

HSSUSY+ Himalaya

1-, 2-loop w/o $O(v/M_S)$
3-loop w/o $O(v/M_S)$
w/ resummation

FlexibleEFTHiggs

1-loop w/ $O(v/M_S)$
w/ resummation

add $O(v/M_S)$

1-loop: FlexibleEFTHiggs - HSSUSY$|_{\text{1-loop}}$

2-loop: FlexibleSUSY$|_{\text{2-loop}}$ - $M_h^2\,|_{\text{EFT}\to\text{FO},2-\text{loop}}$

So far: 1-loop: complete, 2-loop: $\alpha_t\alpha_s$, α_t^2

Combination

FlexibleSUSY+
Himalaya

1-, 2-loop w/ O(v/MS)
3-loop w/o O(v/MS)
w/o resummation

HSSUSY+
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1-, 2-loop w/o O(v/MS)
3-loop w/o O(v/MS)
w/ resummation

FlexibleEFTHiggs

1-loop w/ O(v/MS)
w/ resummation

add O(v/MS)

1-loop: FlexibleEFTHiggs - HSSUSY|1-loop

2-loop: FlexibleSUSY|2-loop - $M_h^2|_{\mathrm{EFT\to FO,2-loop}}$

So far: 1-loop: complete, 2-loop: $\alpha_t\alpha_s, \alpha_t^2$

Uncertainty starts at 2-loop $\alpha_t\alpha_b$ etc.

Uncertainty estimate

$$\Delta M_h^{\rm hyb} = \min\left\{\Delta M_h^{\rm FO}, \Delta M_h^{\rm EFT}\right\}.$$

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$$\Delta M_h^{\mathrm{hyb}} = \min\left\{\Delta M_h^{\mathrm{FO}}, \Delta M_h^{\mathrm{EFT}}\right\}.$$

$$\Delta M_h^{\mathrm{FO}} = \Delta^{(Q_S)} M_h^{\mathrm{FO}} + \Delta^{(g_3)} M_h^{\mathrm{FO}}$$

$$\Delta^{(Q_S)} M_h^{\mathrm{FO}} = \max_{Q_S \in [M_S/2, 2M_S]} \left| M_h^{\mathrm{FO}}(Q_S) - M_h^{\mathrm{FO}}(M_S) \right|,$$

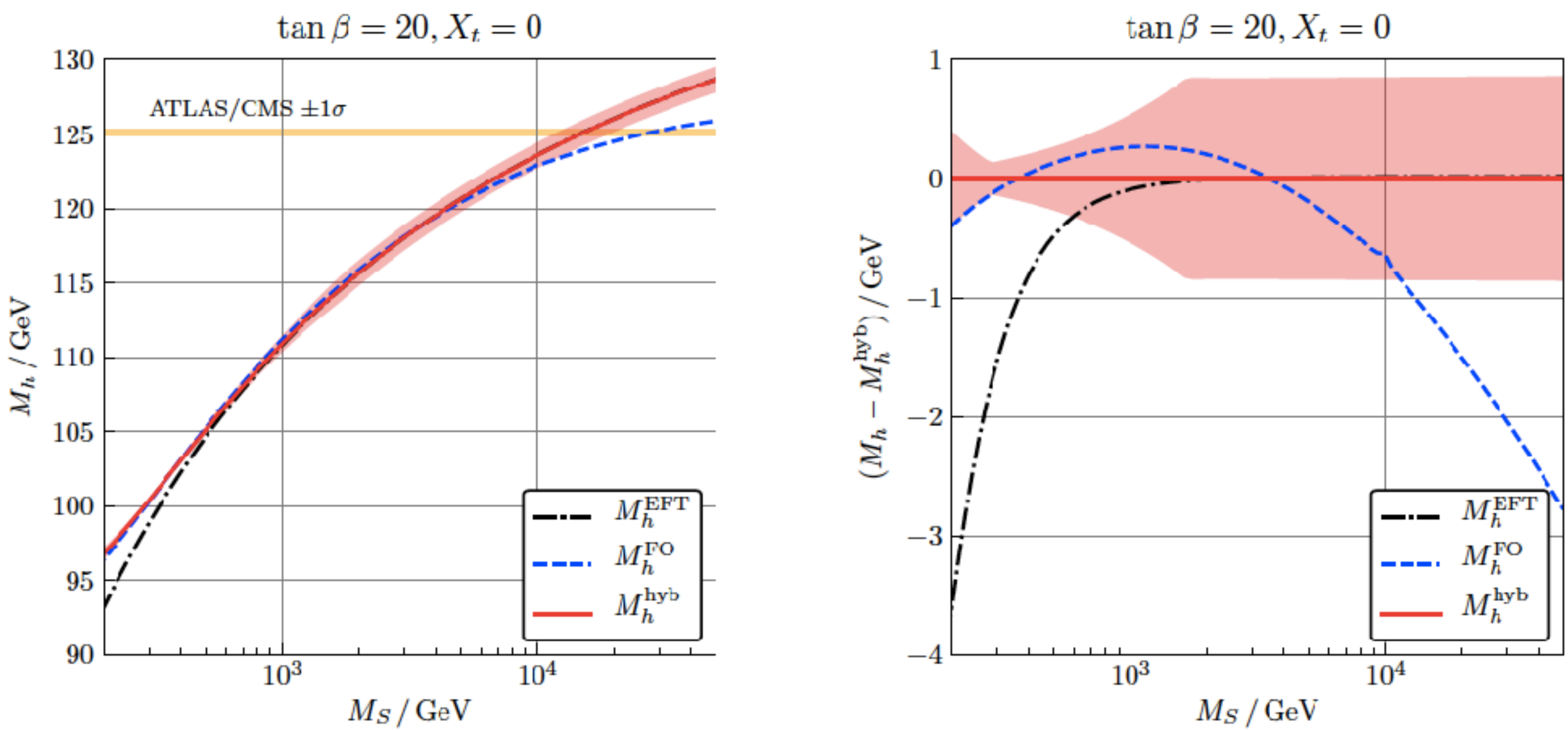
$$\Delta^{(g_3)} M_h^{\mathrm{FO}} = \left| M_h^{\mathrm{FO}}(g_3^{1\ell}) - M_h^{\mathrm{FO}}(g_3^{2\ell}) \right|.$$

Uncertainty estimate

$$\Delta M_h^{\rm hyb} = \min\left\{\Delta M_h^{\rm FO}, \Delta M_h^{\rm EFT}\right\}.$$

$$\Delta M_h^{\rm EFT} = \Delta^{(Q_t)} M_h^{\rm EFT} + \Delta^{(Q_S)} M_h^{\rm EFT} + \Delta^{(y_t^{\rm SM})} M_h^{\rm EFT} + \Delta^{(v^2/M_S^2)} M_h^{\rm EFT}.$$

$$\begin{aligned}
\Delta^{(Q_t)} M_h^{\rm EFT} &= \max_{Q\in[M_t/2,2M_t]} \left| M_h^{\rm EFT}(Q) - M_h^{\rm EFT}(M_t)\right|, \\
\Delta^{(Q_S)} M_h^{\rm EFT} &= 0.5\,{\rm GeV}, \\
\Delta^{(y_t^{\rm SM})} M_h^{\rm EFT} &= \left| M_h^{\rm EFT}(y_t^{{\rm SM},3\ell}) - M_h^{\rm EFT}(y_t^{{\rm SM},4\ell})\right|, \\
\Delta^{(v^2/M_S^2)} M_h^{\rm EFT} &= \left| M_h^{\rm EFT} - M_h^{\rm EFT}(v^2/M_S^2)\right|.
\end{aligned}$$



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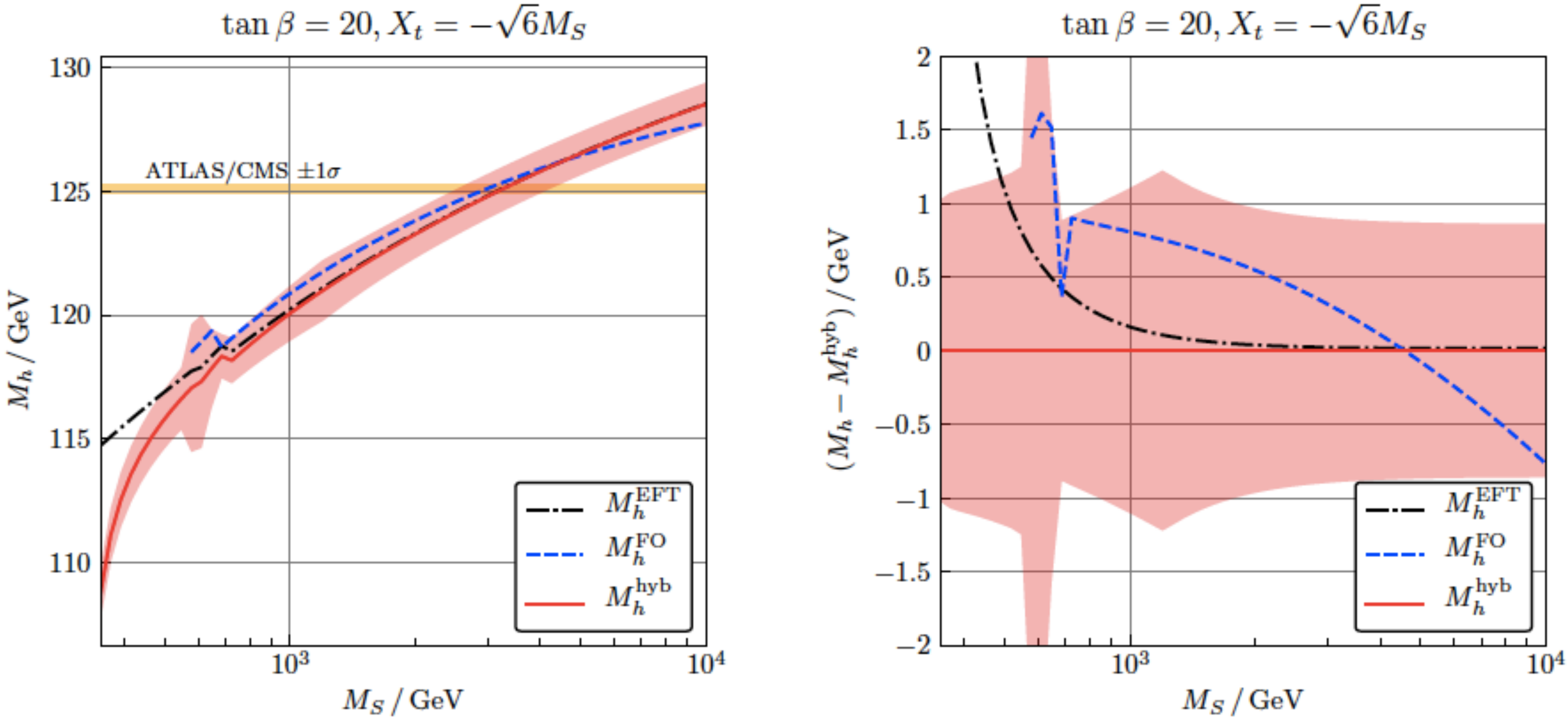
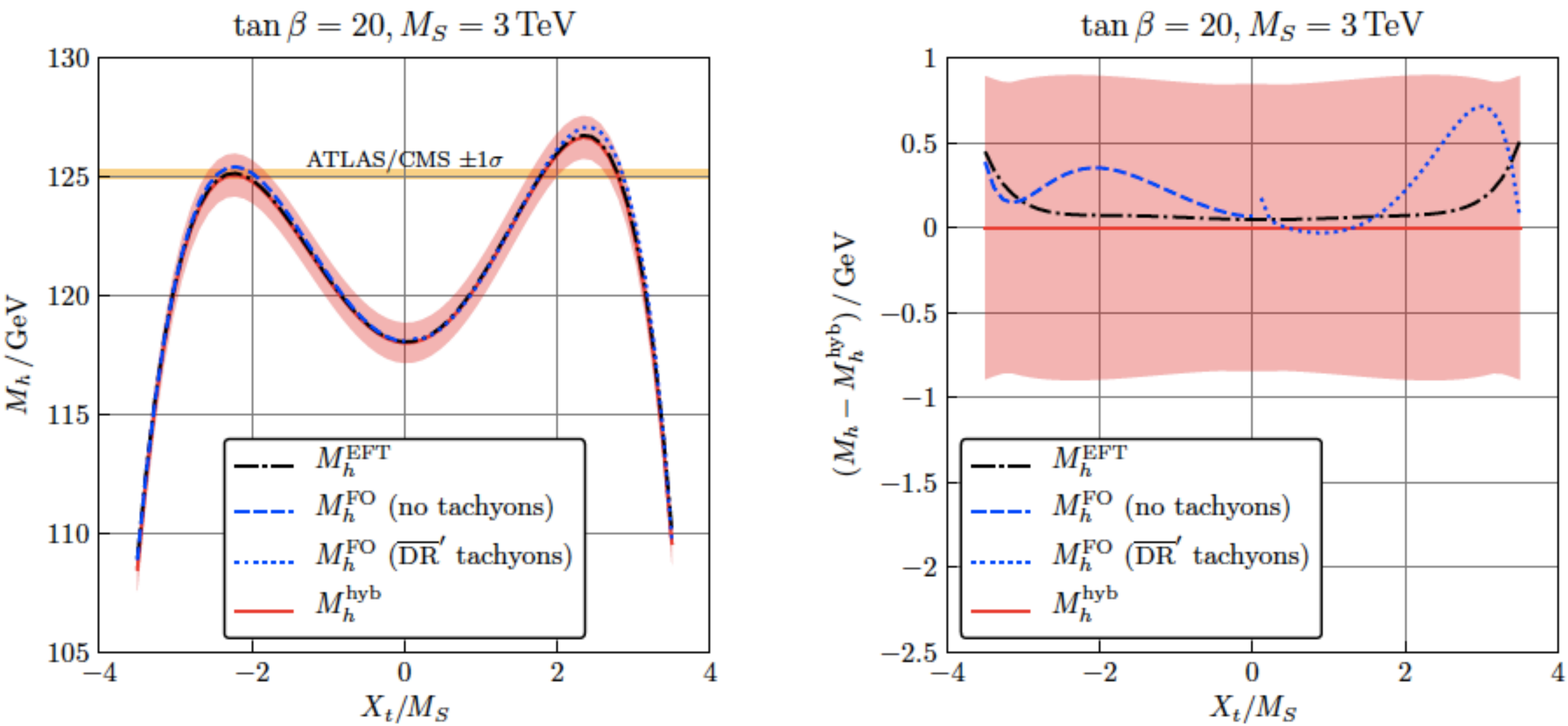


$\tan\beta = 20, X_t = -\sqrt{6}M_S$
130
125
120
115
110
M_h / GeV
ATLAS/CMS $\pm 1\sigma$
10^3
10^4
M_S / GeV
M_h^{EFT}
M_h^{FO}
M_h^{hyb}
$\tan\beta = 20, X_t = -\sqrt{6}M_S$
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
$(M_h - M_h^{\mathrm{hyb}})$ / GeV
10^3
10^4
M_S / GeV
M_h^{EFT}
M_h^{FO}
M_h^{hyb}



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Tachyonic Higgs masses

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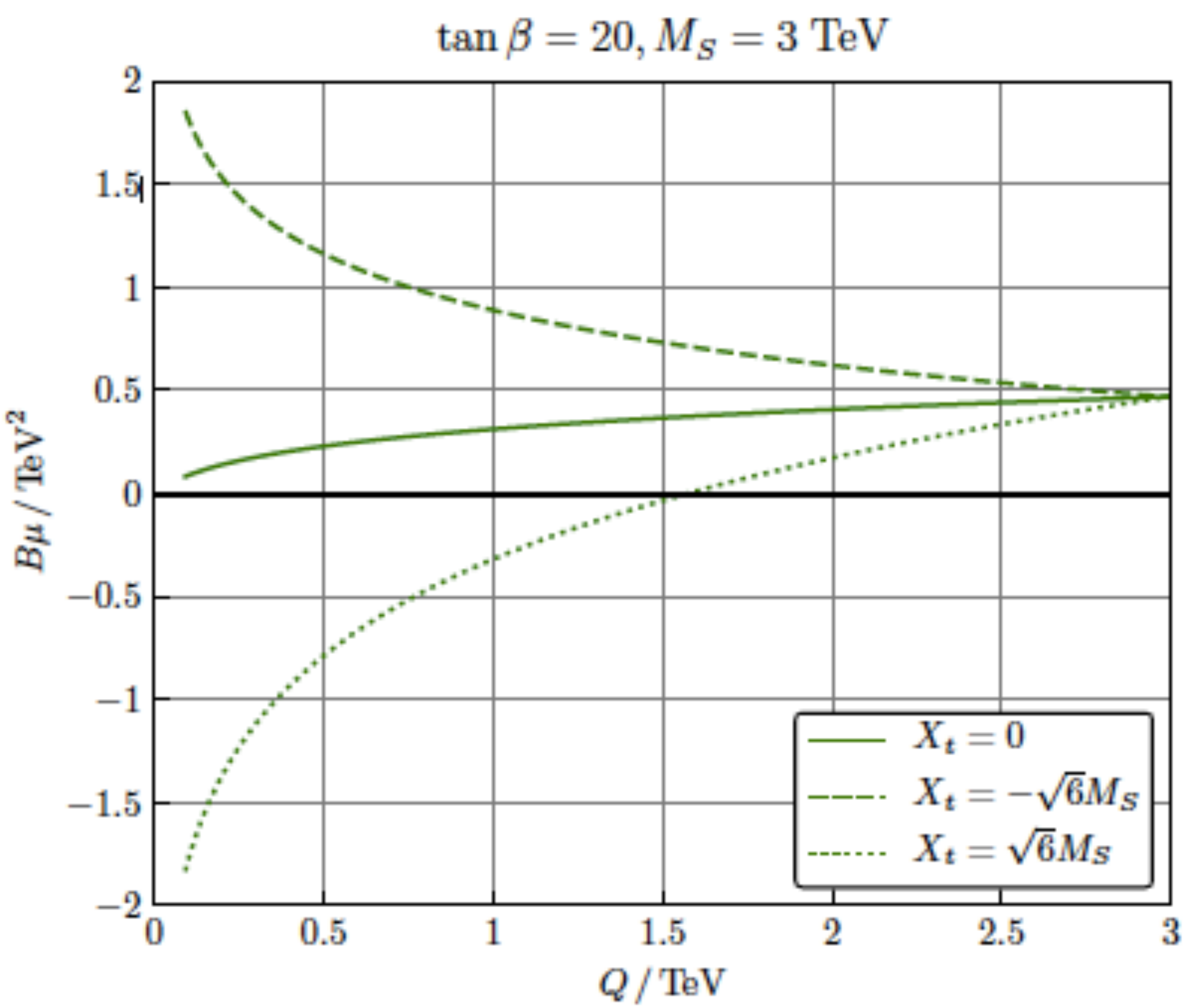
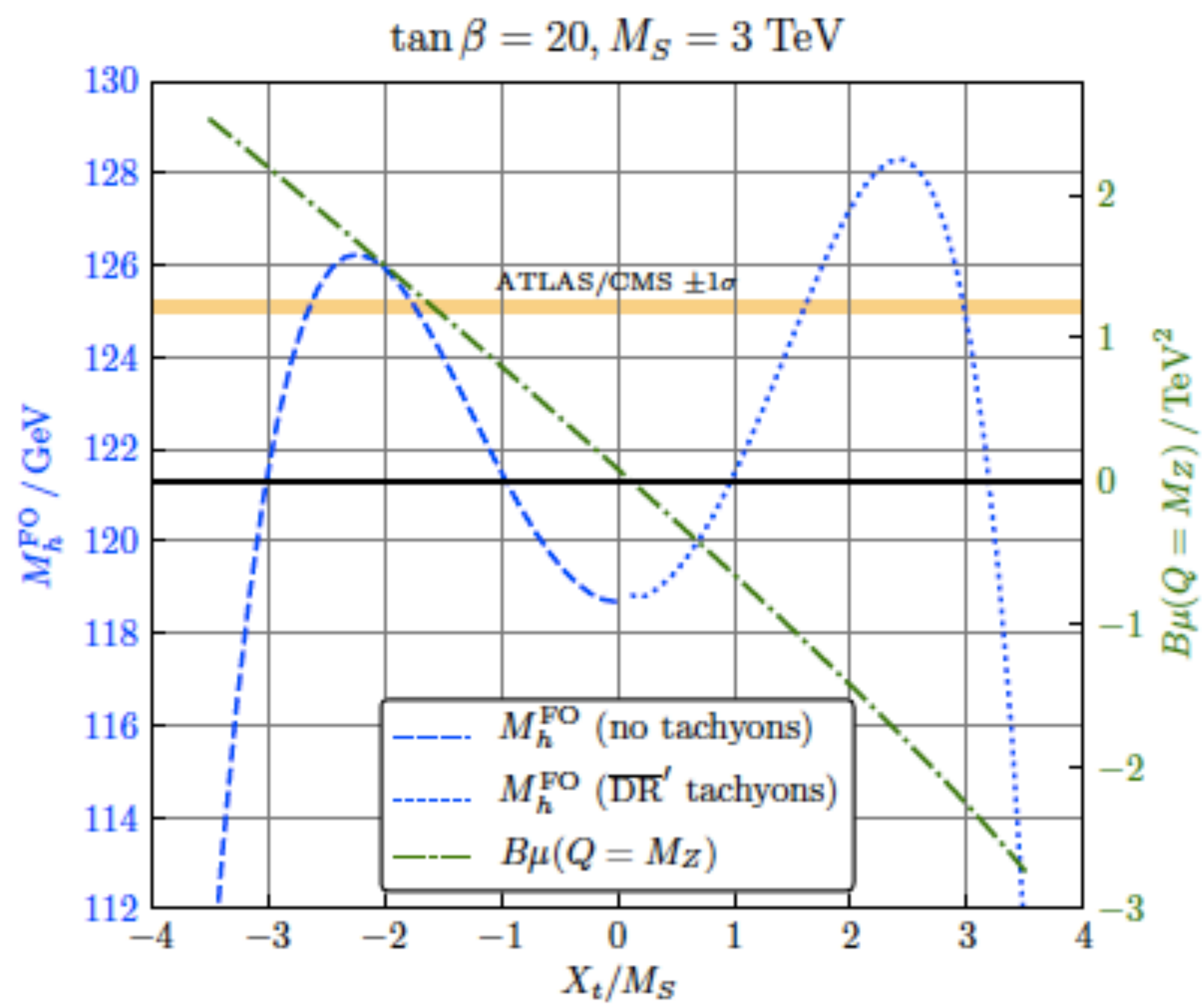
$$M_h^2 = M_h^2(x(\mu \sim M_Z))$$

$$B\mu(M_S) = \frac{1}{2}\sin[2\beta(M_S)]\, m_A^2(M_S)$$

$$m_H^2(M_Z) \approx m_{H^\pm}^2(M_Z) \approx m_A^2(M_Z) = \frac{2B\mu(M_Z)}{\sin[2\beta(M_Z)]} < 0.$$

Tachyonic Higgs masses





Conclusions

- hybrid FO+EFT result in DRbar scheme
- includes
  - tree-level + LL resummation
  - full 1-loop + NLL resummation
  - full $\alpha_t\alpha_s + \alpha_t^2$ + NNLL resummation
  - $\alpha_t\alpha_s^2$ w/o O(v/M_S) + N^3LL resummation
- missing higher orders sizable at low M_S
- EFT sufficient above 1-2 TeV