Complexity + Simplicity in Theoretical Particle Physics

Robert Harlander
RWTH Aachen University

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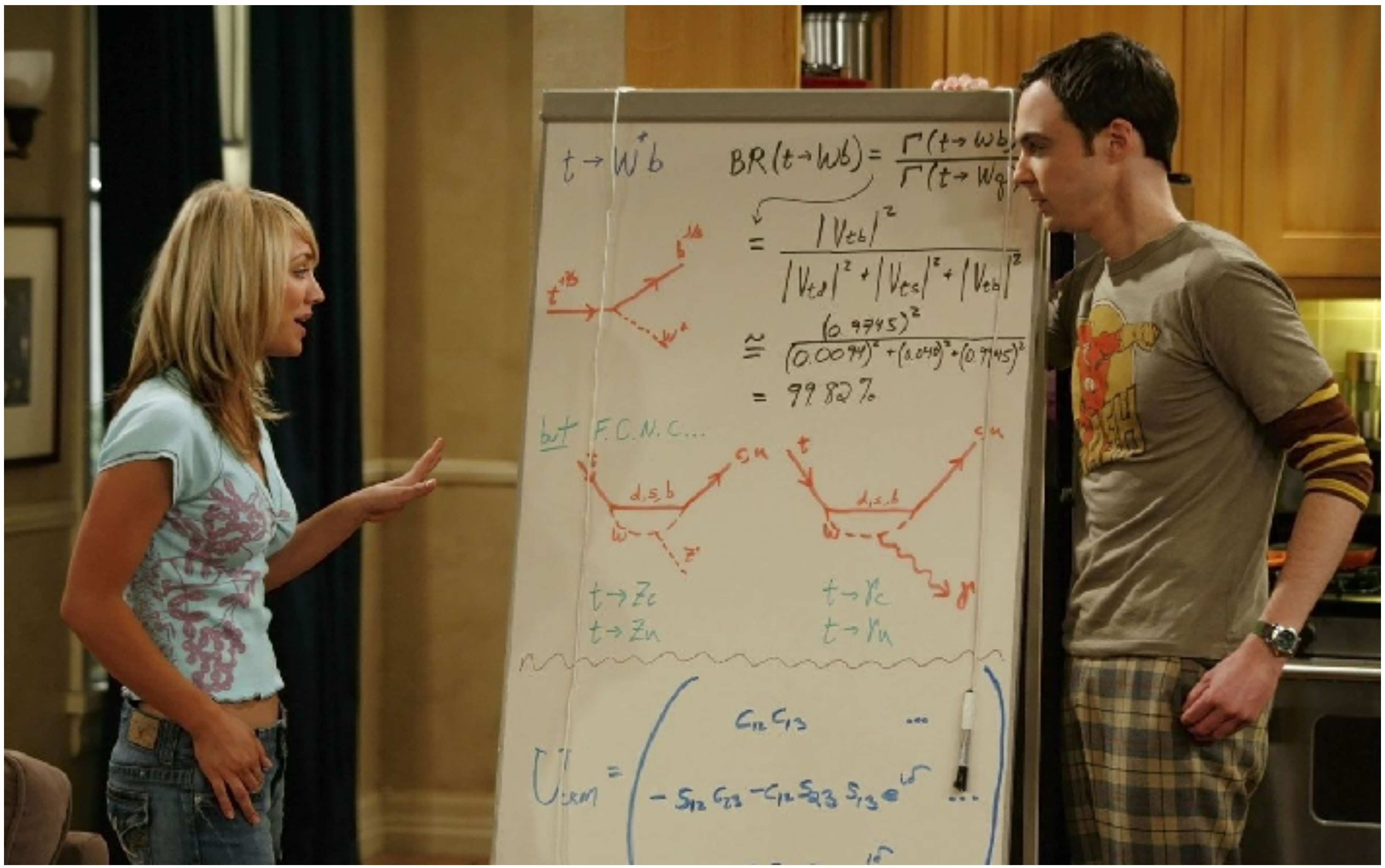
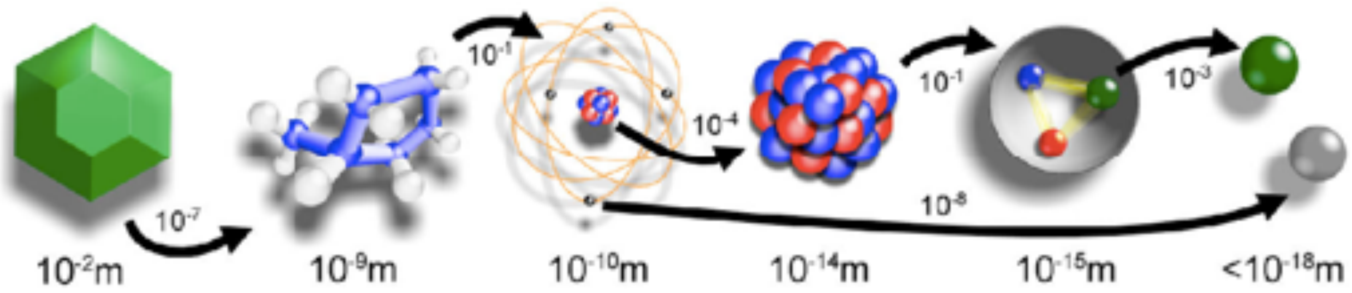
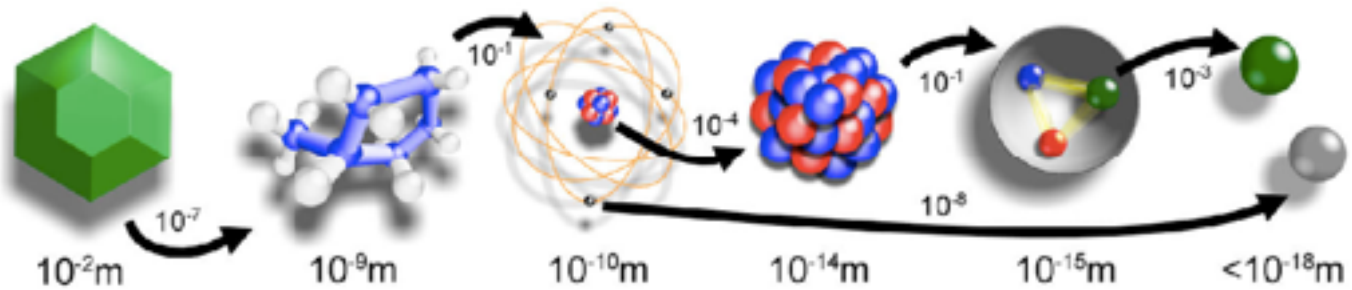
EPISTEMOLOGY LHC

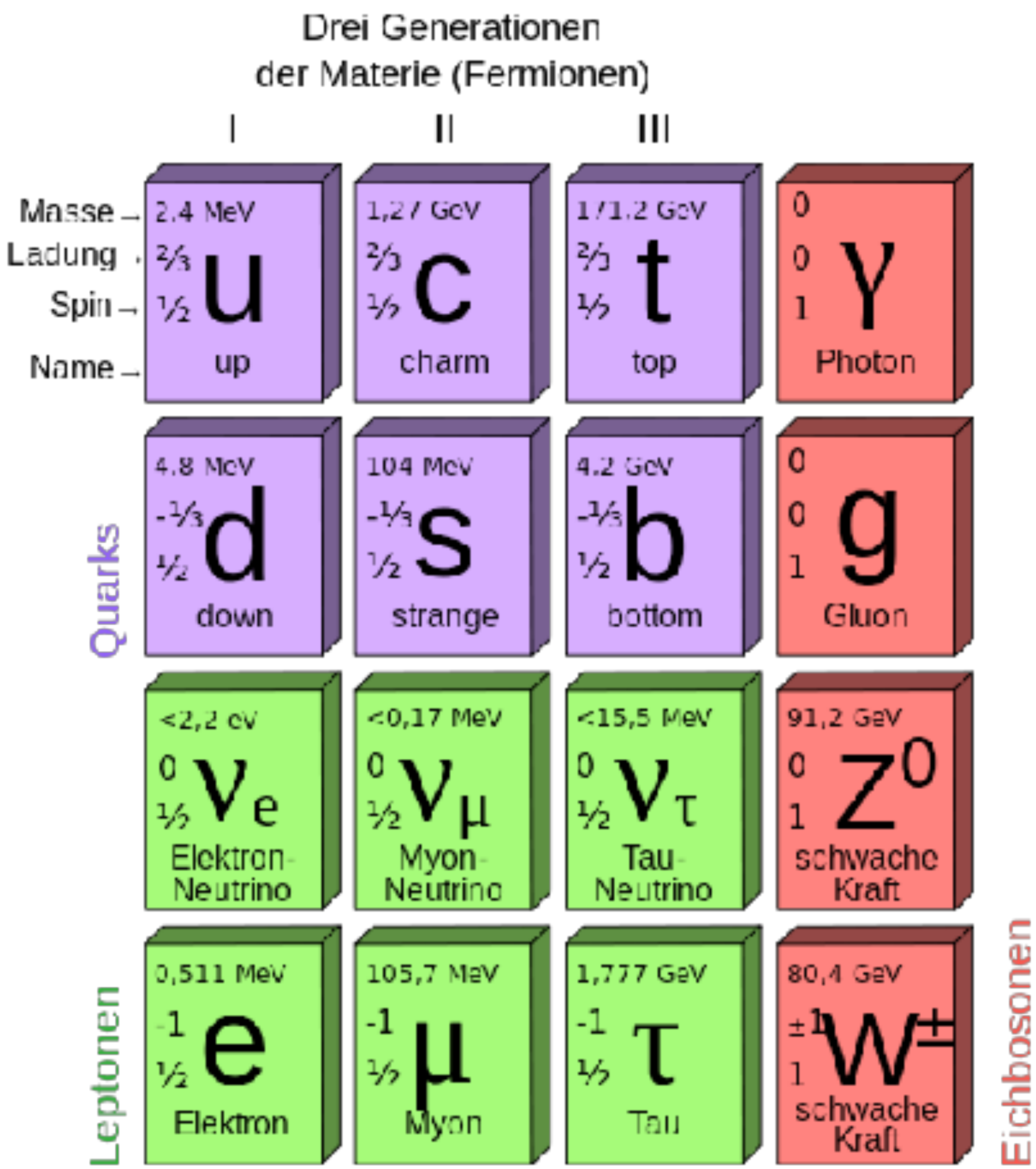
The complexity and simplicity of Feynman diagrams

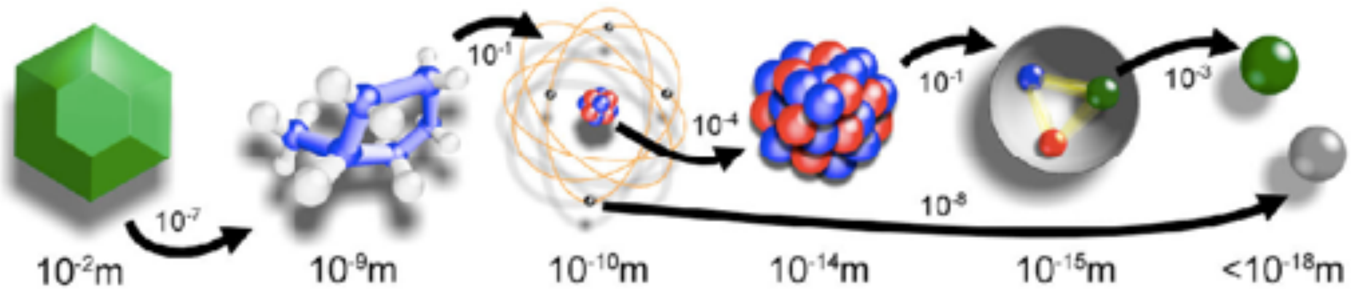


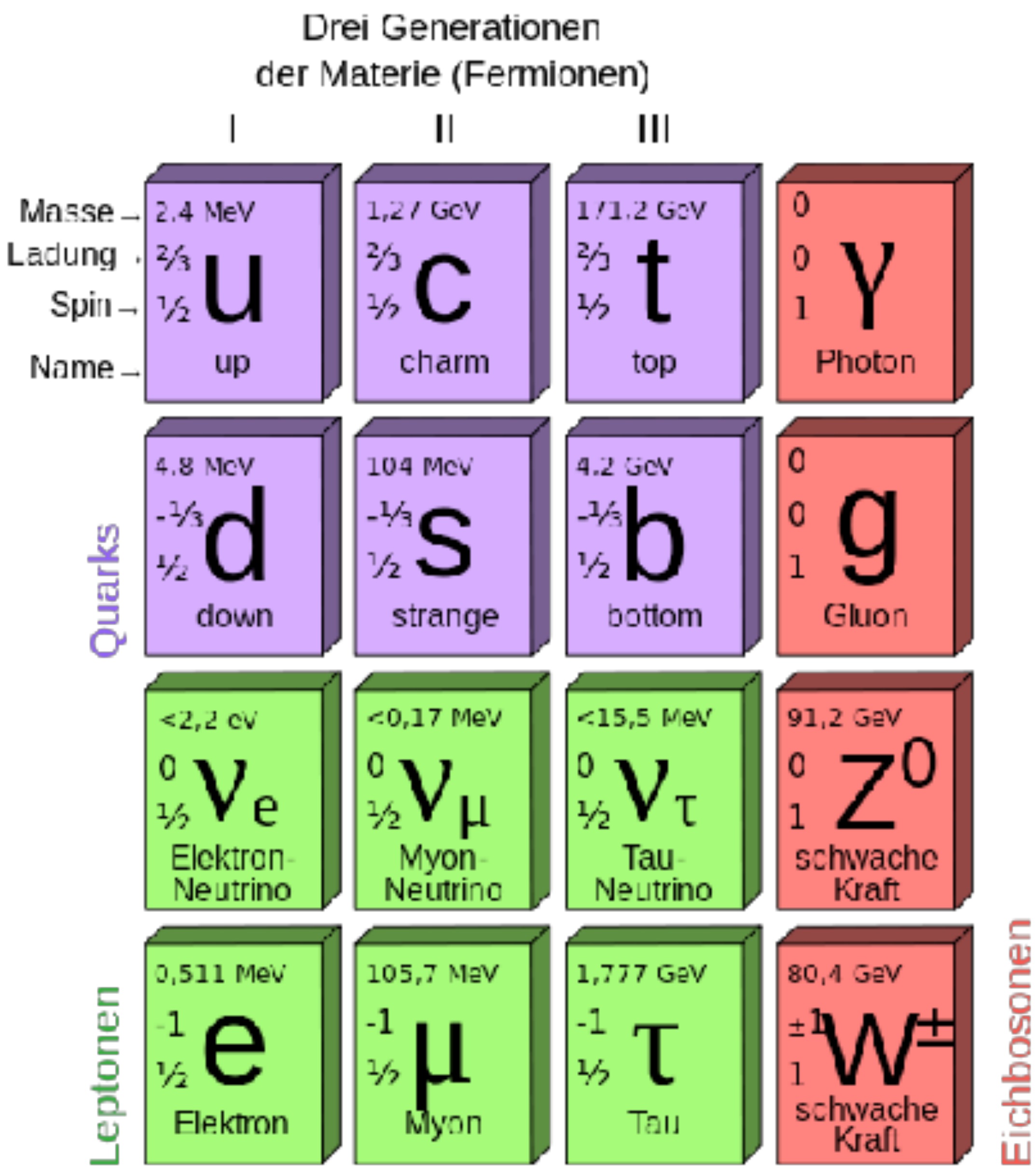


10^{-2}m
10^{-7}
10^{-9}m
10^{-1}
10^{-10}m
10^{-4}
10^{-14}m
10^{-1}
10^{-15}m
10^{-3}
10^{-8}
$<10^{-18}$m

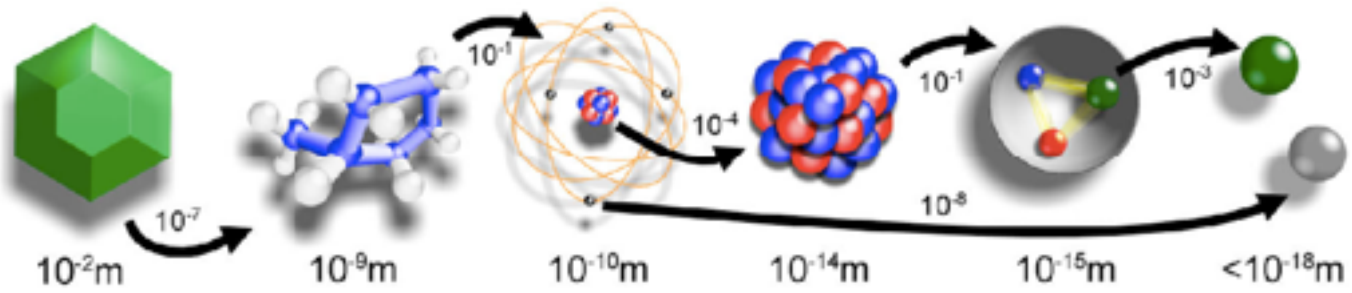

10-2m
10-9m
10-10m
10-14m
10-15m
<10-18m
10-7
10-1
10-4
10-1
10-3
10-8


Drei Generationen
der Materie (Fermionen)
I
II
III
Masse
Ladung
Spin
Name
Quarks
Leptonen
Eichbosonen
2,4 MeV
2/3
1/2
u
up
1,27 GeV
2/3
1/2
c
charm
171,2 GeV
2/3
1/2
t
top
0
0
1
γ
Photon
4,8 MeV
-1/3
1/2
d
down
104 MeV
-1/3
1/2
s
strange
4,2 GeV
-1/3
1/2
b
bottom
0
0
1
g
Gluon
<2,2 eV
0
1/2
νe
Elektron-Neutrino
<0,17 MeV
0
1/2
νμ
Myon-Neutrino
<15,5 MeV
0
1/2
ντ
Tau-Neutrino
91,2 GeV
0
1
Z0
schwache Kraft
0,511 MeV
-1
1/2
e
Elektron
105,7 MeV
-1
1/2
μ
Myon
1,777 GeV
-1
1/2
τ
Tau
80,4 GeV
±1
1
W±
schwache Kraft





$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + i \bar{\psi} \not{D} \psi + h.c. \\
& - \bar{\psi} y \psi \phi + h.c. \\
& + |D_\mu \phi|^2 - V(\phi)
\end{aligned}
$$



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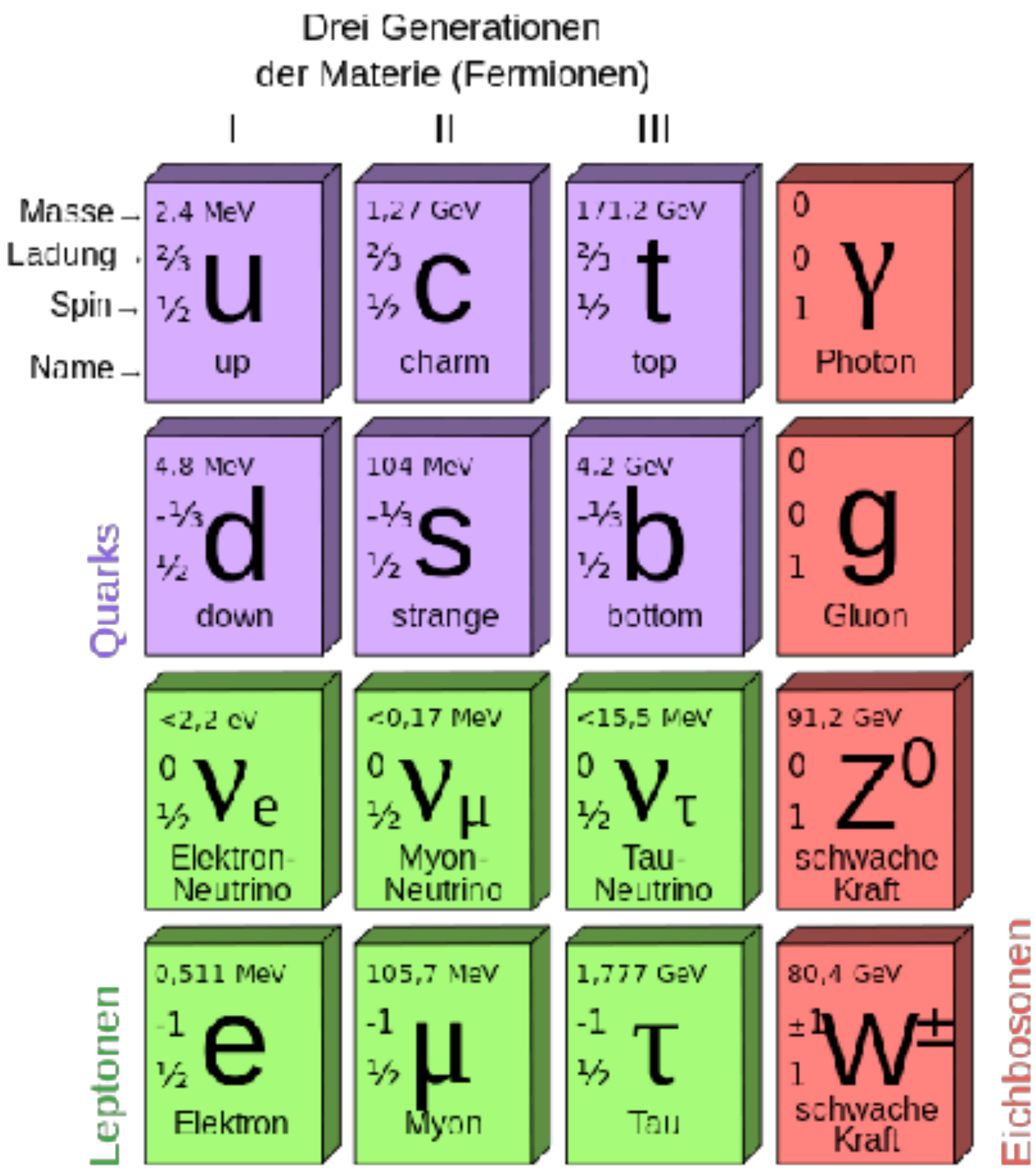
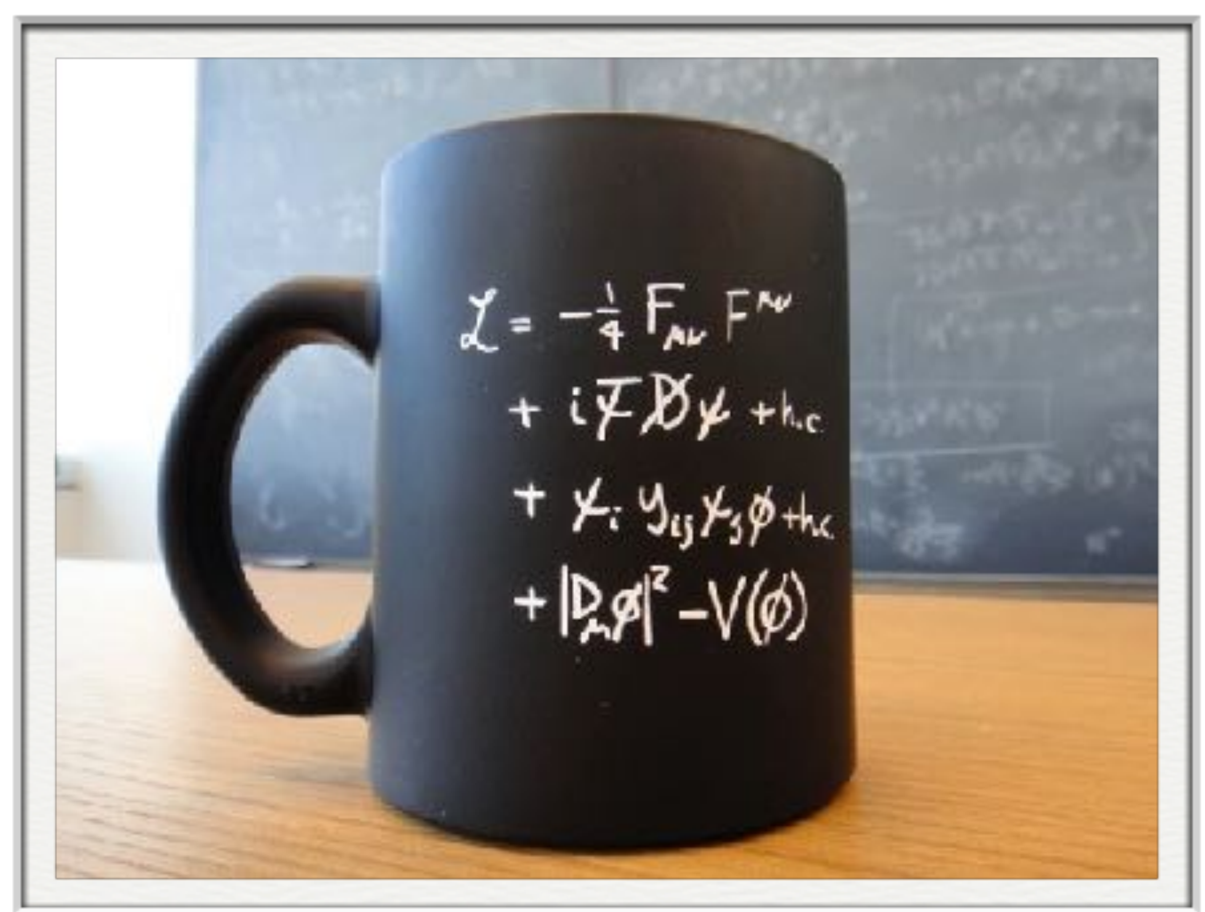
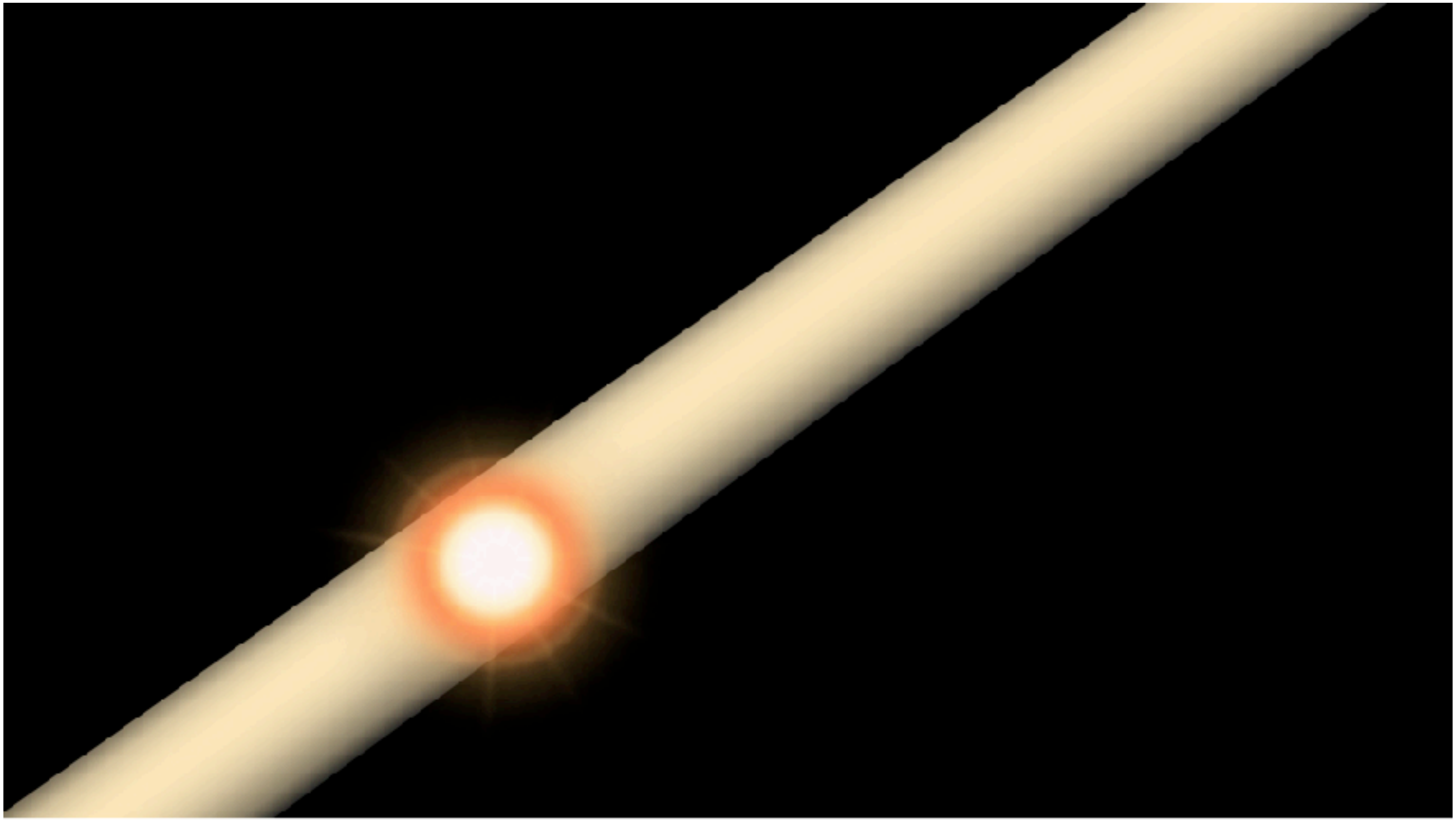
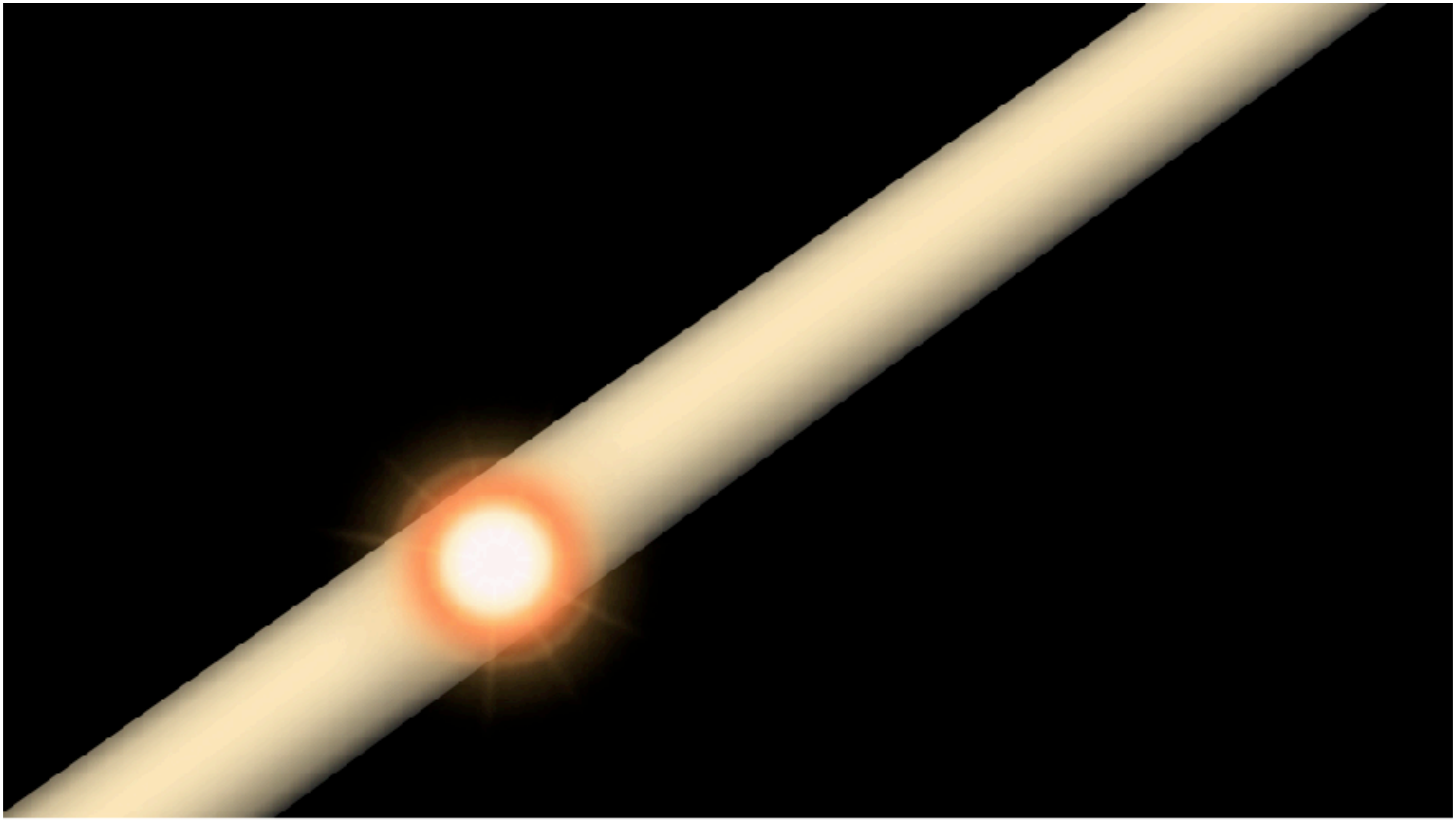
& + i \bar{\psi} \not{D} \psi + h.c. \\
& - \bar{\psi} {\color{red} y} \psi {\color{blue} \phi} + h.c. \\
& + |D_\mu {\color{blue} \phi}|^2 - V({\color{blue} \phi})
\end{aligned}
$$



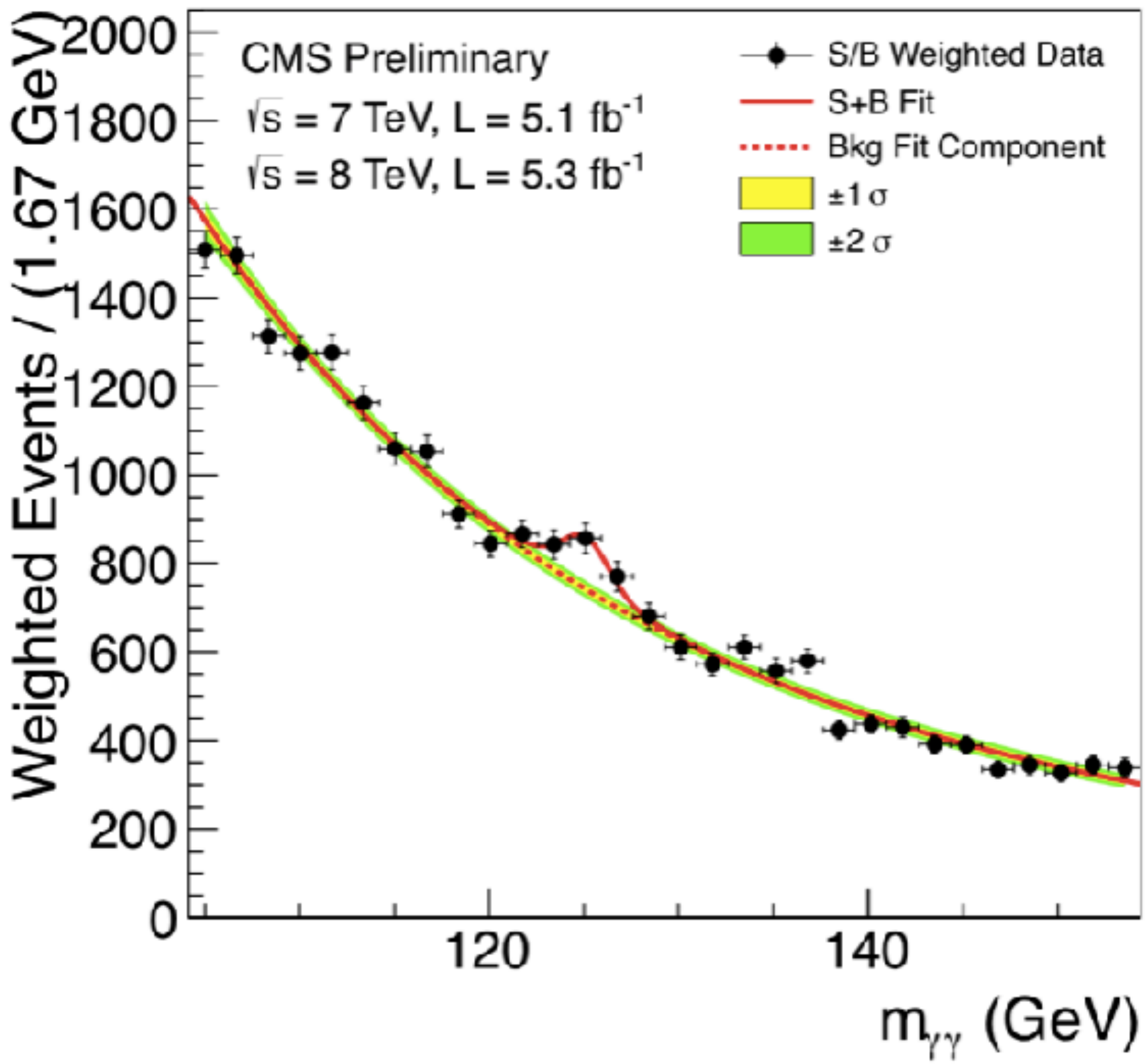


Large Hadron Collider





We are good…


CMS Preliminary
√s = 7 TeV, L = 5.1 fb^-1
√s = 8 TeV, L = 5.3 fb^-1
S/B Weighted Data
S+B Fit
Bkg Fit Component
±1 σ
±2 σ
Weighted Events / (1.67 GeV)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
120
140
m_γγ (GeV)

Electron (g-2)

Electron in magnetic field: $E = -\vec{\mu} \cdot \vec{B}$

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

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$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Dirac (1928): g = 2

Electron (g-2)

Electron in magnetic field: $E = -\vec{\mu} \cdot \vec{B}$

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Dirac (1928): g = 2

Schwinger (1948): g = 2.002

Electron (g-2)

Electron in magnetic field: $E = -\vec{\mu} \cdot \vec{B}$

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

Dirac (1928): g = 2

Schwinger (1948): g = 2.002

theory: g = 2.002319304363…

Electron (g-2)

Electron in magnetic field: $E = -\vec{\mu} \cdot \vec{B}$

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Dirac (1928): g = 2

Schwinger (1948): g = 2.002

theory: g = 2.0023193043633…

experiment: g = 2.0023193043611…

Model building

What is Dark Matter?
What is Dark Energy?
Naturalness?

Model building

What is Dark Matter?
What is Dark Energy?
Naturalness?

e.g. Technicolor

New interaction instead of Higgs

Model building

What is Dark Matter?
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New interaction instead of Higgs

Conflict with experiment.

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Model building

What is Dark Matter?
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Naturalness?

e.g. ~~Technicolor~~

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Extended Technicolor

Model building

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New interaction instead of Higgs

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~~Extended Technicolor~~
Walking Technicolor

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~~Extended Technicolor~~
~~Walking Technicolor~~
Topcolor

Model building

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~~Extended Technicolor~~
~~Walking Technicolor~~
~~Topcolor~~
Topcolor-assisted walking Technicolor

Model building

What is Dark Matter?
What is Dark Energy?
Naturalness?

e.g. ~~Technicolor~~

New interaction instead of Higgs

Conflict with experiment.

~~Extended Technicolor~~
~~Walking Technicolor~~
~~Topcolor~~
~~Topcolor-assisted walking Technicolor~~

Theoretical Particle Physics is complex.

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Classical Mechanics (i.e. pre-QM)

t=0: Particle at x with momentum p.

Question: where is the particle at t > 0?

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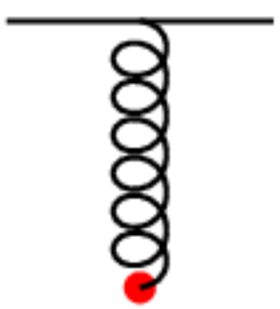

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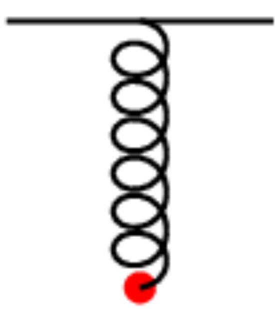

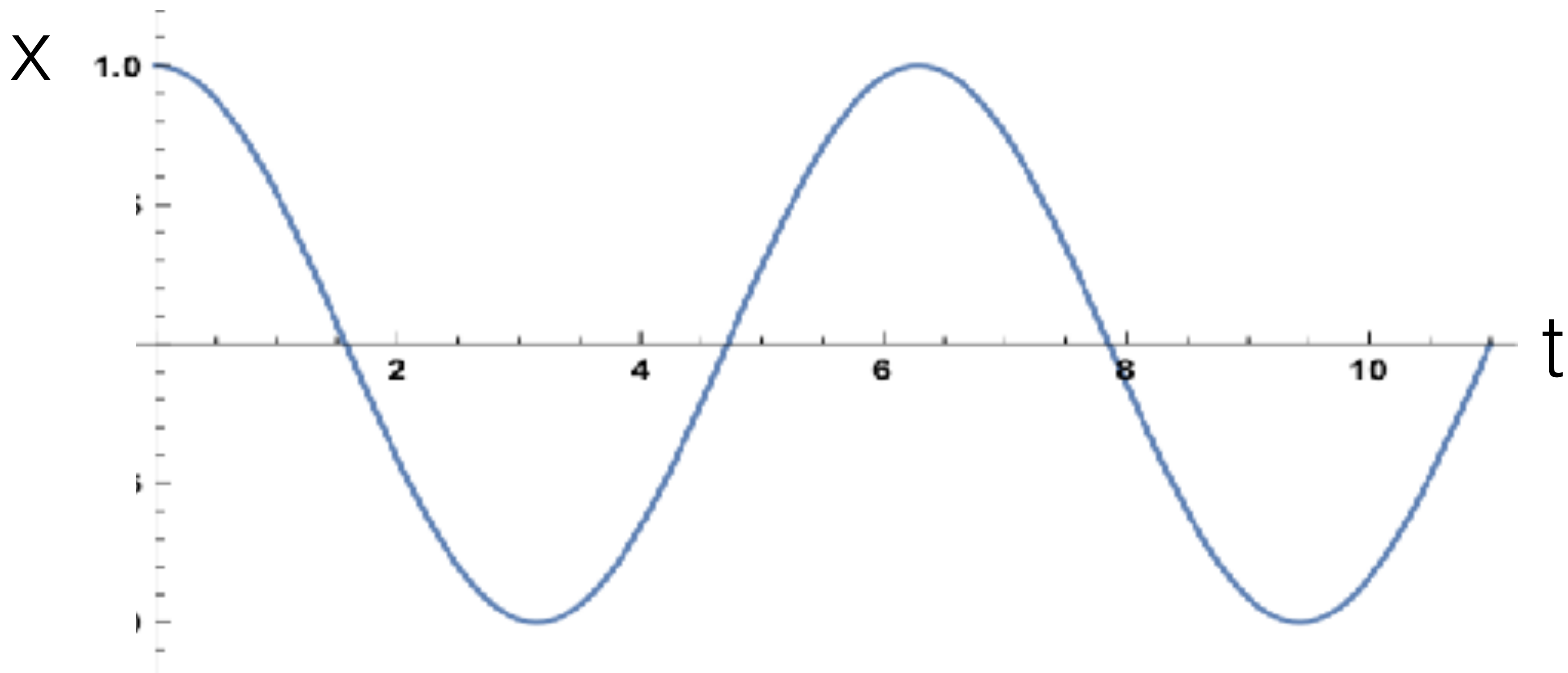

Classical Mechanics (i.e. pre-QM)

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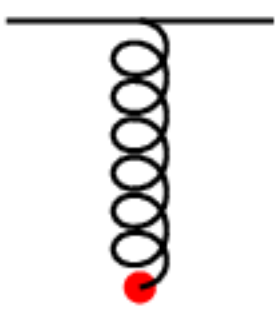
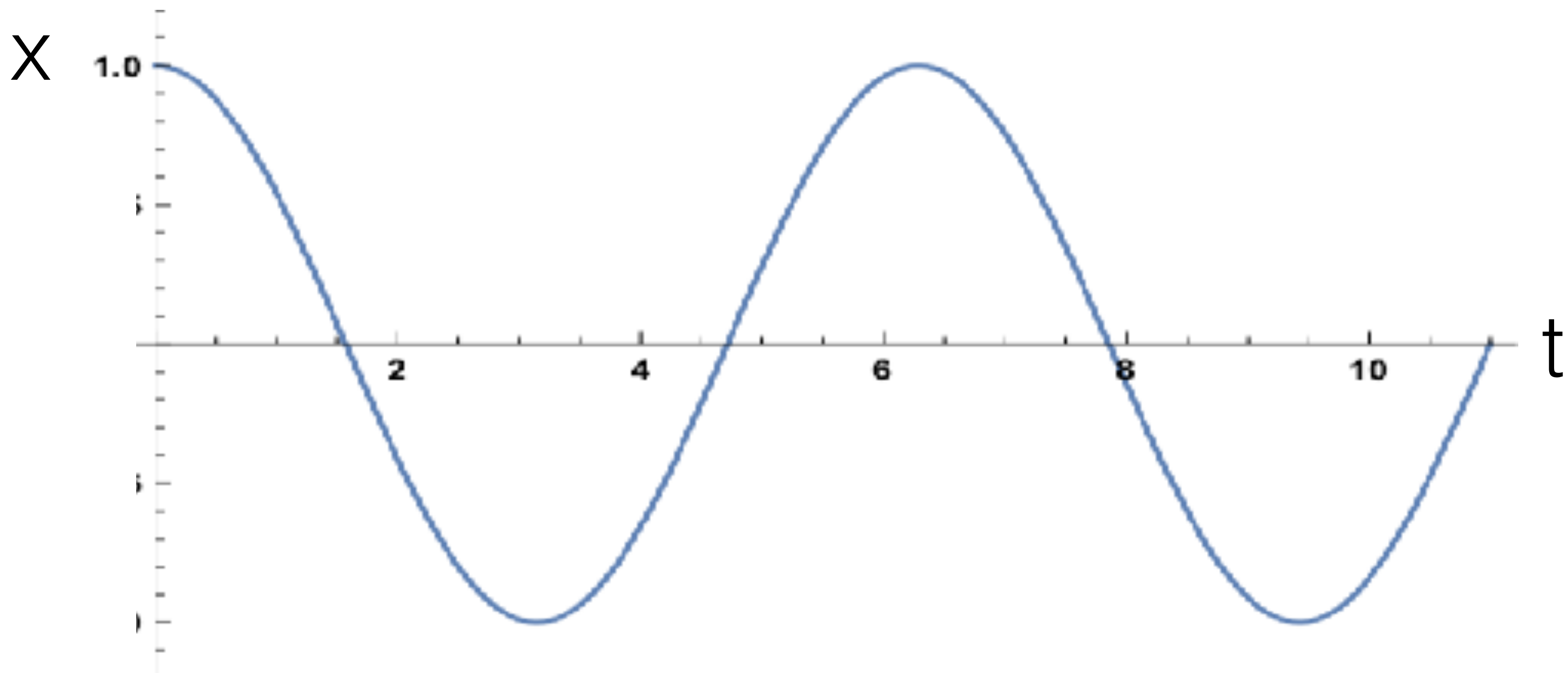
Question: where is the particle at t > 0?

$$\vec{F} = m\vec{a} = \frac{d}{dt}\vec{p}$$





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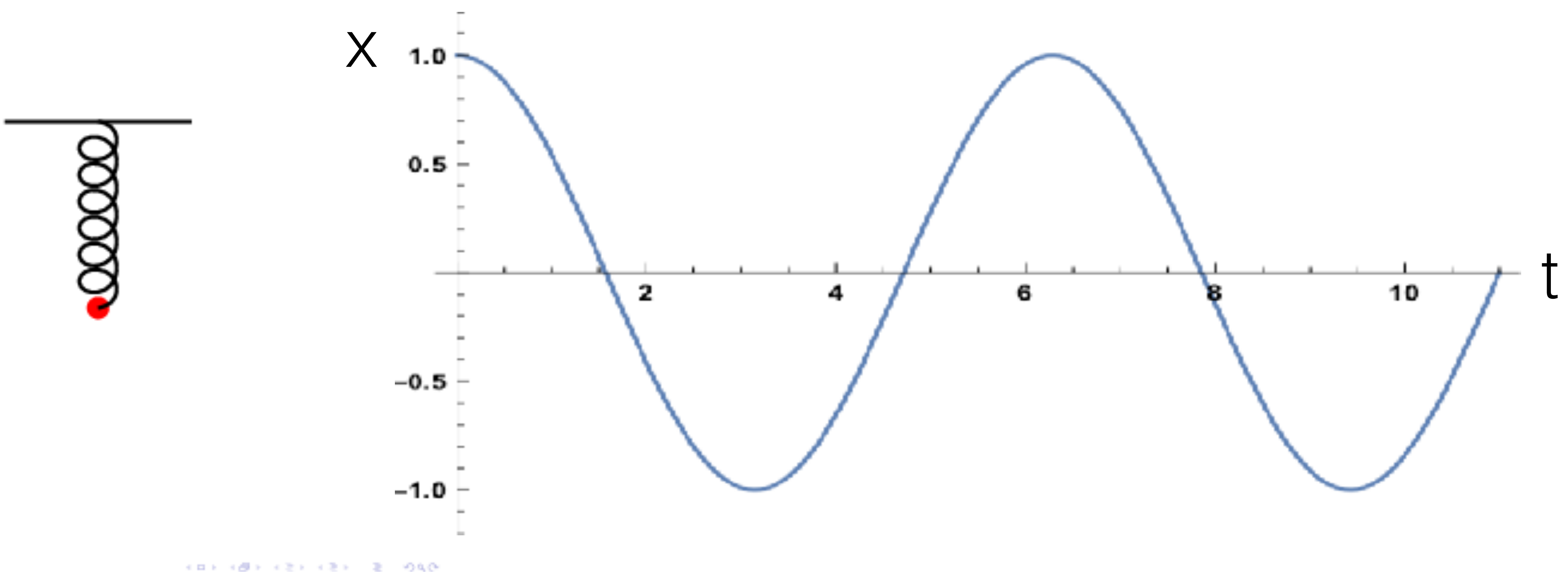
Least-action principle



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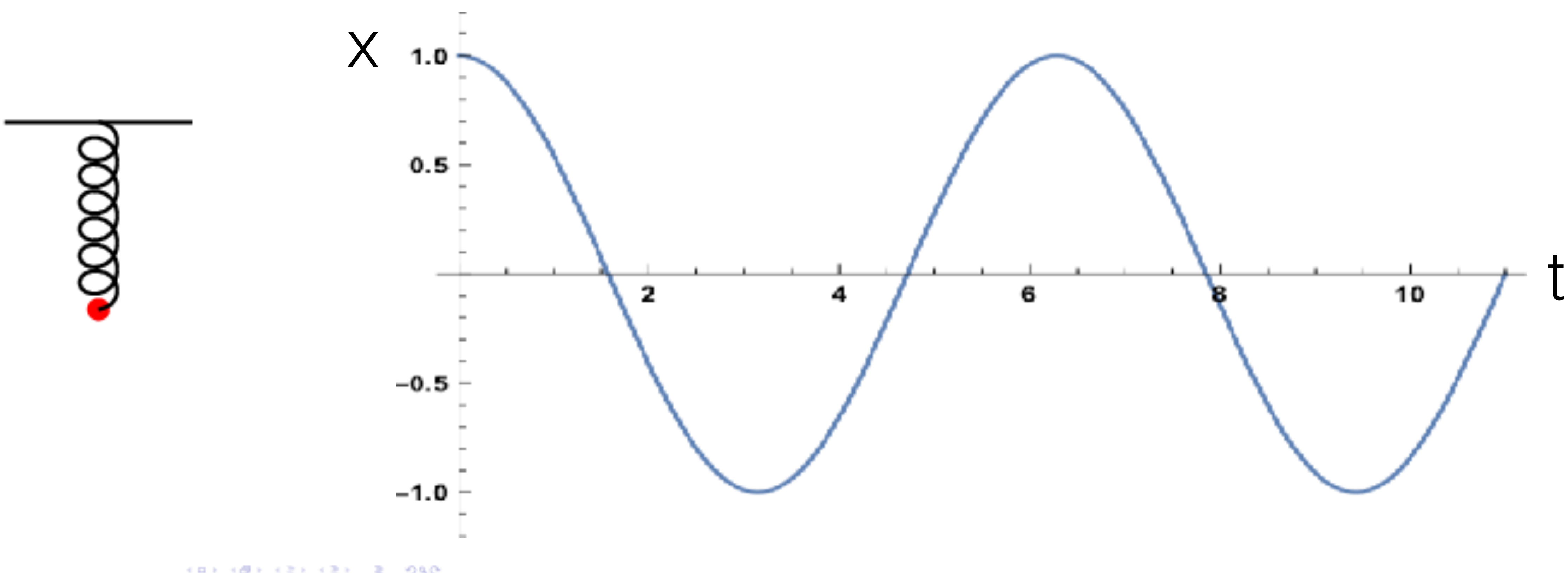
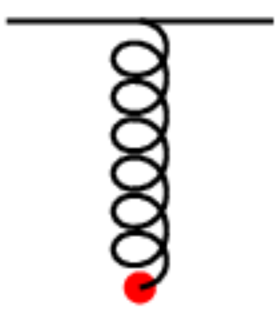
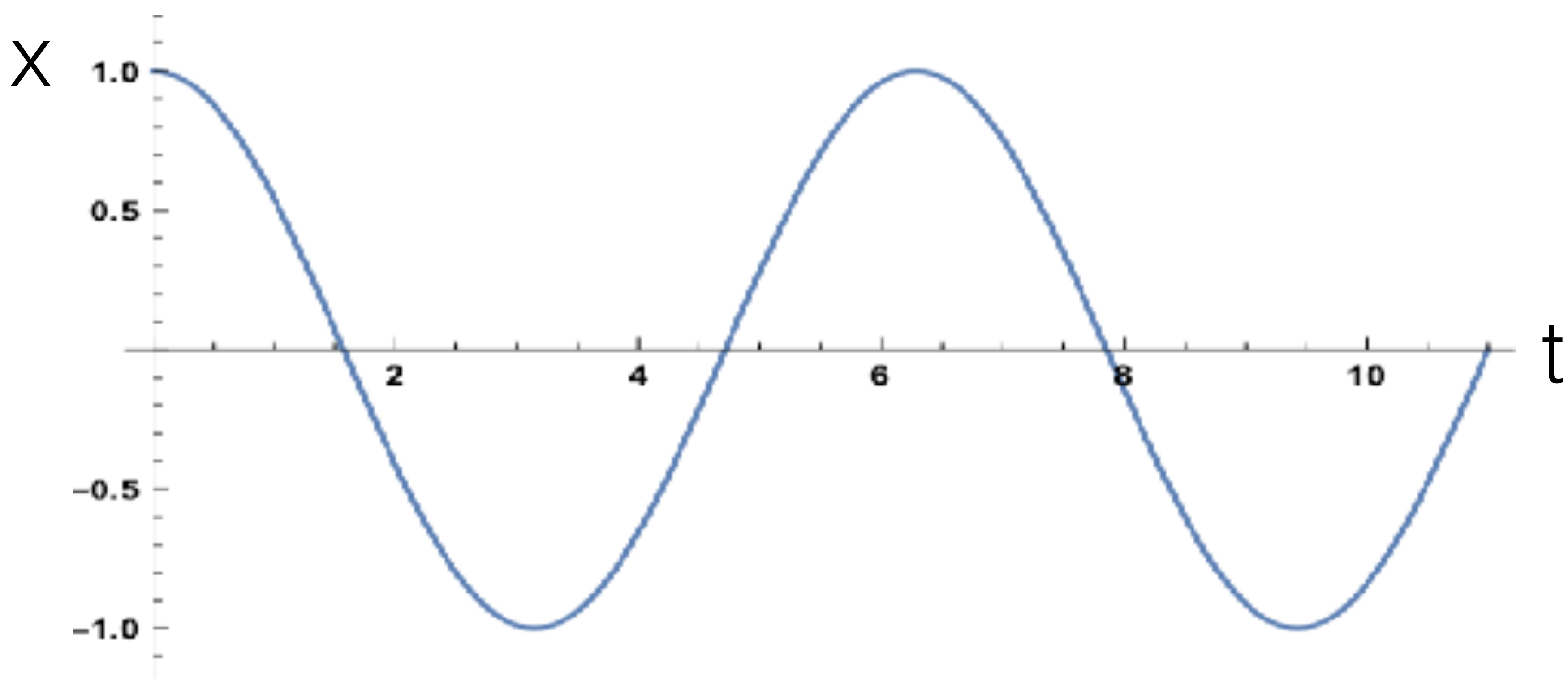


Least-action principle

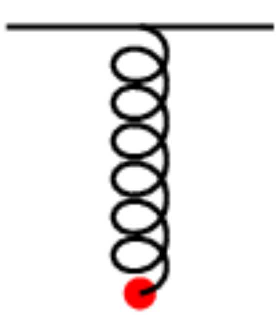
$L(t) = E_{kin}(t) - E_{pot}(t)$

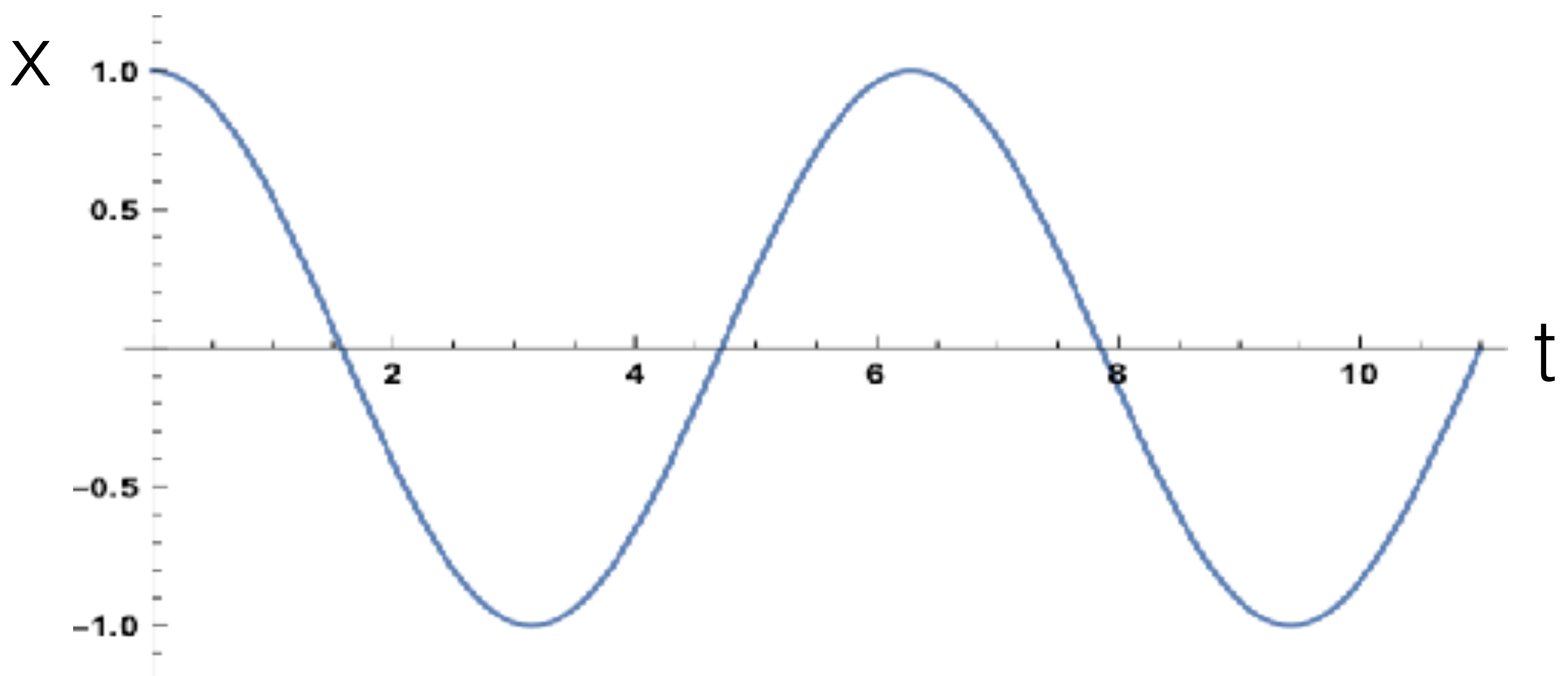




Least-action principle

$$L(t) = E_{kin}(t) - E_{pot}(t) = \frac{1}{2}mv^2(t) - \frac{1}{2}kx^2(t)$$

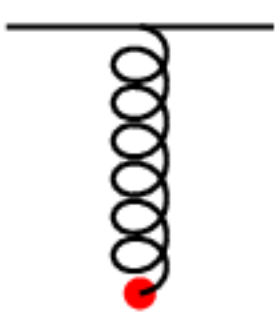


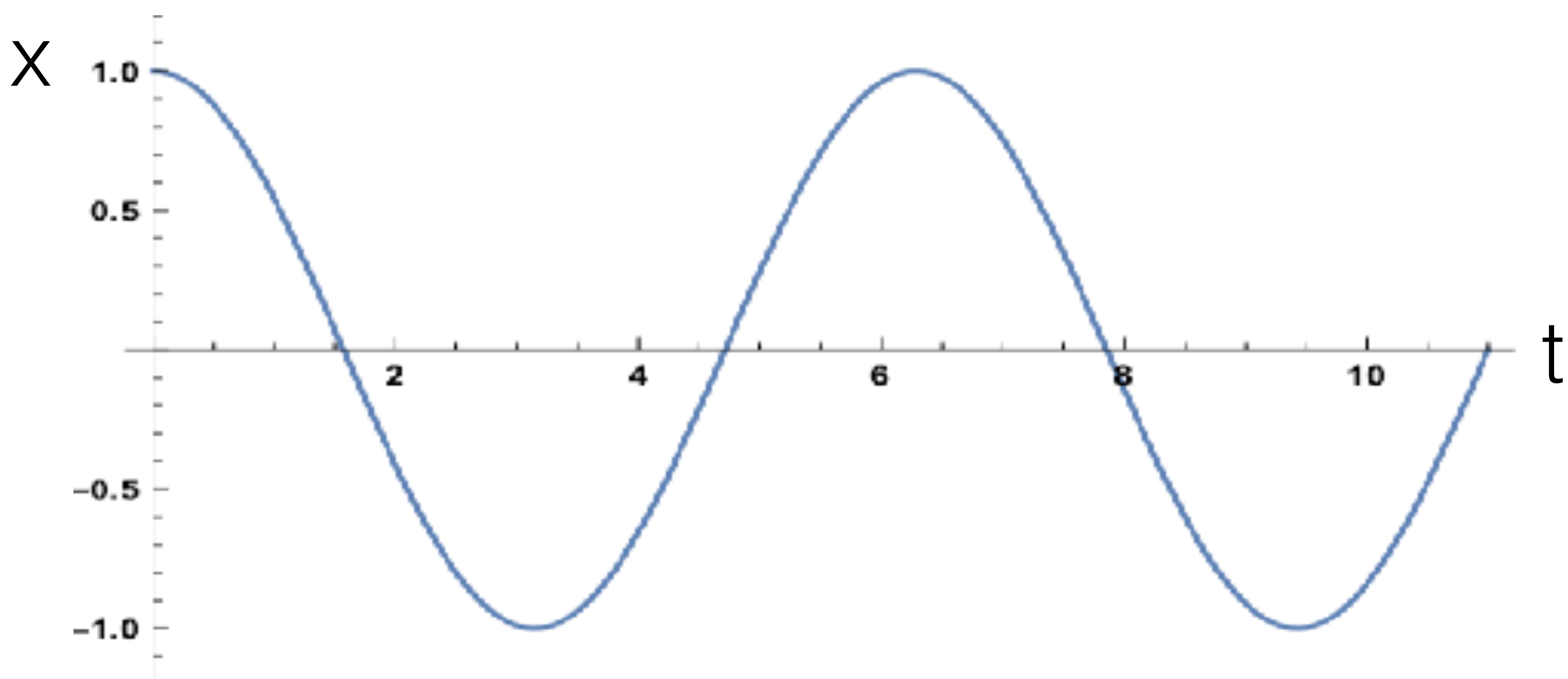


Least-action principle

$$L(t) = E_{kin}(t) - E_{pot}(t) \;= \frac{1}{2}mv^2(t) - \frac{1}{2}kx^2(t)$$

action: $$S[\text{path}] = \int dt\, L(t)$$





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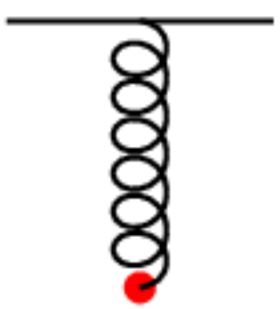

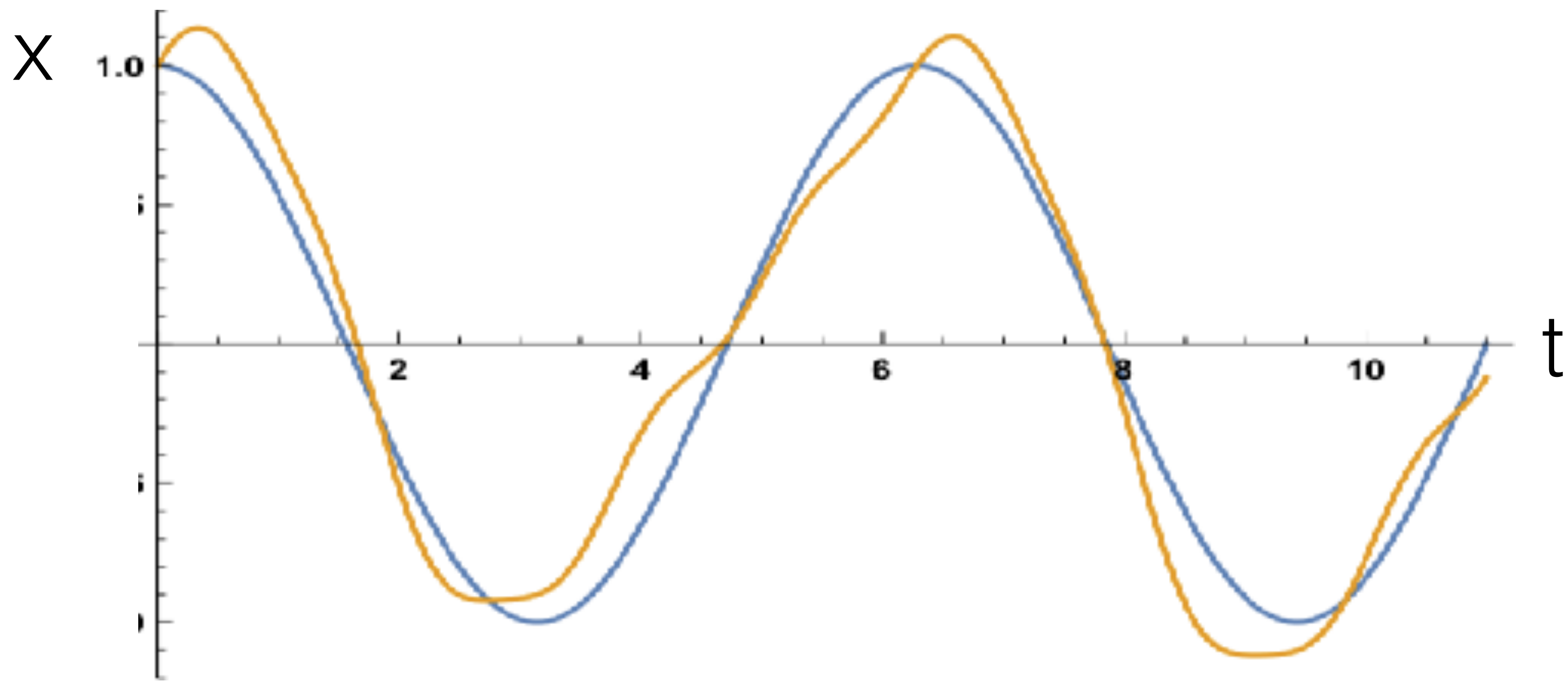

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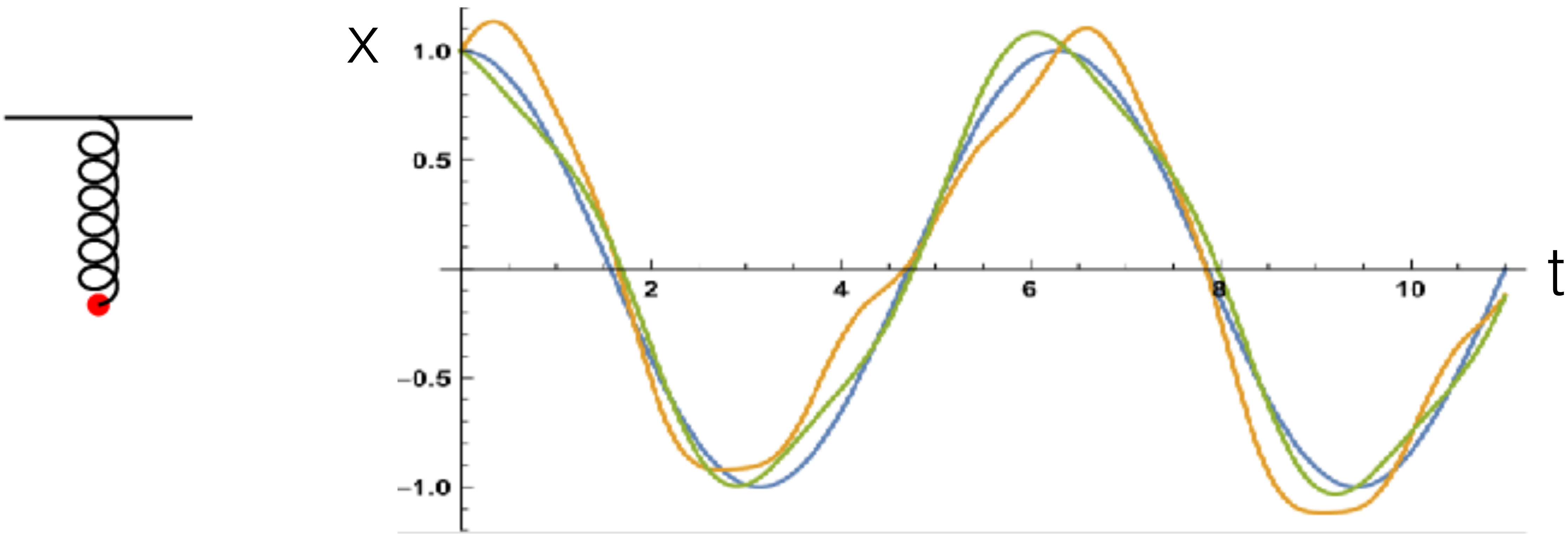

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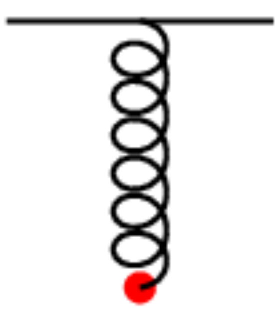
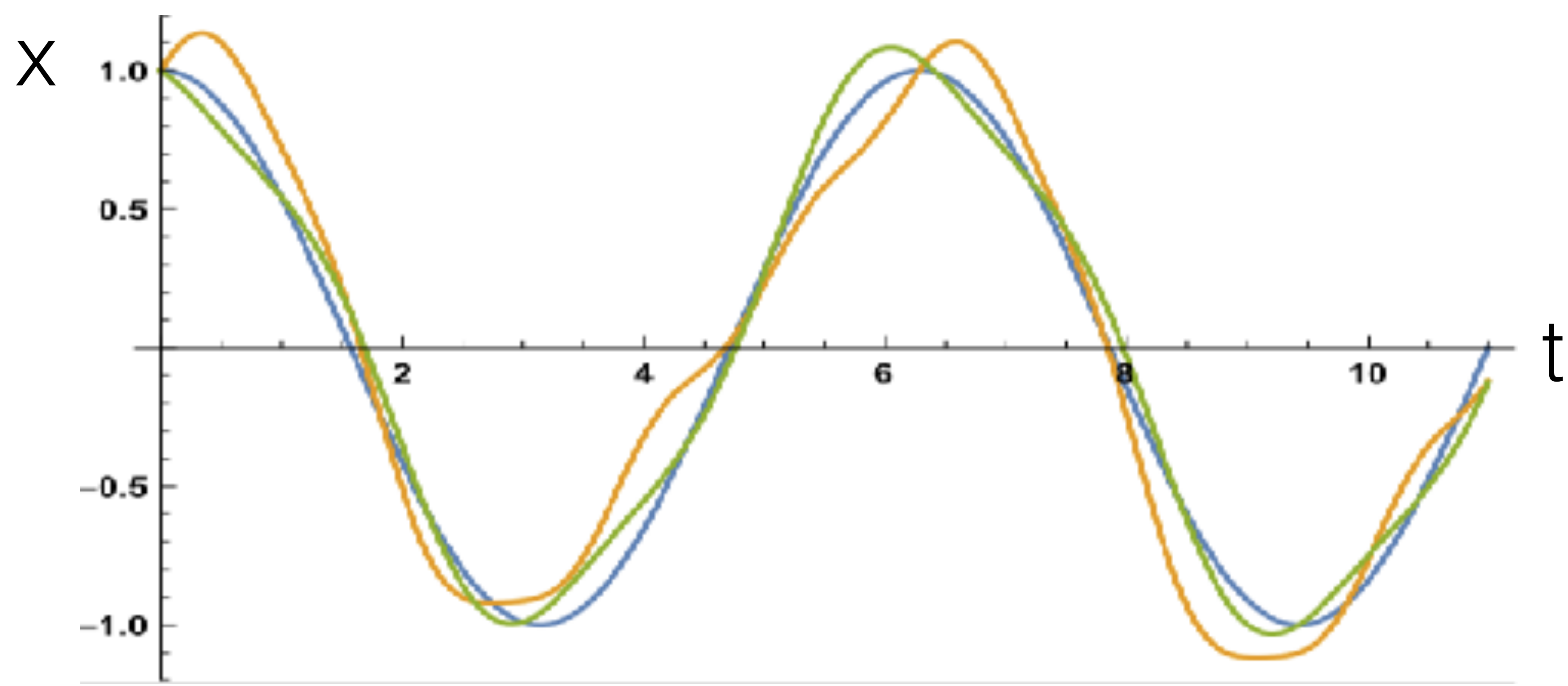
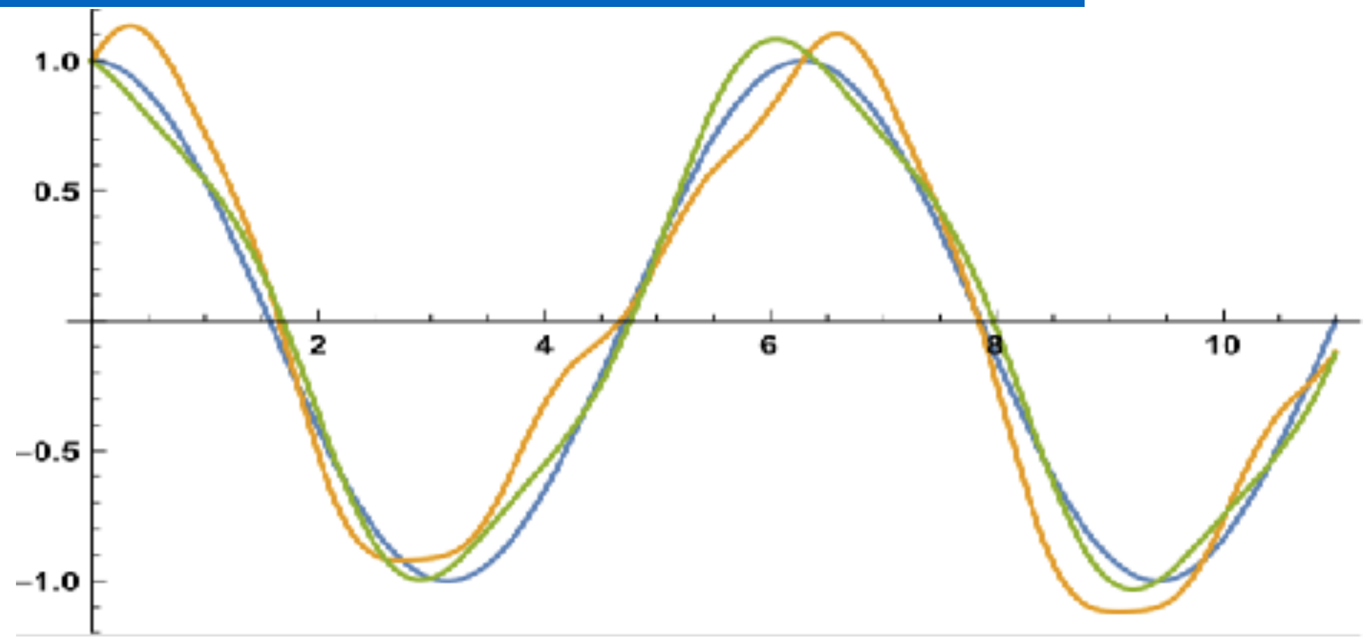
action: $$S[\text{path}] = \int dt\, L(t) \quad \Rightarrow \quad \vec{F} = m\,\vec{a}$$





Quantum Mechanics



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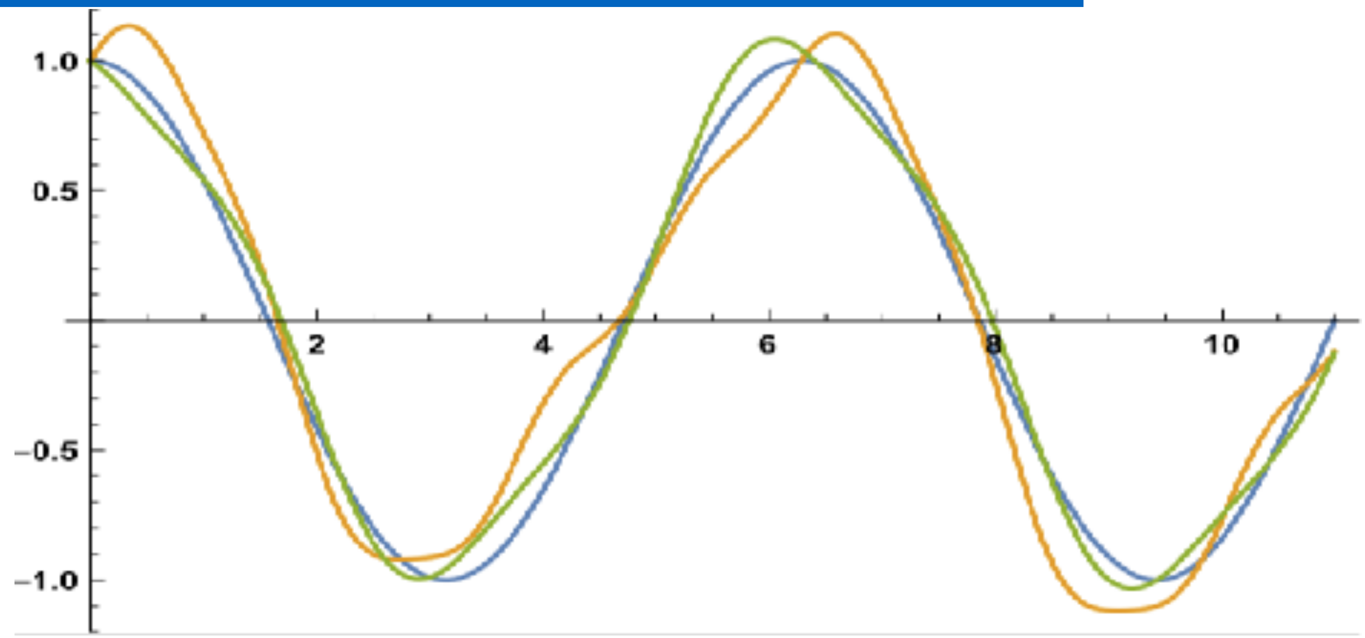

Each path is weighted by exp(iS[path]).

$$S[\text{path}] = \int dt\, L(t) \in \mathbb{R}$$

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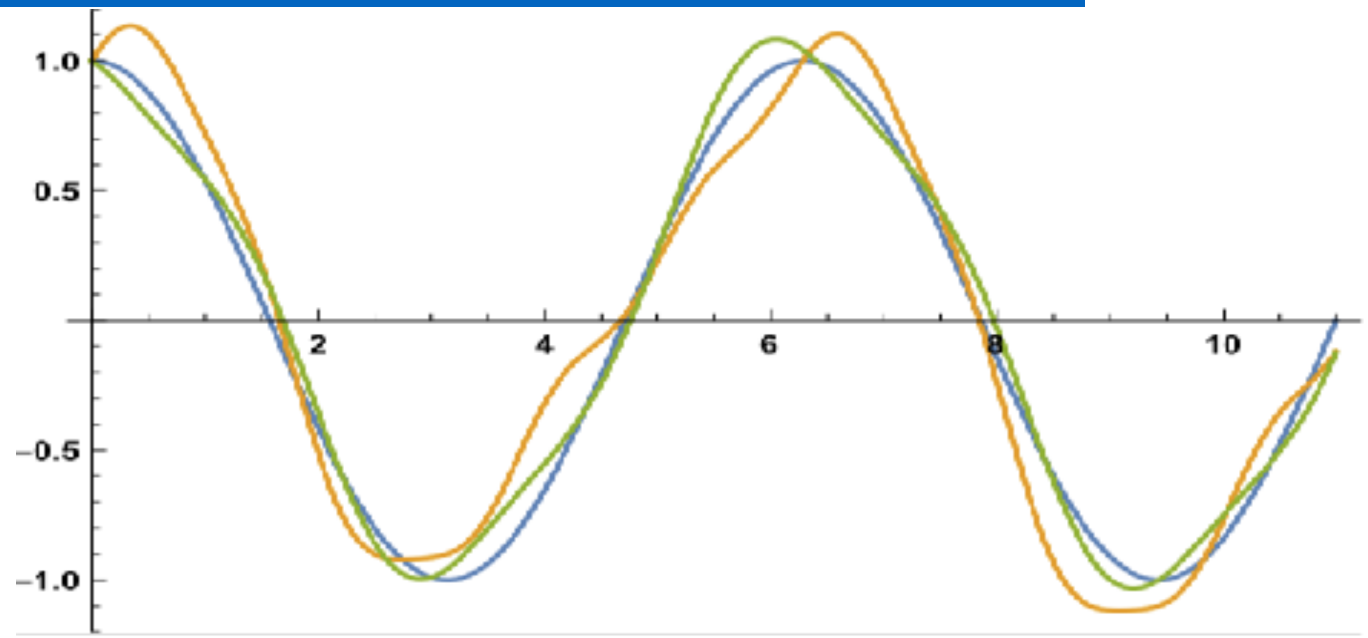
Quantum Mechanics



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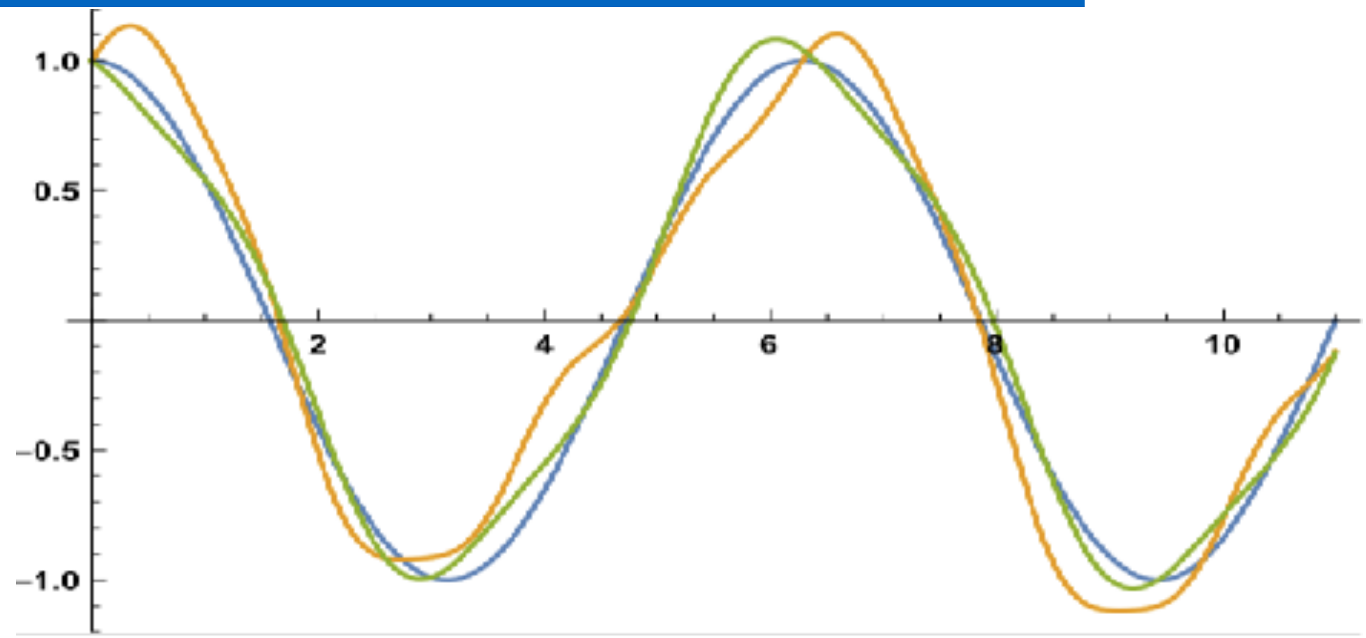
$$A(x,0;x',t) = \int_{P:\, x(0)\to x'(t)} dP\, e^{iS[P]} \in \mathbb{C}$$

probability amplitude

"Path integral"

Quantum Mechanics



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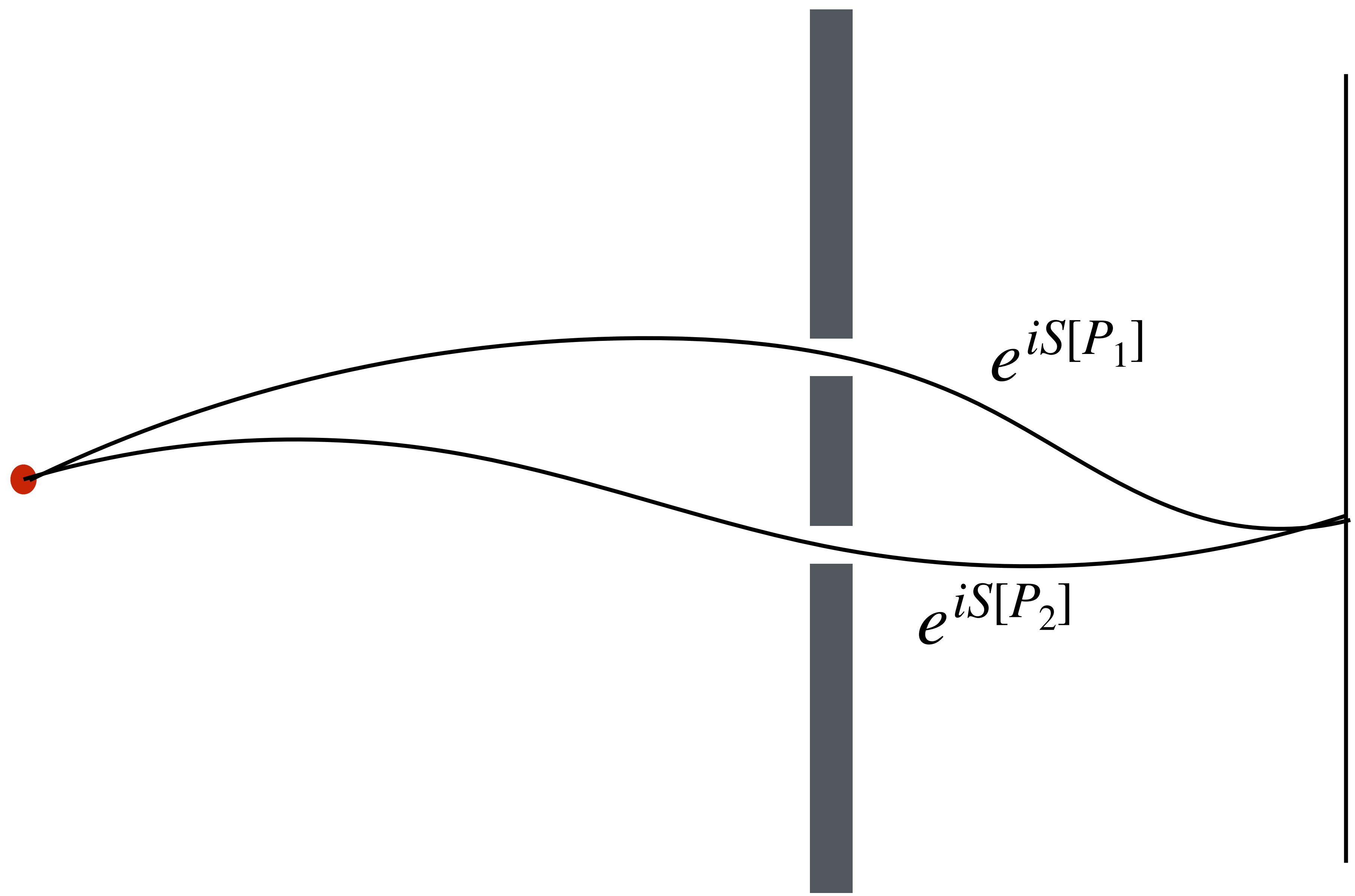
probability amplitude

"Path integral"

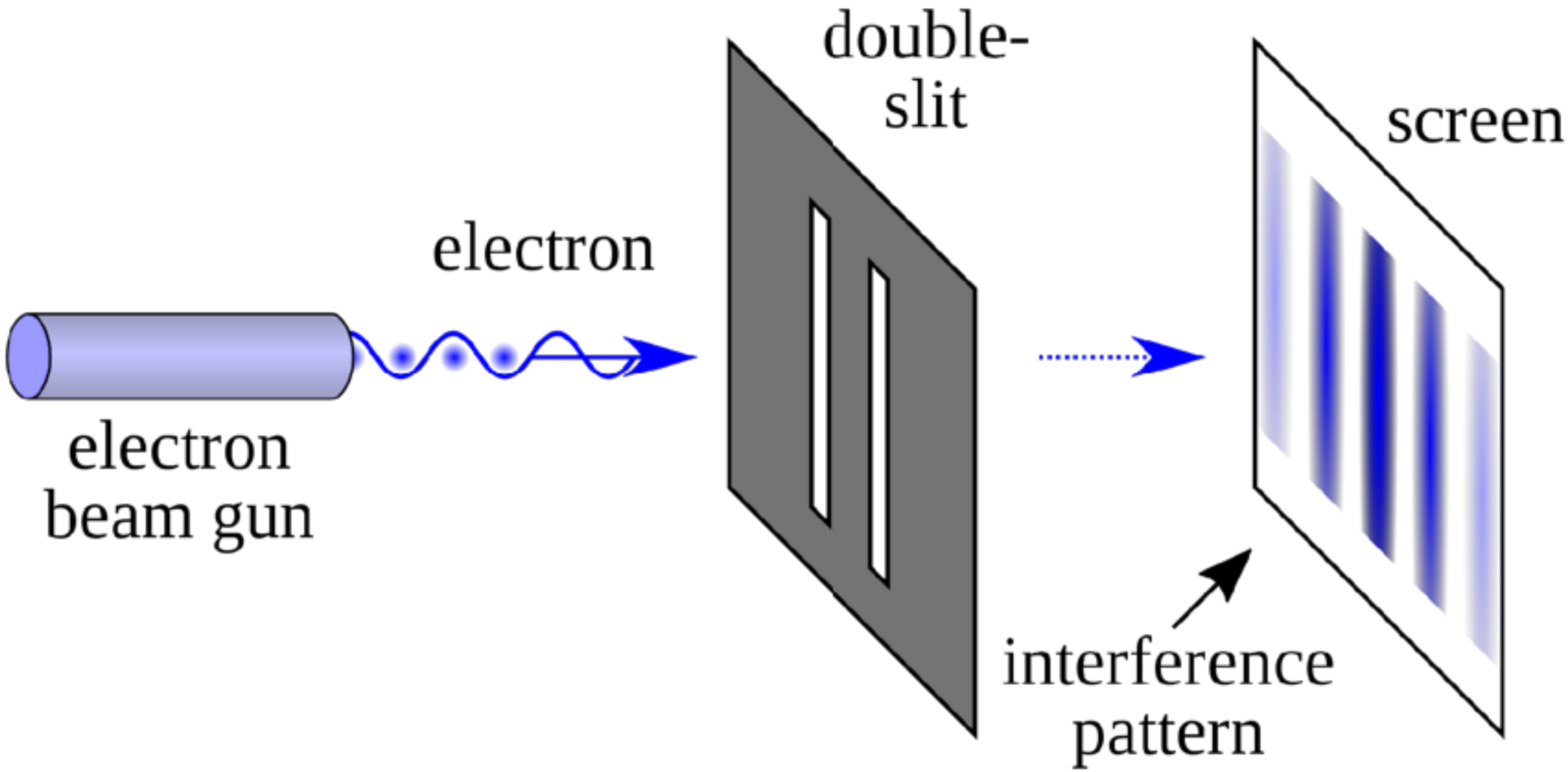
probability: $\left|A(x,0;x',t)\right|^2$

Double slit experiment



probability: $$\left| e^{iS[P_1]} + e^{iS[P_2]} \right|^2$$

Double slit experiment



source: https://en.wikipedia.org/wiki/Double-slit_experiment

Quantum Mechanics

Recall classical mechanics:

t=0: Particle at x with momentum p.

Question: where is the particle at t > 0?

Quantum Mechanics

Recall classical mechanics:

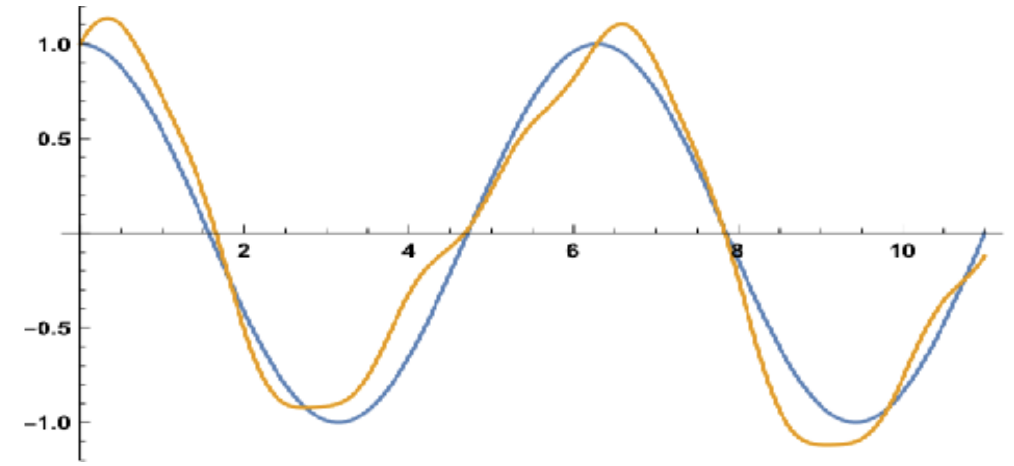
t=0: Particle at x with momentum p.

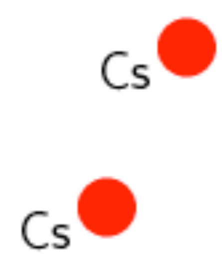
Question: where is the particle at t > 0?

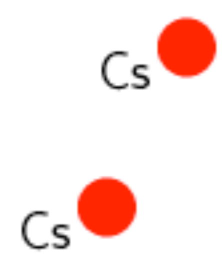
Quantum Mechanics:

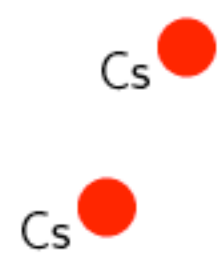
t=0: Particle at x with momentum p.

Question: what is the probability to find particle at x' at t > 0?




Cs
Cs


Cs
Cs

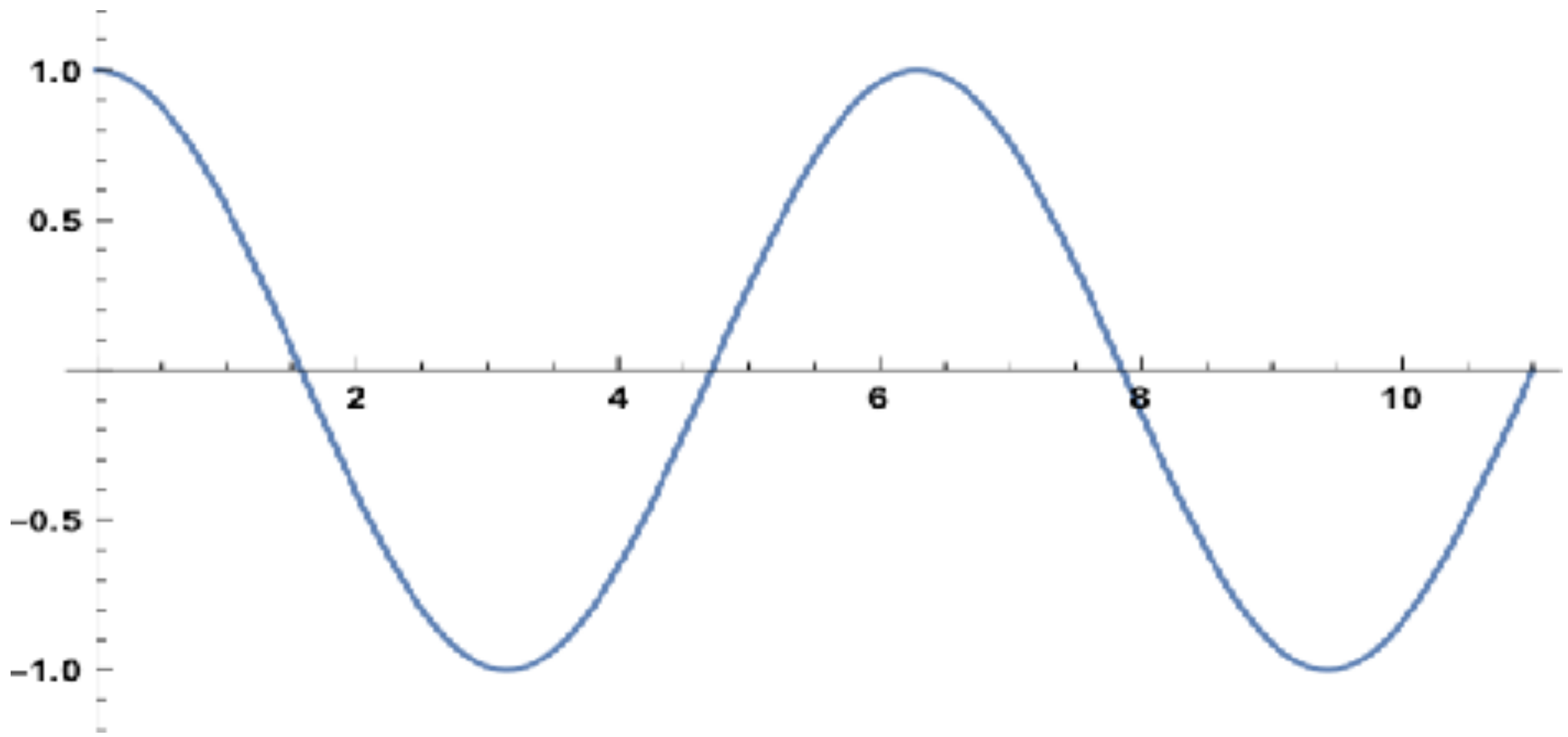

Cs
Cs

Wave function

Heisenberg uncertainty principle: $(\Delta x)(\Delta p) > \hbar/2$

Classical state cannot be sharply defined!

ψ(x,t=0)



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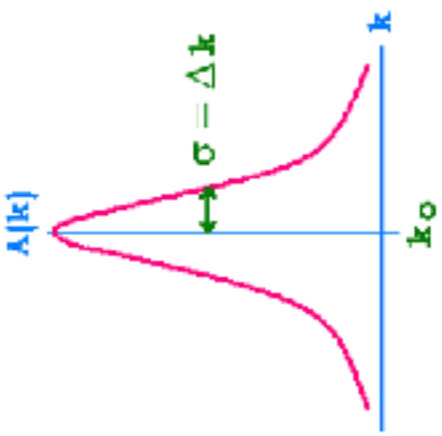

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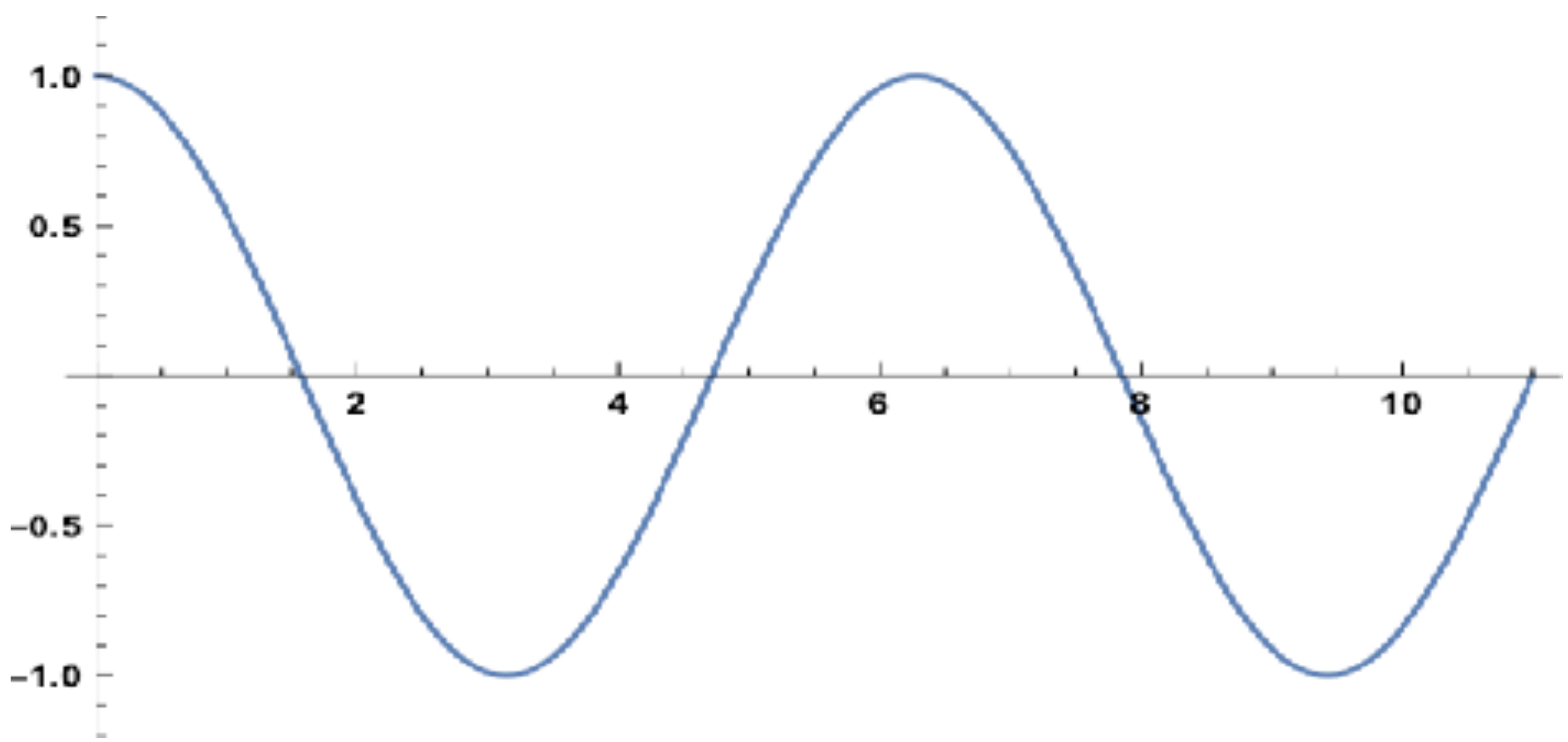

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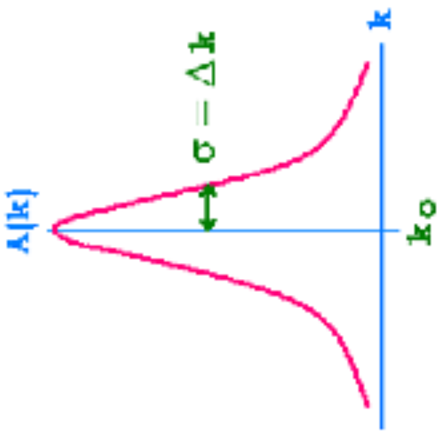

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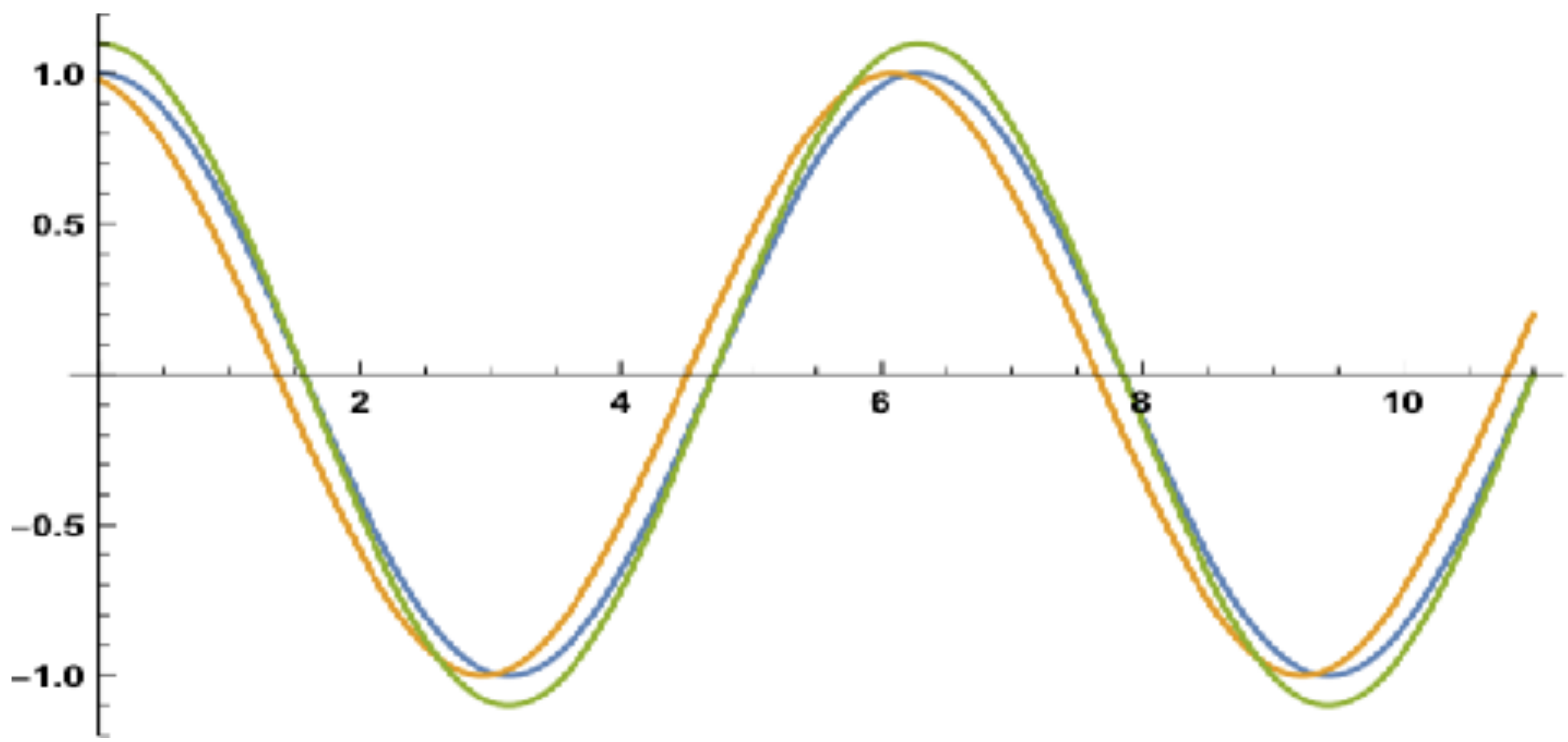

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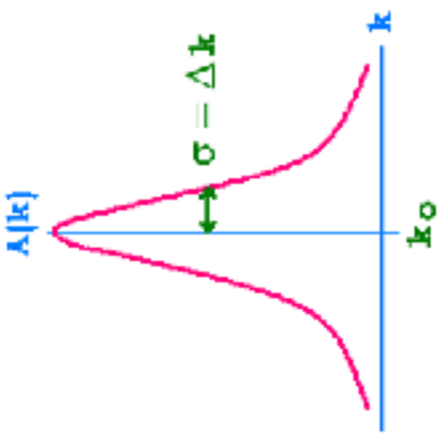
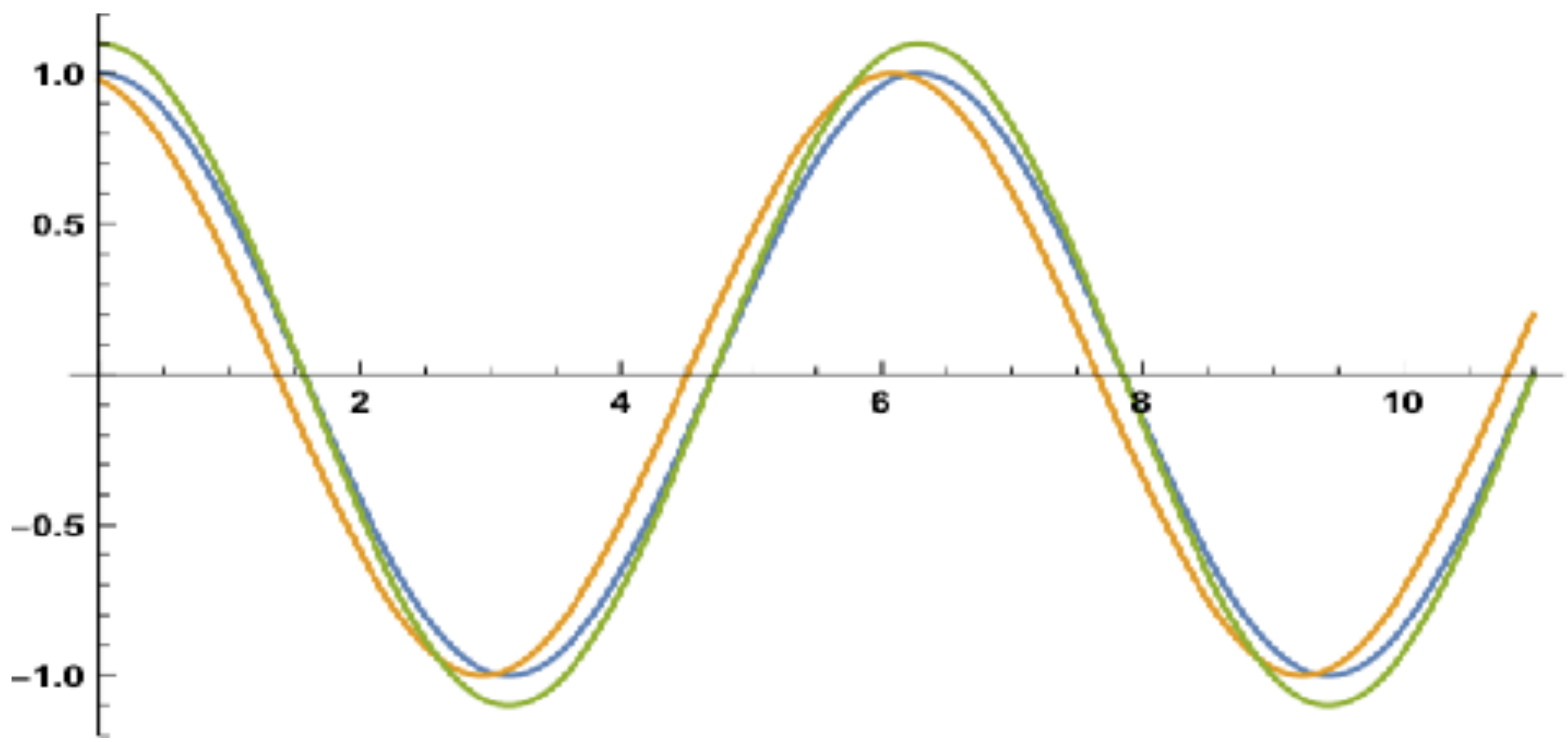


$\psi(x,t=0)$

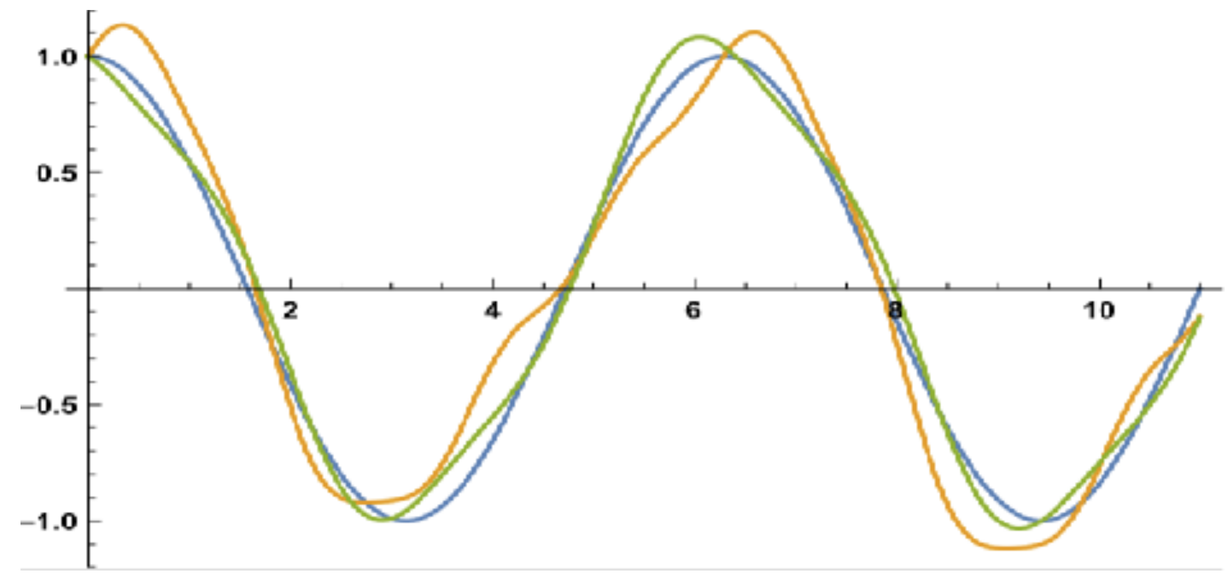


Even classically only probability can be calculated, but this is systematics!

Evolution of ψ(x)

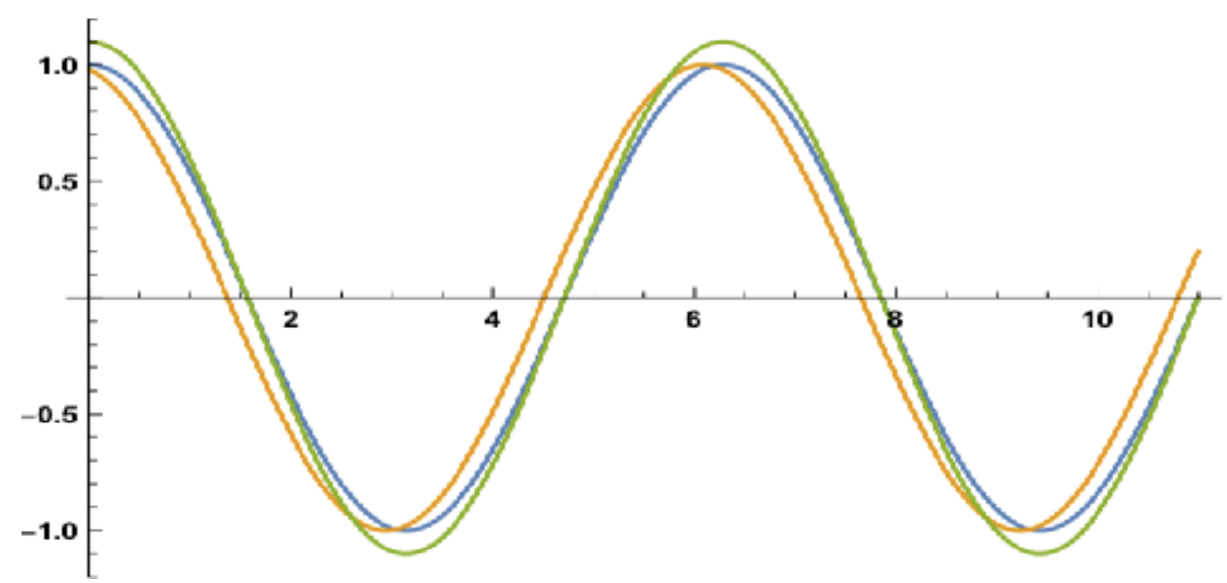
Evolve each x along all possible paths.

one x along
any path



$$\psi(x,t) = \int dx' \int_{P:\,x'(0)\to x(t)} dP\, e^{iS[P]}\, \psi(x', t=0)$$

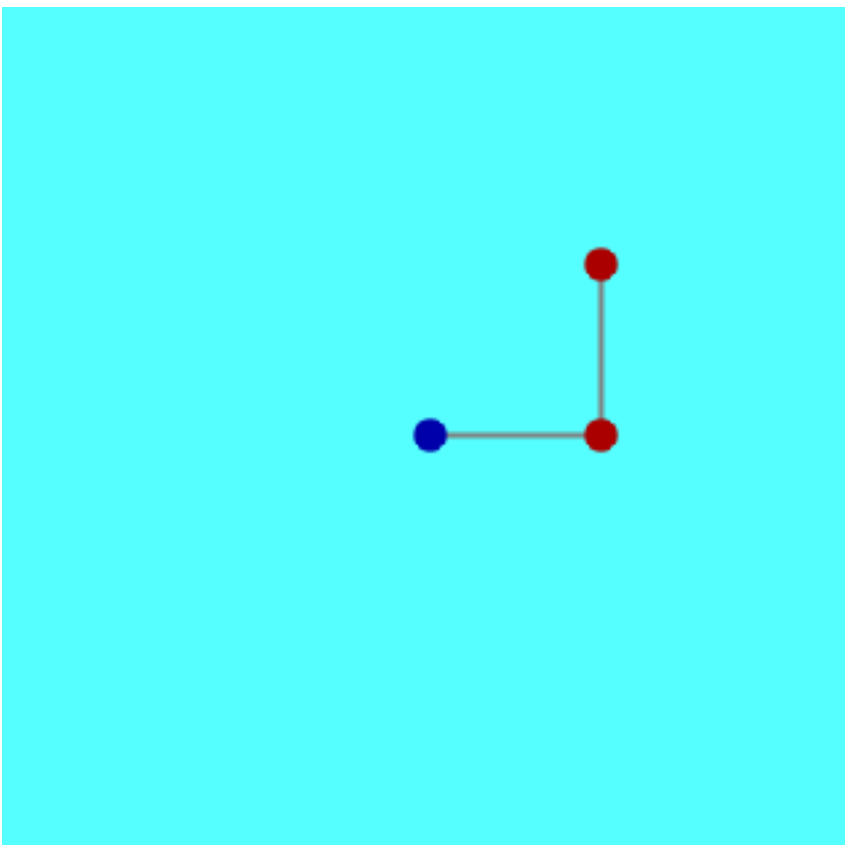
different x
along
classical path



Field theory

So far: one particle in one space dimension: 1 d.o.f.

Things may change with coupled (interacting) d.o.f.'s:



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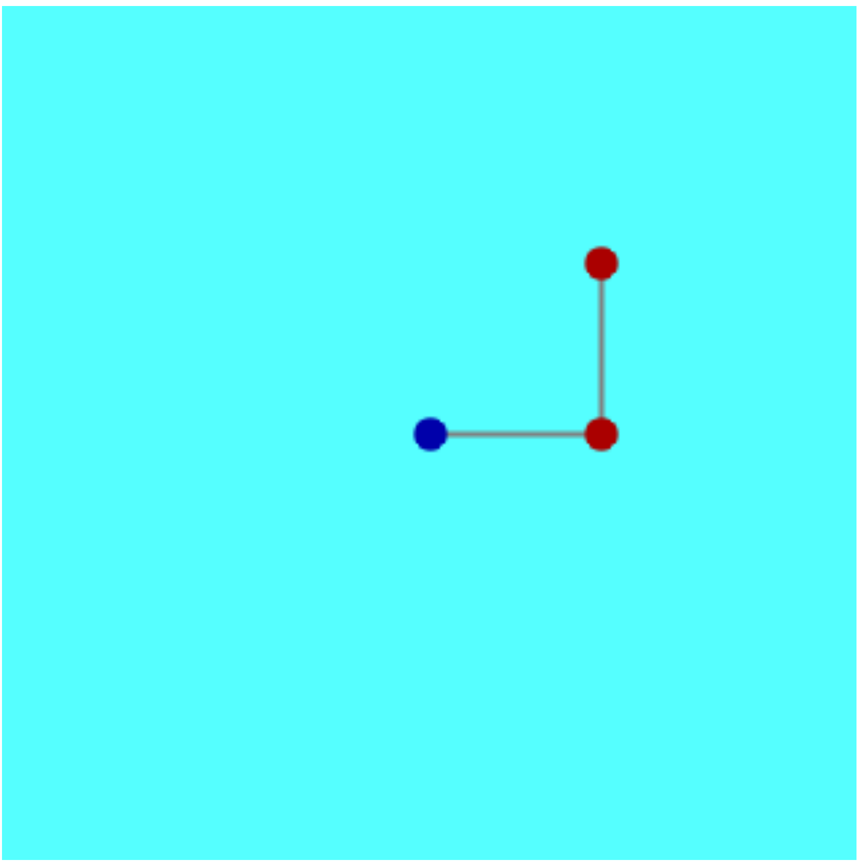

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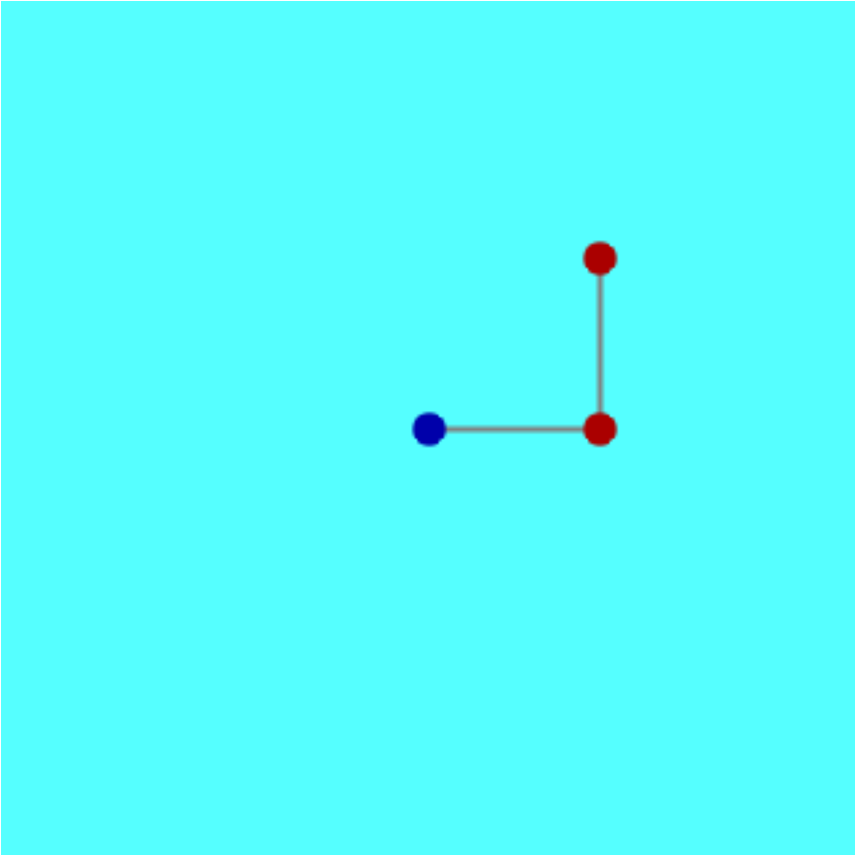

action:

$$S[P_1,P_2] = S_1[P_1] + S_2[P_2] + S_{int}[P_1,P_2]$$

Field theory

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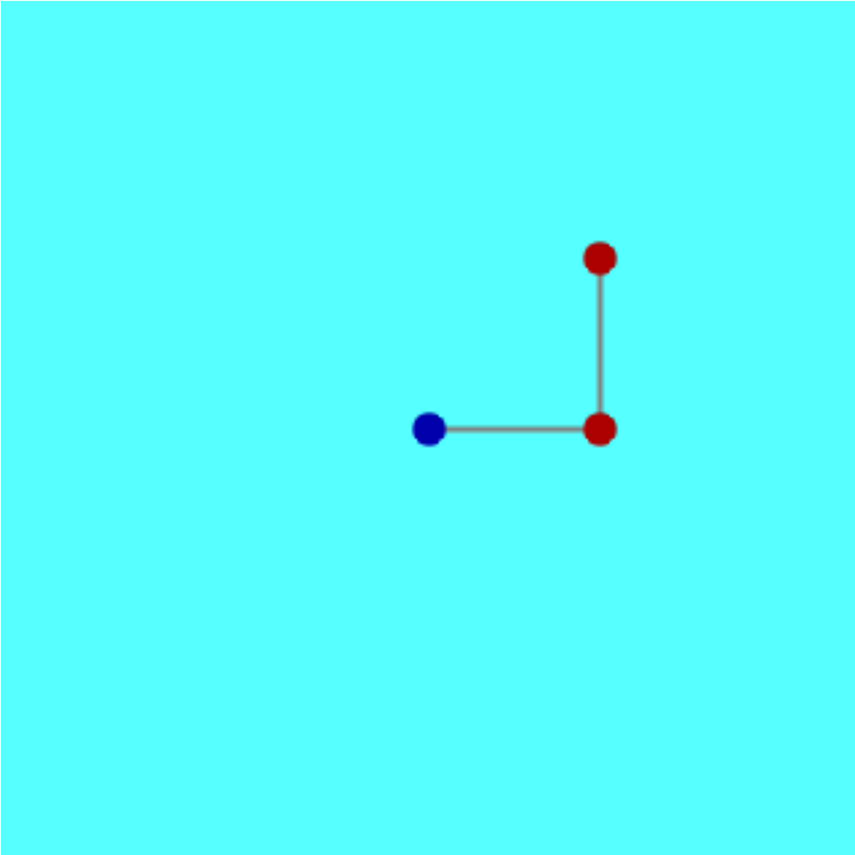

action:

$$S[P_1,P_2] = S_1[P_1] + S_2[P_2] + S_{int}[P_1,P_2]$$

$$\int_{P_1:\, x_1(0)\to x_1'(t)} \int_{P_2:\, x_2(0)\to x_2'(t)} dP_1 dP_2\, e^{iS[P_1,P_2]}$$

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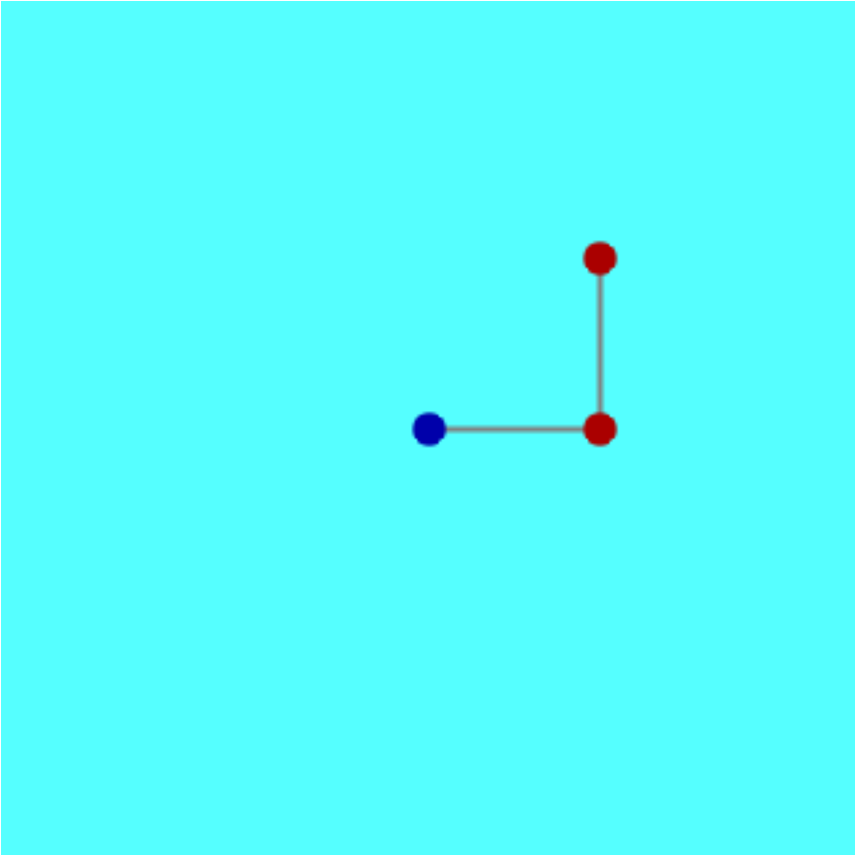

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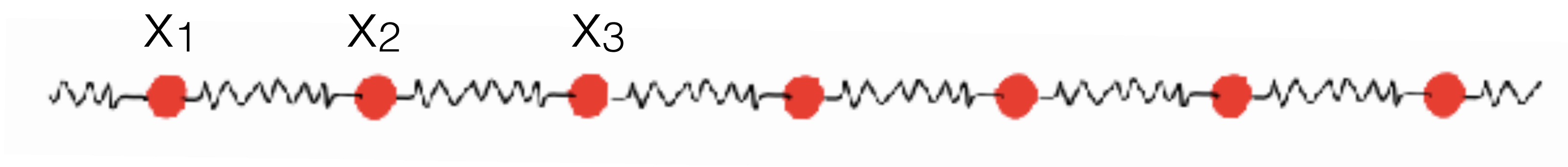
But: principles remain the same (with a few additions…).

Field theory

infinite d.o.f.

dislocations: $\phi(x_i, 0) \to \phi'(x_i, t)$



$$\cdots \int dP_0 \int dP_1 \int dP_2 \cdots e^{iS[\ldots, P_0, P_1, P_2, \ldots]}$$

Field theory

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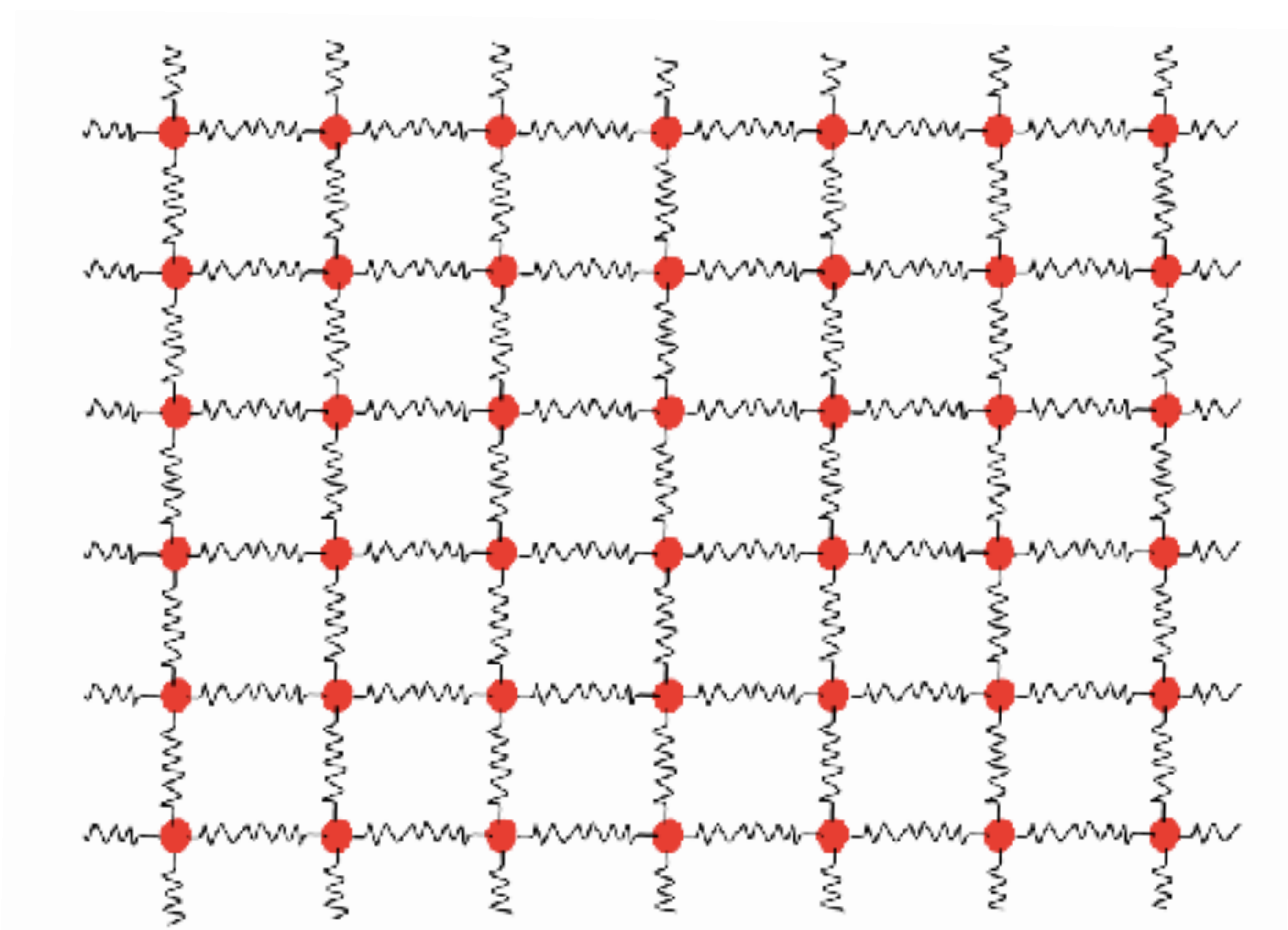

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Field theory

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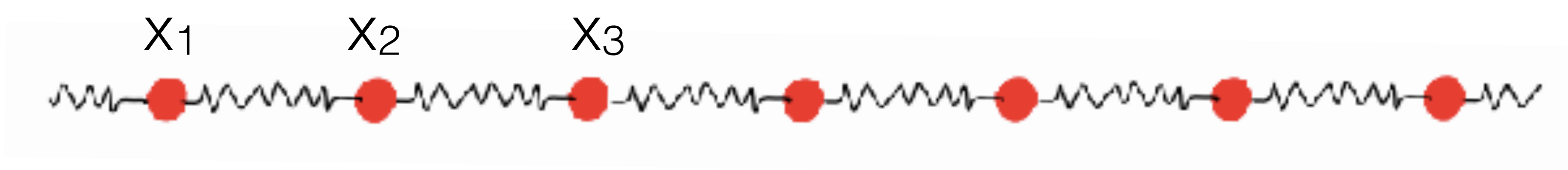

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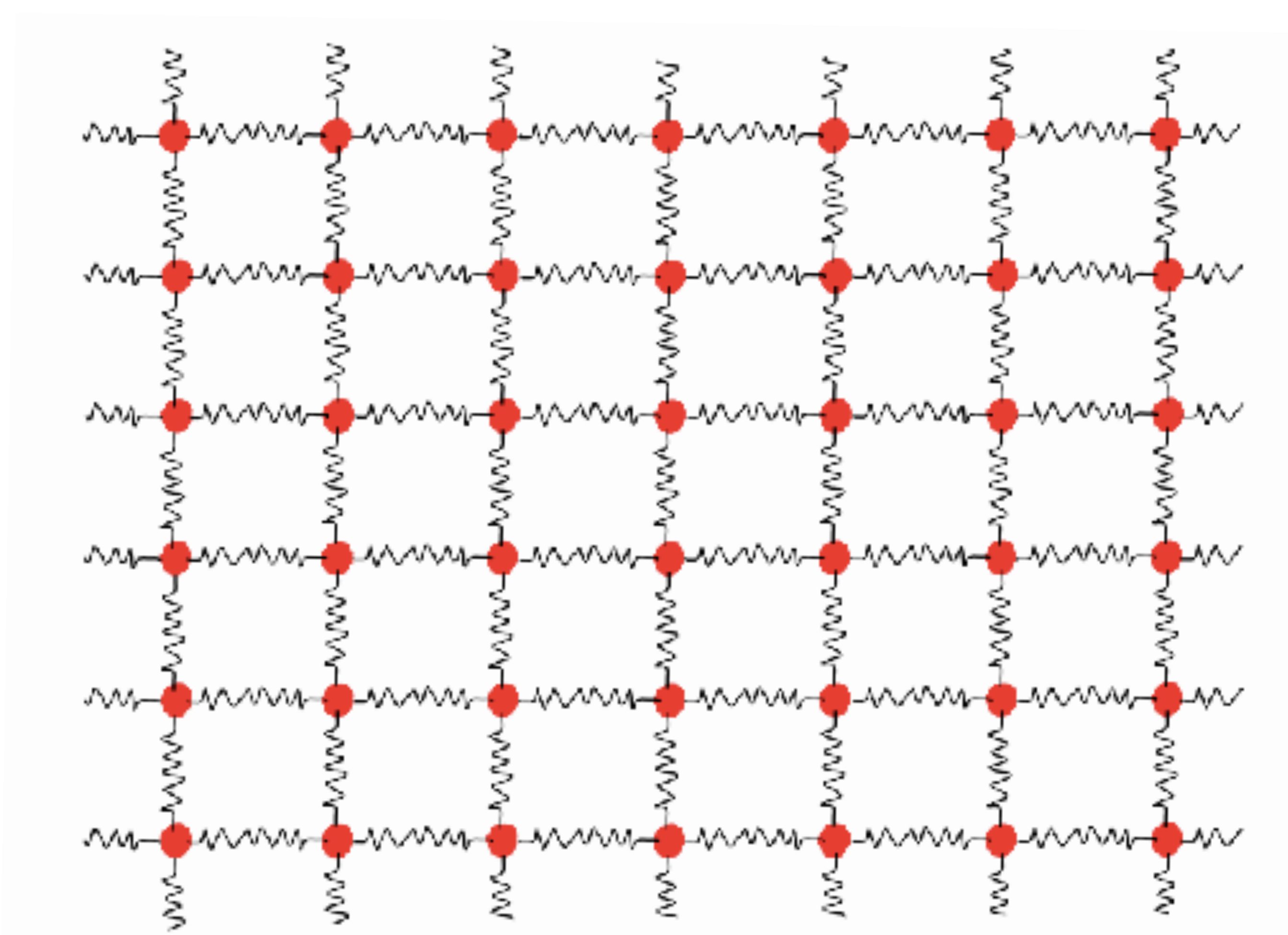

Field theory:
continuum limit

$\phi(x,0) \to \phi'(x,t)$

Field theory

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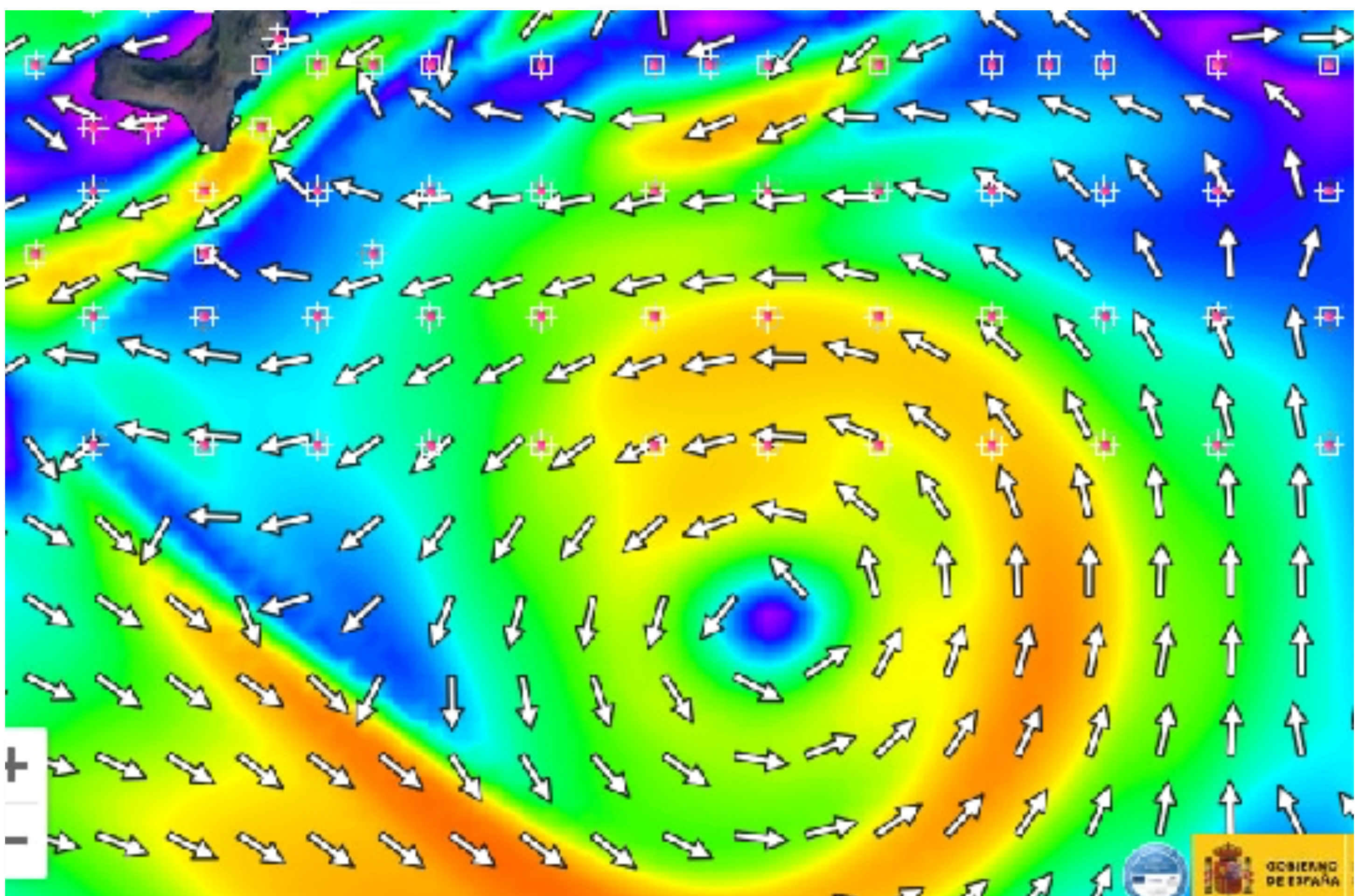

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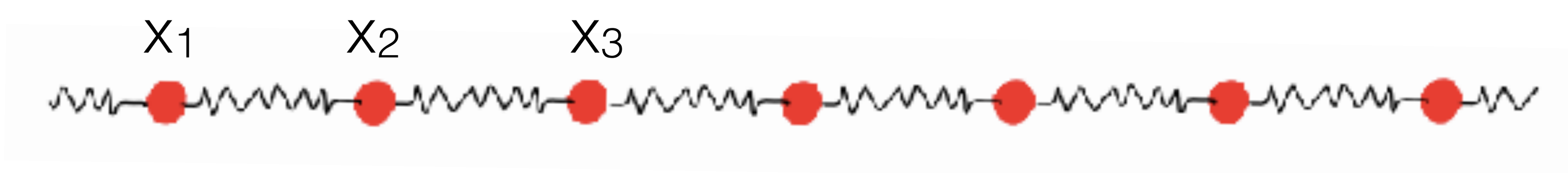
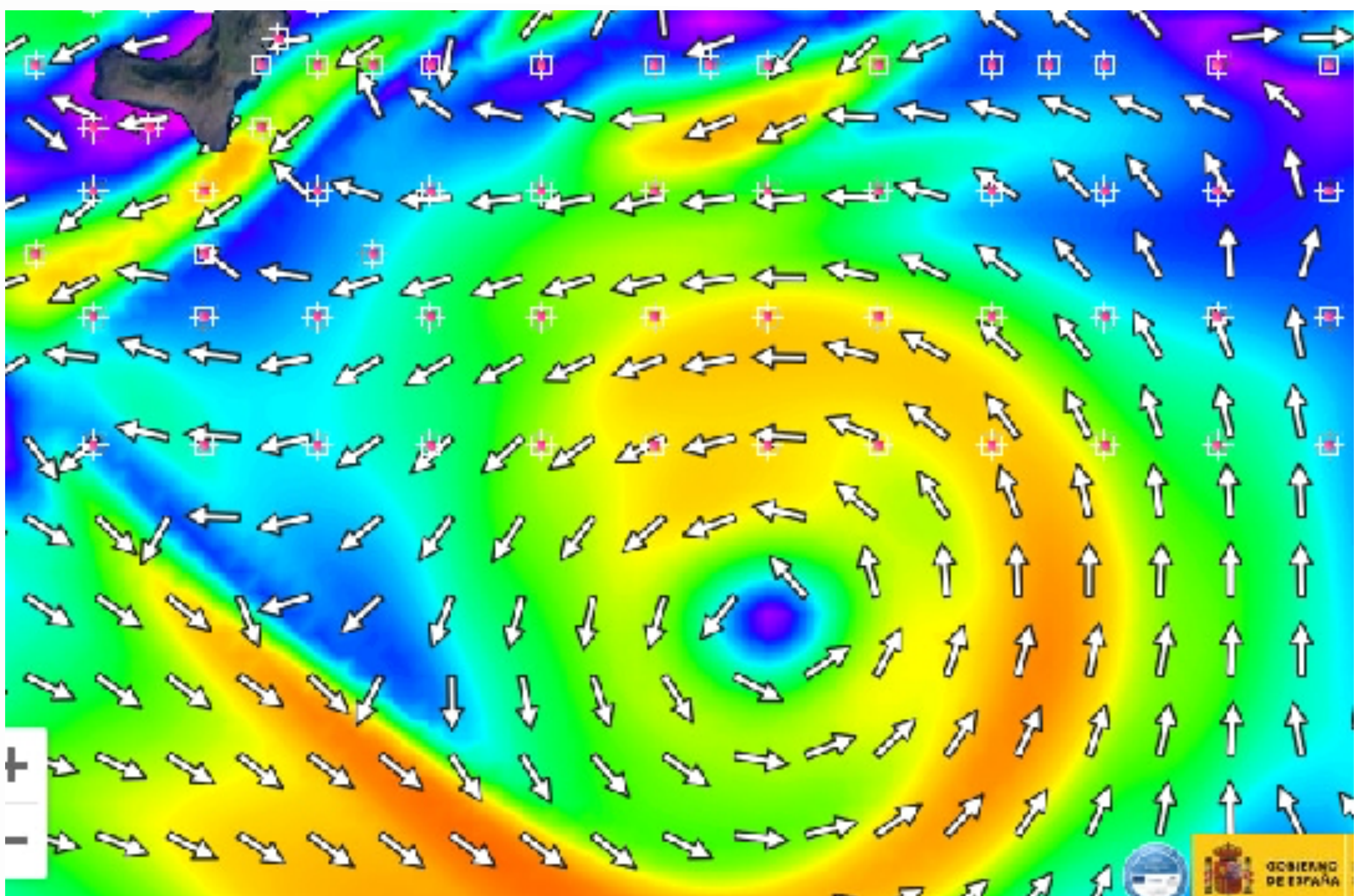


$$\cdots \int dP_0 \int dP_1 \int dP_2 \cdots e^{iS[\ldots,P_0,P_1,P_2,\ldots]}$$



Field theory:
continuum limit

$$\phi(x,0) \to \phi'(x,t)$$

$$\int \mathscr{D}\phi \, e^{iS[\phi]}$$

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Quantum Field Theory

$$\int \mathscr{D}\phi\, e^{iS[\phi]}$$

how can we calculate that?

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Perturbation theory: $S[\phi] = S_{\text{free}}[\phi] + S_{\text{int}}[\phi]$

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small

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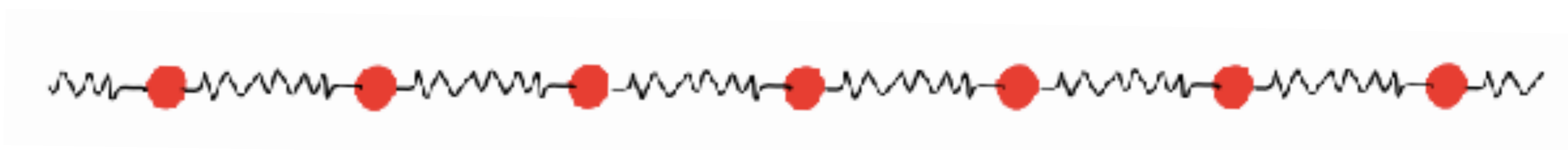
small



free: no springs

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Using $\displaystyle \int_{-\infty}^{\infty} dx \, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$

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… let's say it's 1.

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$$\int \mathscr{D}\phi \, \phi_x \phi_y \phi_y \phi_z \, e^{iS_{\text{free}}[\phi]} \sim \frac{1}{D_{x-y}} \frac{1}{D_{y-z}} + \frac{1}{D_{x-z}} \frac{1}{D_{y-y}}$$

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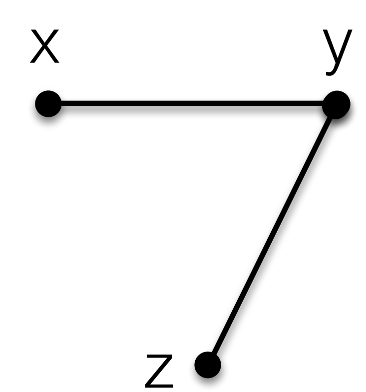

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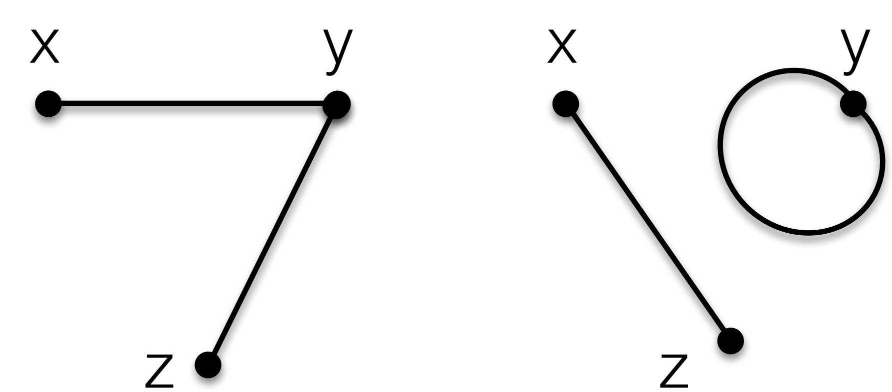

Quantum Field Theory

$$\int \mathscr{D}\phi\, e^{iS[\phi]} = \int \mathscr{D}\phi\, e^{iS_{\text{int}}[\phi]}\, e^{iS_{\text{free}}[\phi]}$$

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Quantum Field Theory

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$$= \int \mathscr{D}\phi \left[1 + \phi_x^3 + \frac{1}{2} \phi_x^3 \phi_y^3 + \frac{1}{3!} \phi_x^3 \phi_y^3 \phi_z^3 + \ldots \right] e^{iS_{\text{free}}[\phi]}$$

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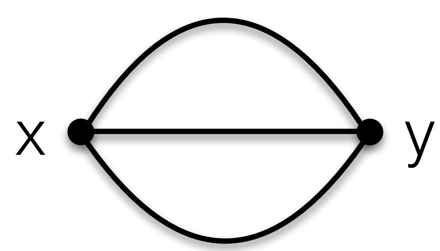

$$\phi_x\phi_x\phi_x\phi_y\phi_y\phi_y$$

Quantum Field Theory

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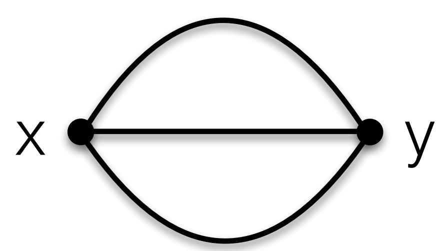

$$\phi_x\phi_x\phi_x\phi_y\phi_y\phi_y$$

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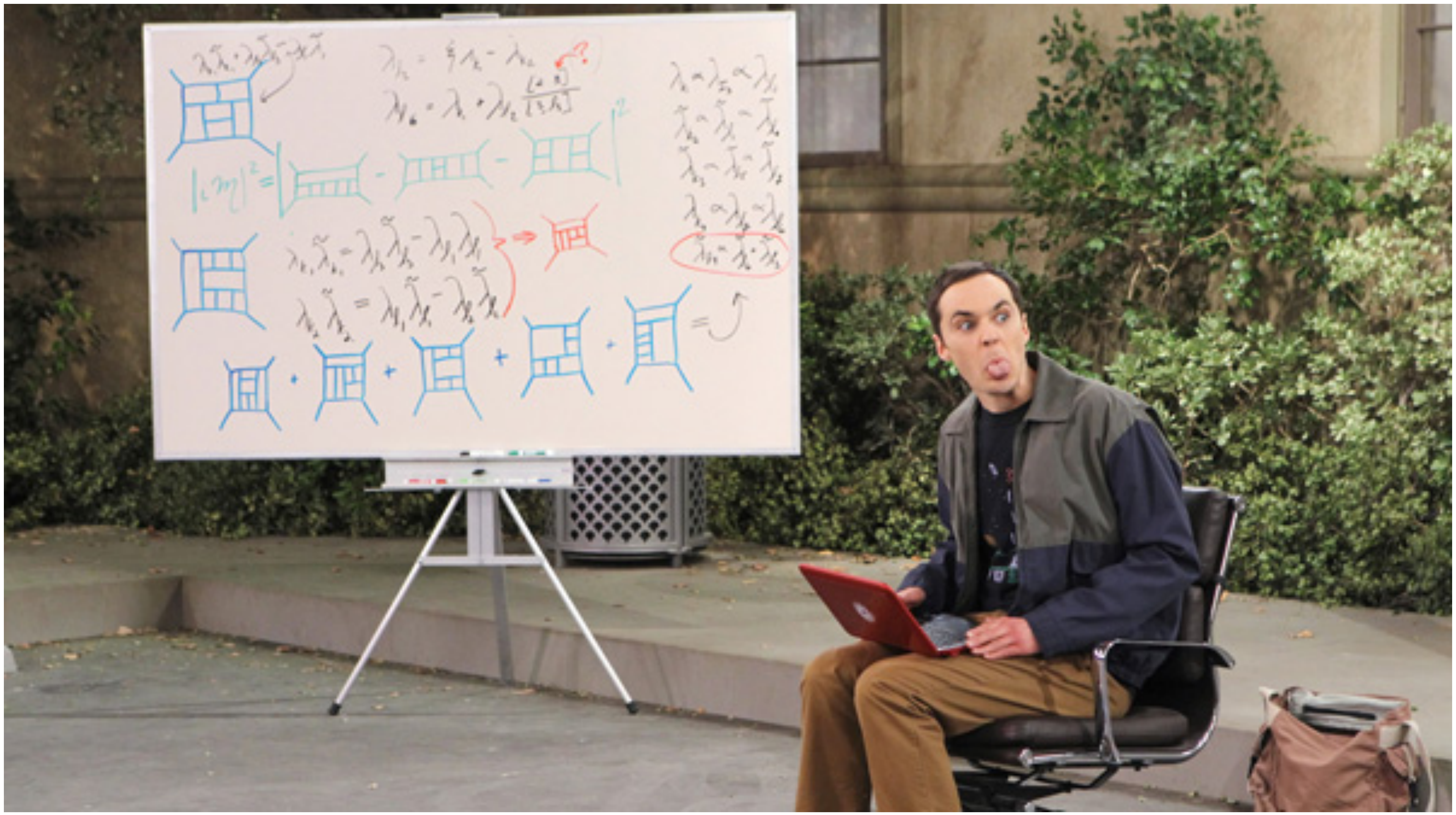
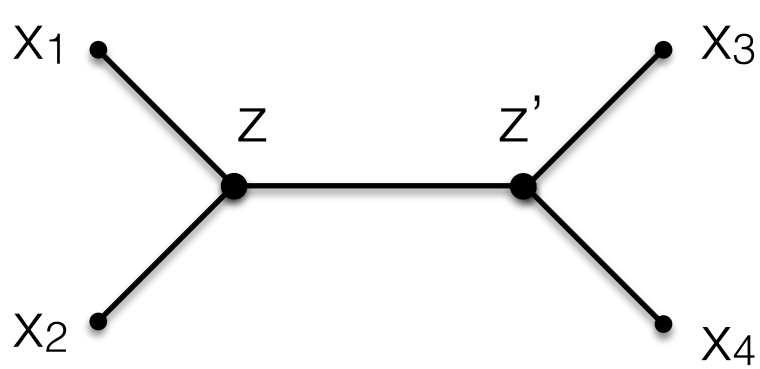


Scattering: external fields

$$\int \mathscr{D}\phi \, \phi_{x_1}\phi_{x_2}\phi_{x_3}\phi_{x_4} \, e^{iS[\phi]}$$

$$\int_z \int_{z'}$$



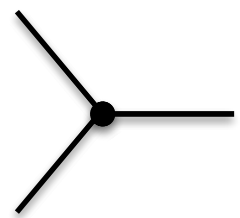
~ g²

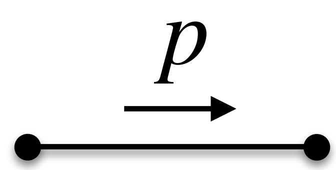
Feynman rules

Forget all of the above!

Feynman rules

Forget all of the above!

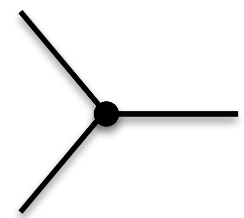
vertex:  $= g$

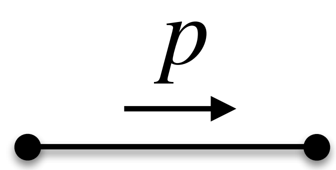
propagator: 

$= \tilde{D}^{-1}(p) = \dfrac{1}{p^2 - m^2}$

Feynman rules

Forget all of the above!

vertex:  $= \mathrm{g}$

propagator: 

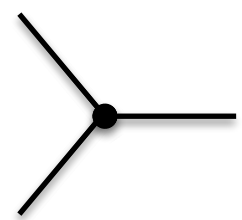
$$= \tilde{D}^{-1}(p) = \frac{1}{p^2 - m^2}$$

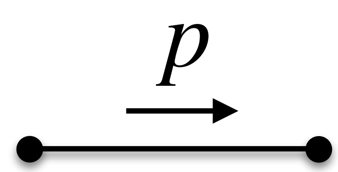
Connect initial and final state with these objects

$\Rightarrow$ probability amplitude

Feynman rules

Forget all of the above!

vertex:  $= g$

propagator: 

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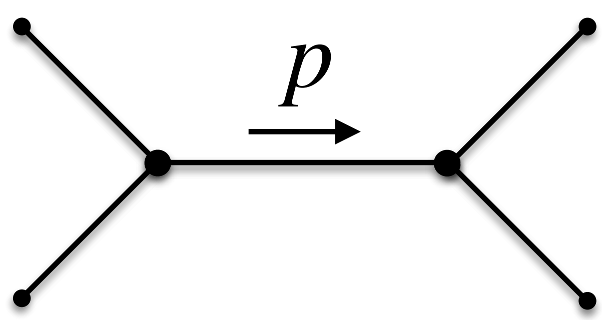
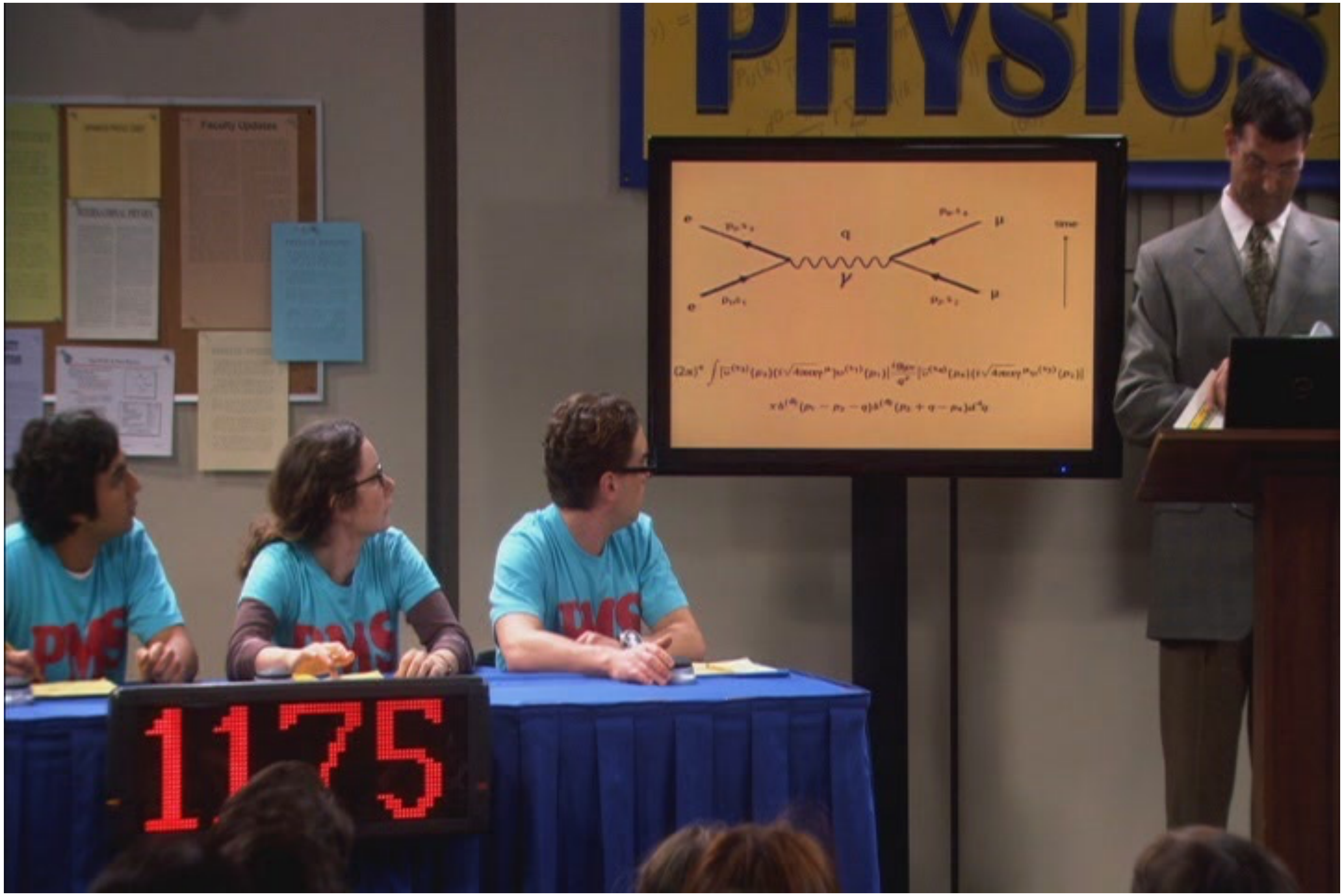
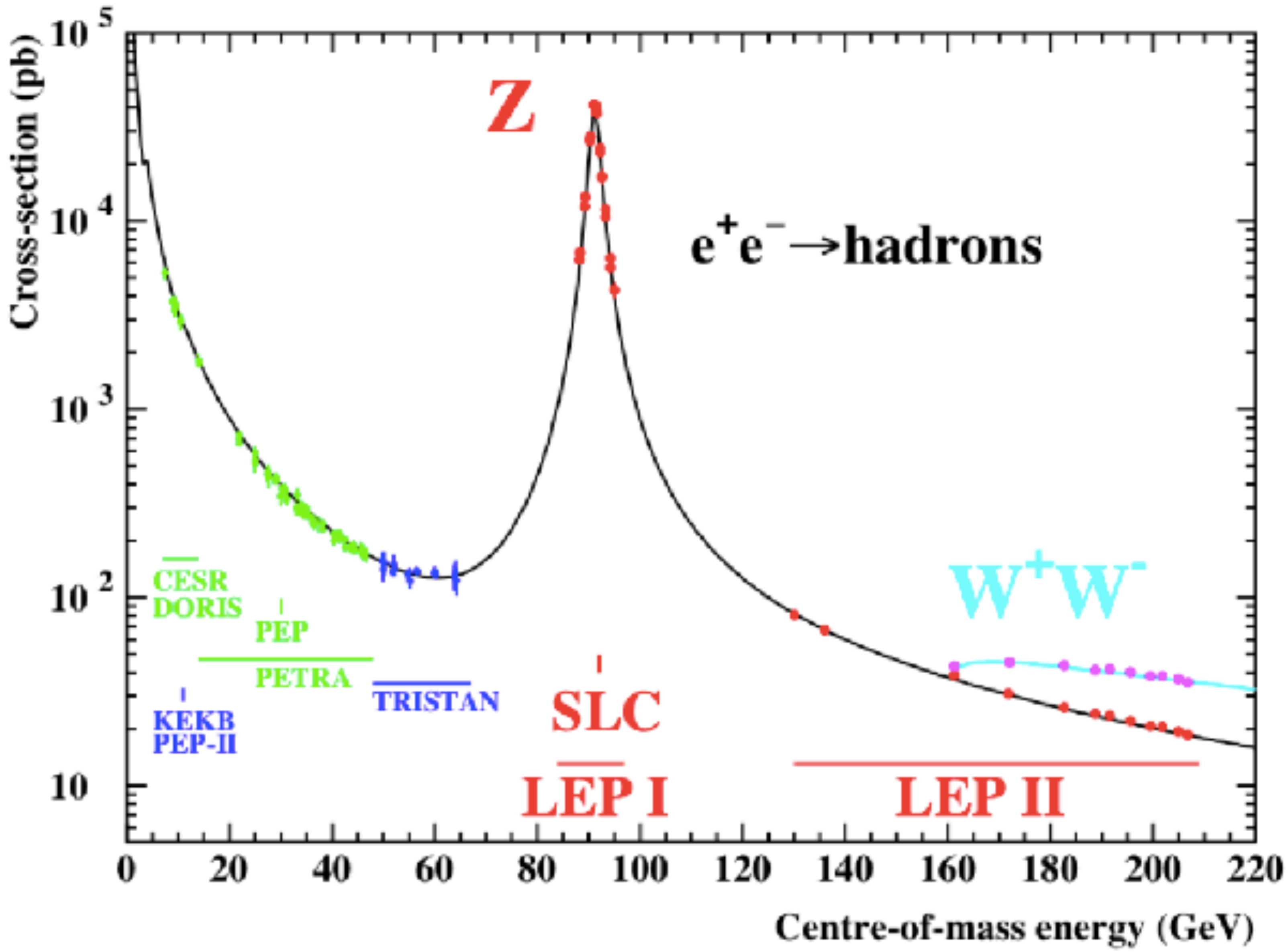


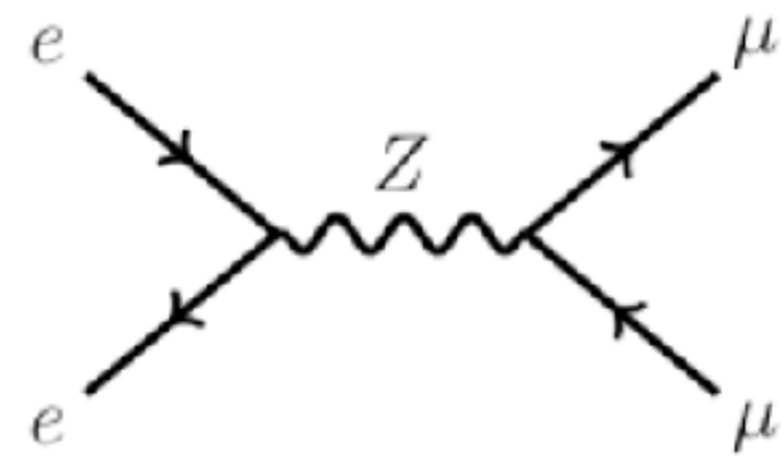
$$\sim \frac{g^2}{p^2 - m^2}$$



$$-8\pi\alpha$$

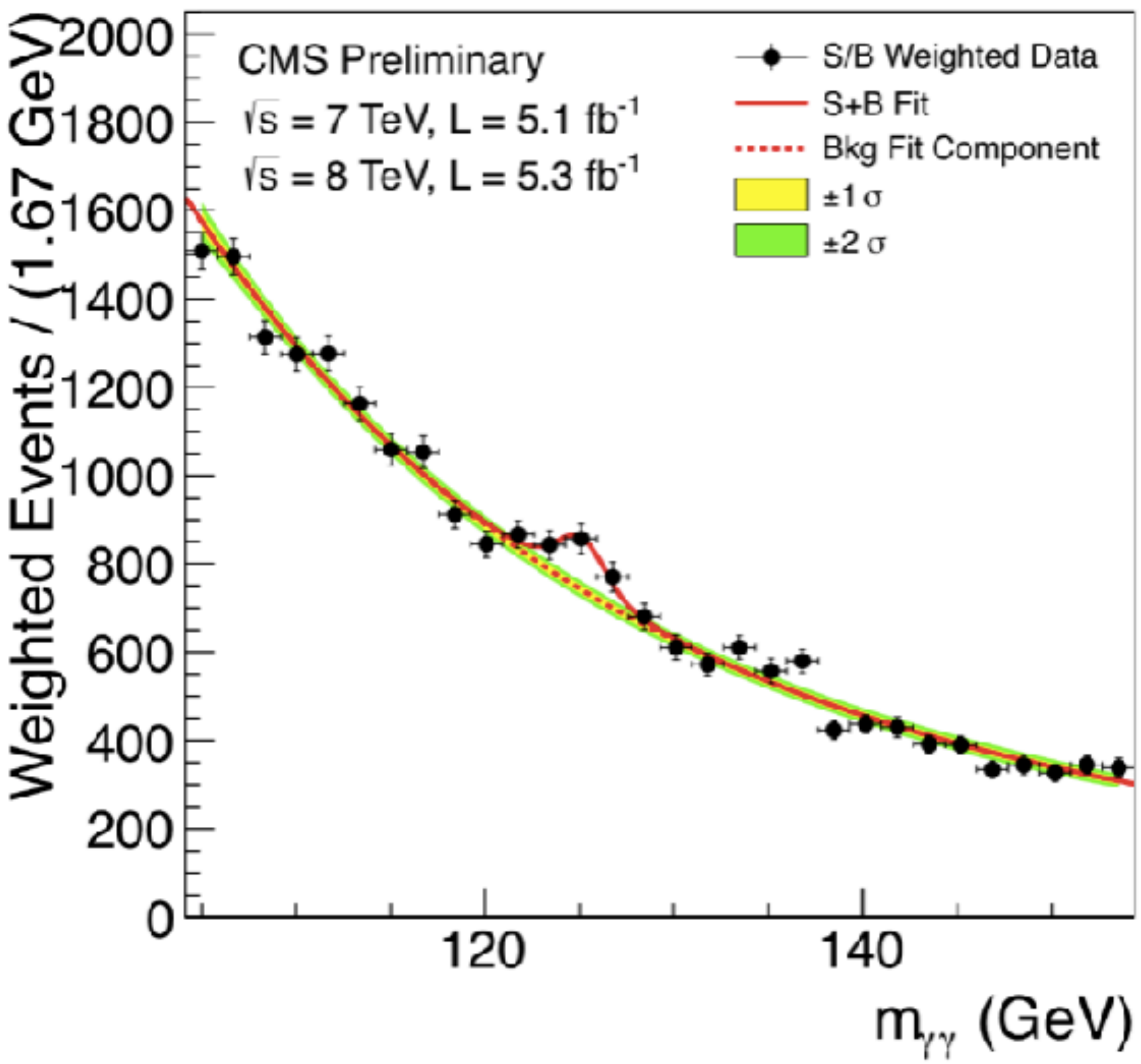
Z-boson production

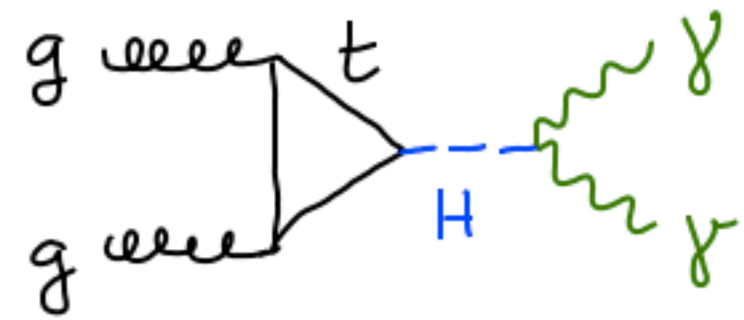




$$\sim \frac{g^2}{p^2 - M_Z^2}$$

Higgs discovery





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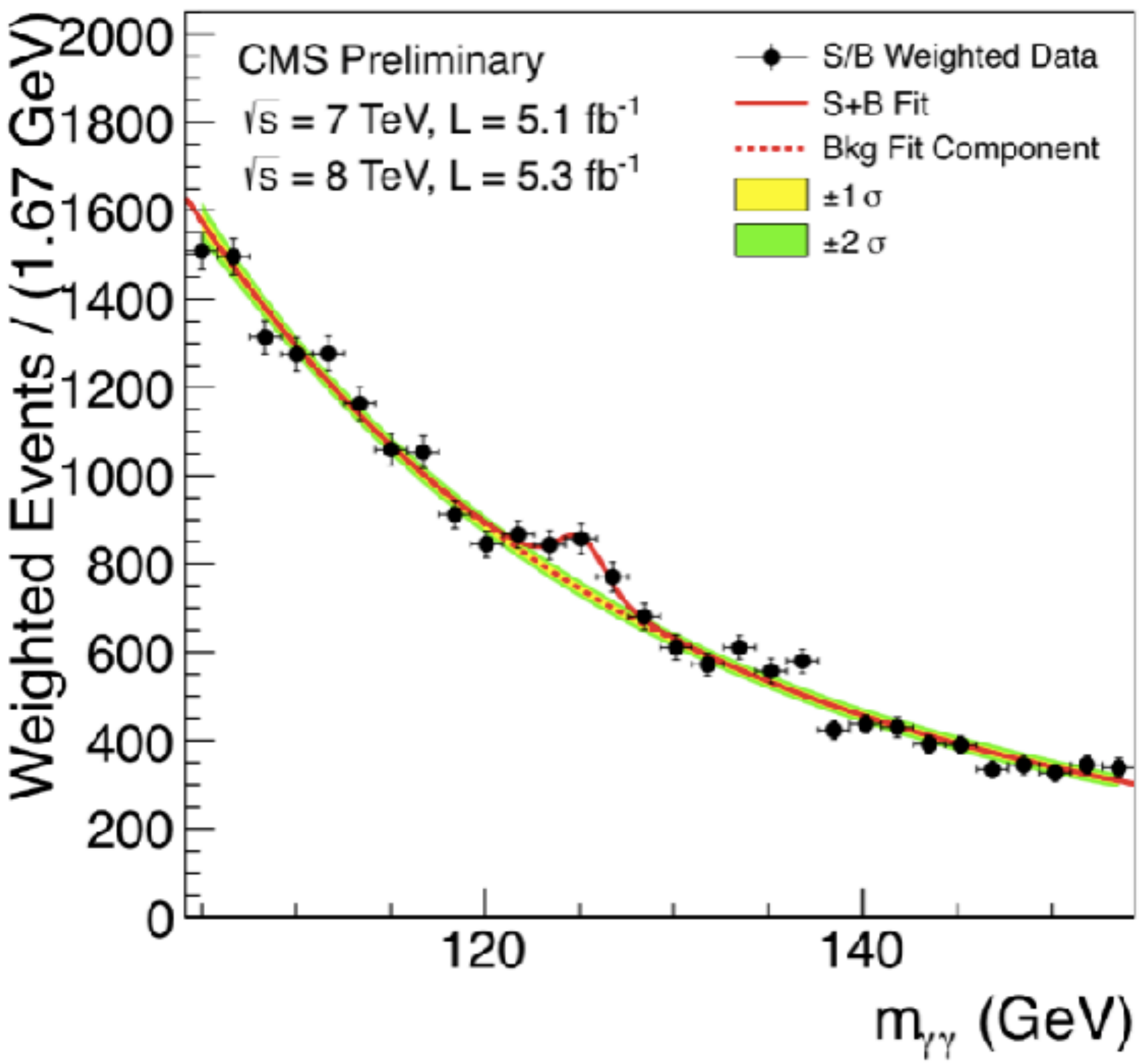
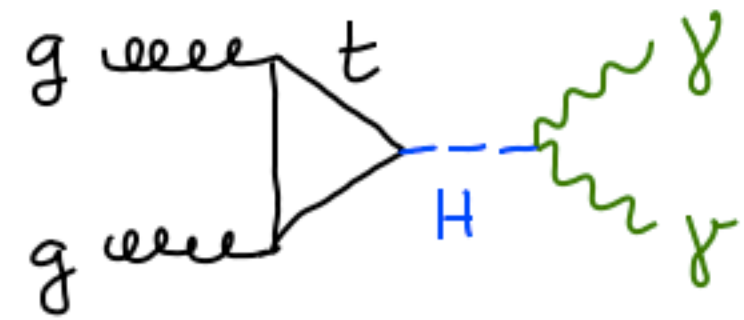
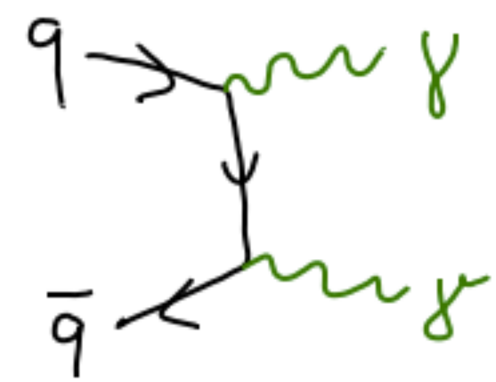
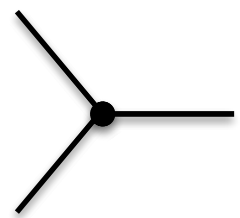


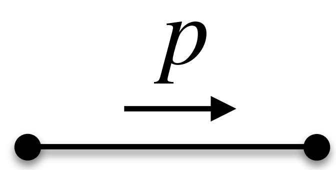




Feynman rules

Forget all of the above!

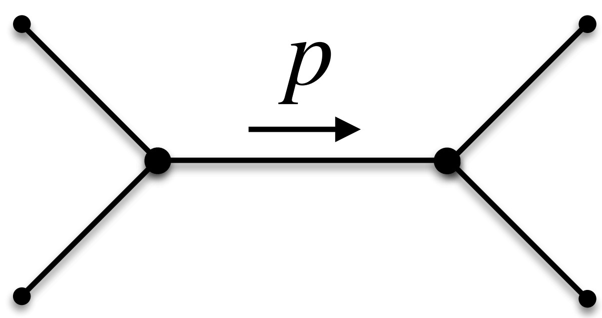
vertex:  = g

propagator: 

$$= \tilde{D}^{-1}(p) = \frac{1}{p^2 - m^2}$$

Connect initial and final state with these objects

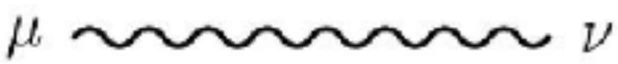
$\Rightarrow$ probability amplitude



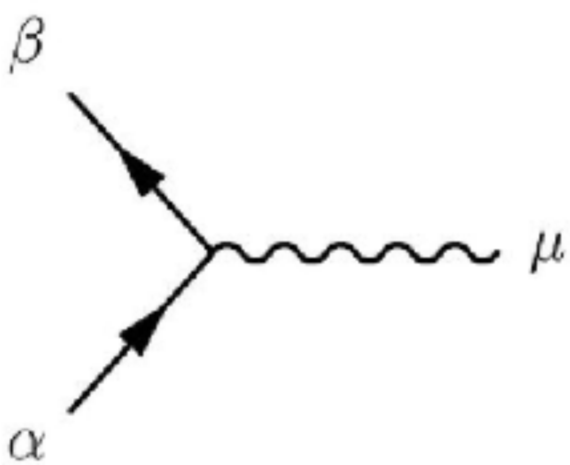
$$\sim \frac{g^2}{p^2 - m^2}$$

Feynman rules for QED

electron e^-
positron e^+



photon γ

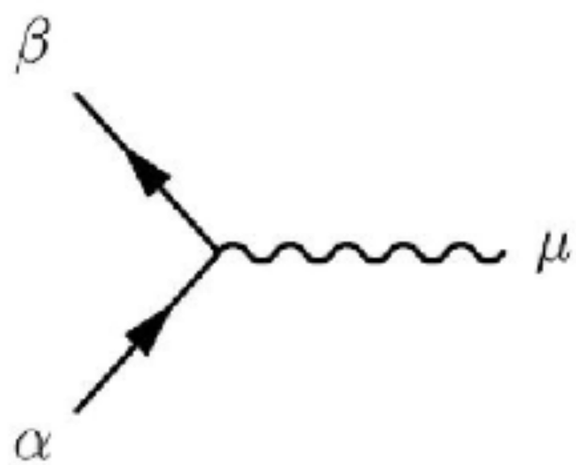


~ charge

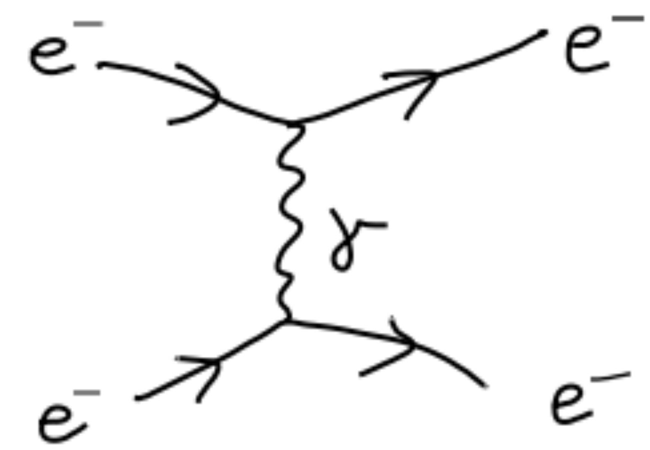
Feynman rules for QED

electron e^-
positron e^+

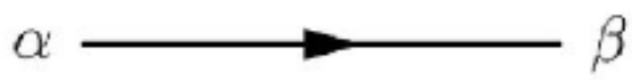
photon γ



~ charge

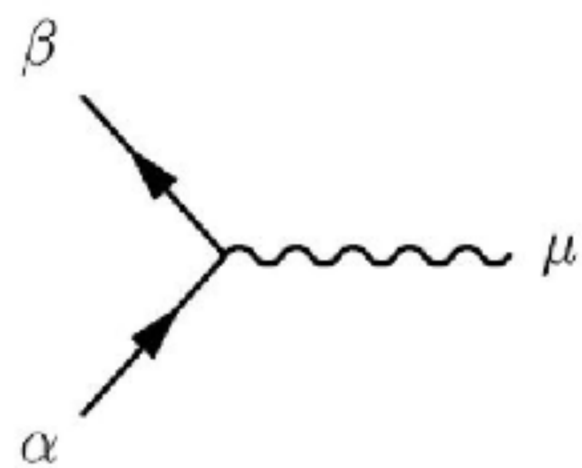


Feynman rules for QED

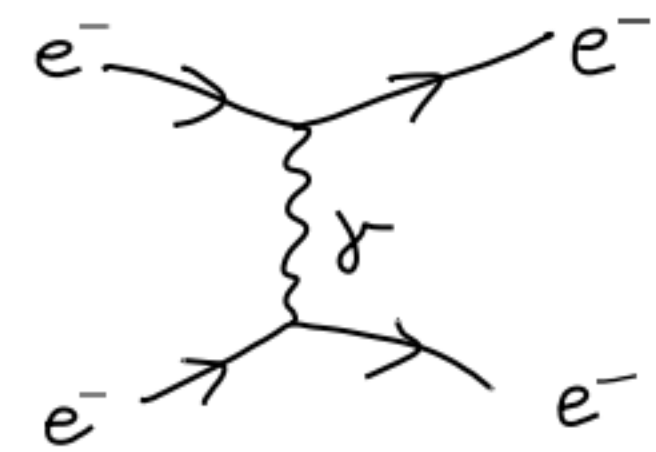


electron e^-
positron e^+

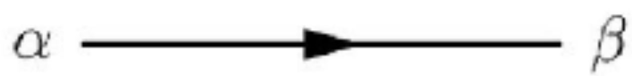
photon γ



~ charge

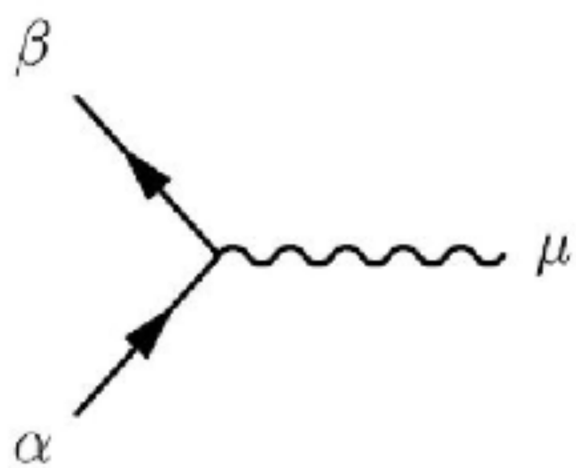


Feynman rules for QED

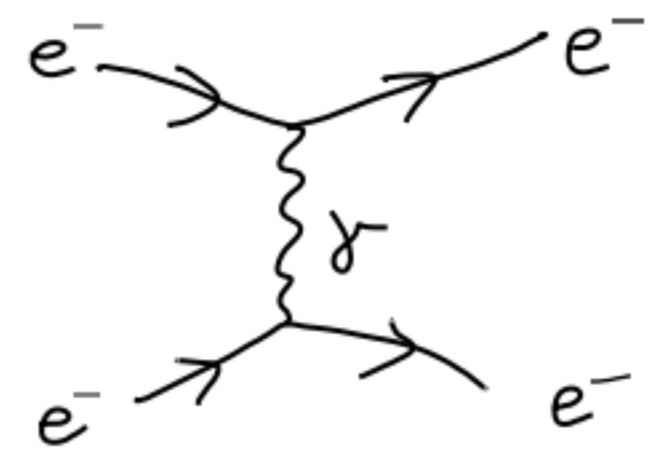


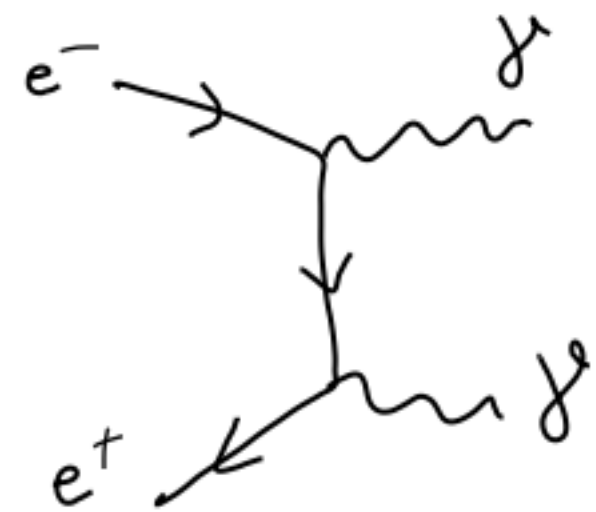
electron e^-
positron e^+

photon γ

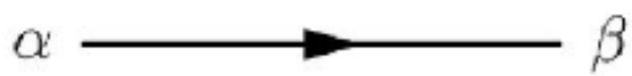


~ charge



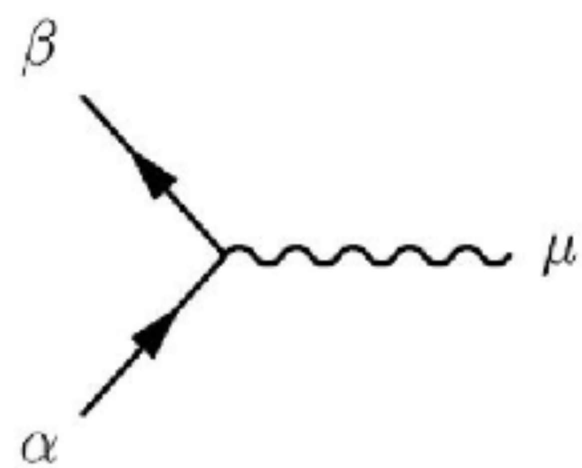


Feynman rules for QED

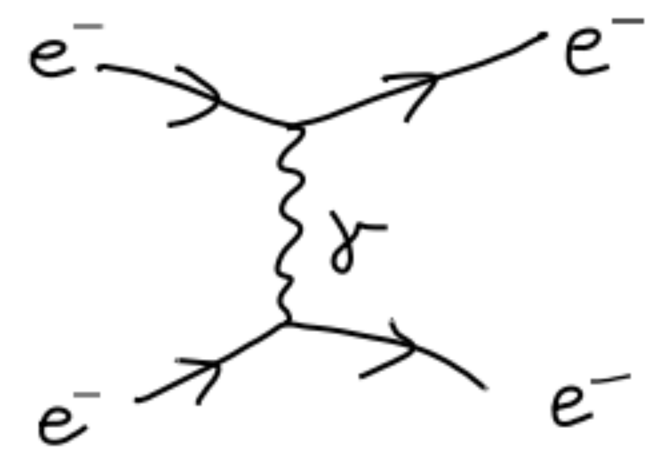


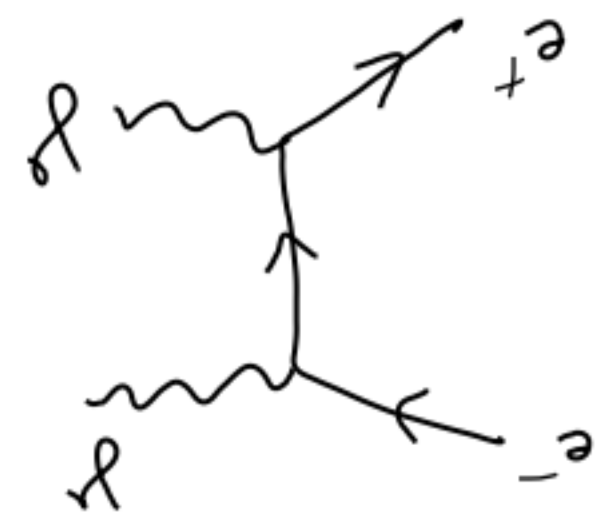
electron e⁻
positron e⁺

photon γ

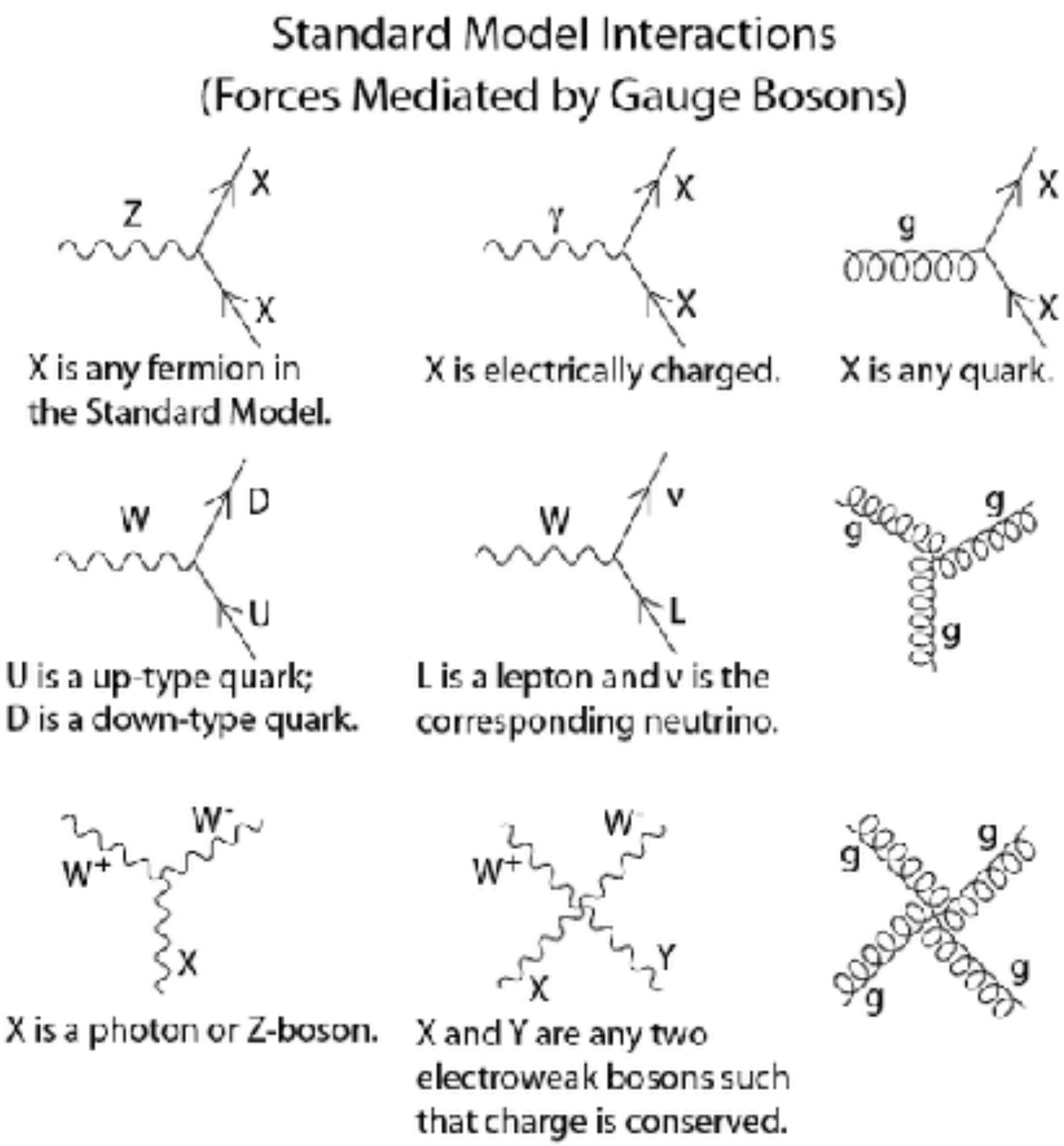


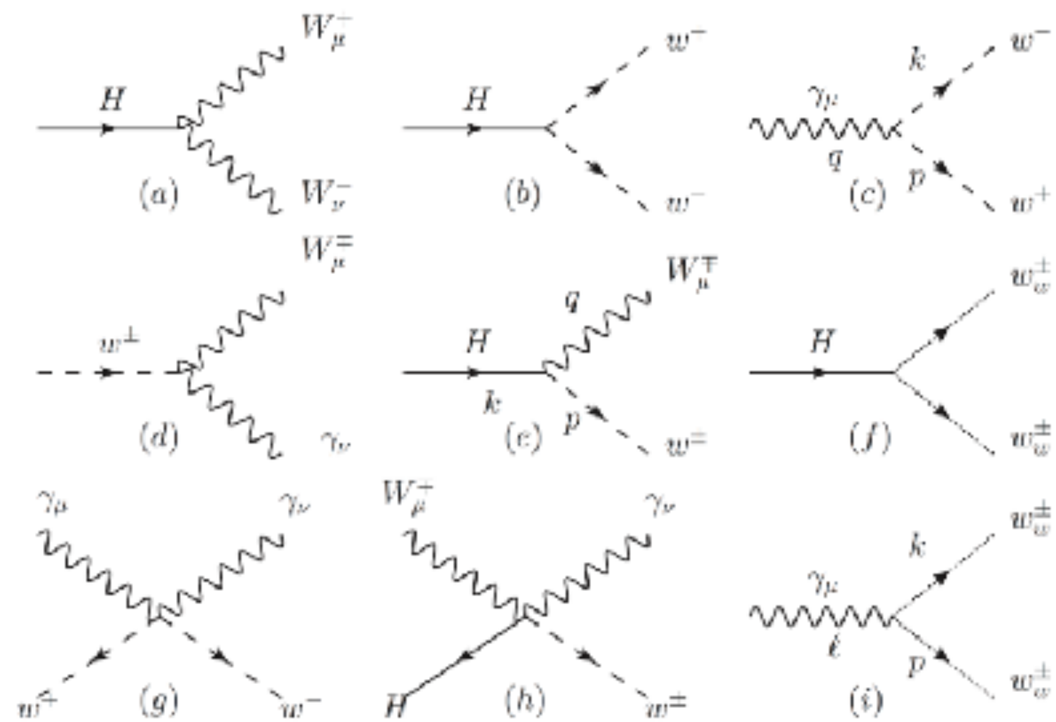
~ charge



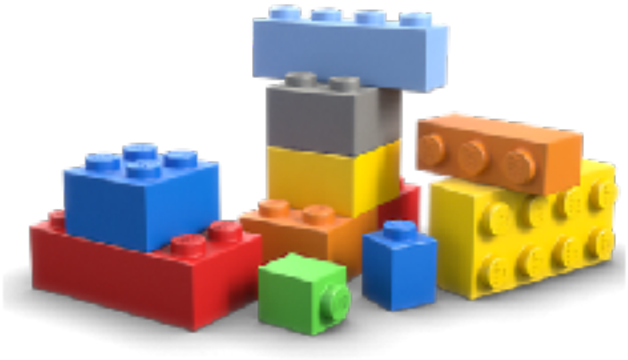


Standard Model



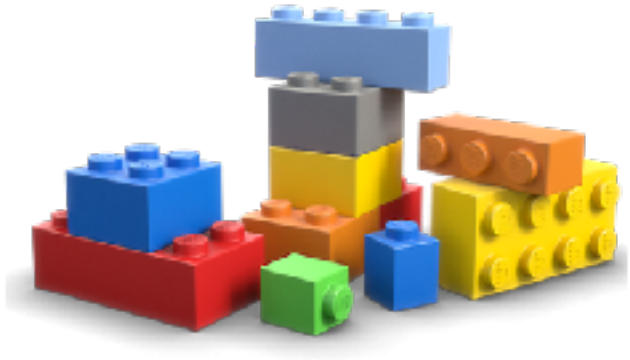


The virtues of Feynman diagrams

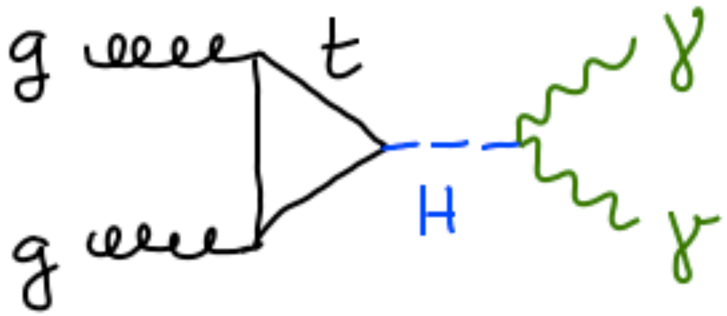


Easy to use.

The virtues of Feynman diagrams



Easy to use.



Easy to talk about.

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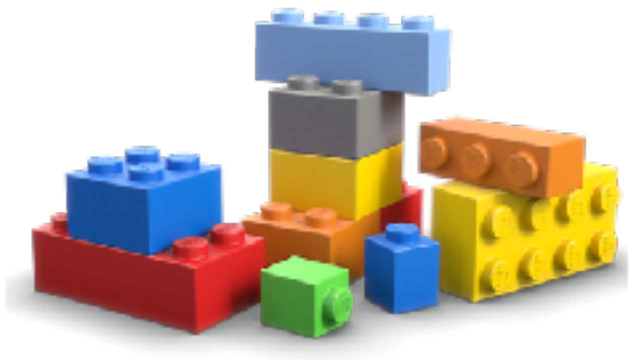
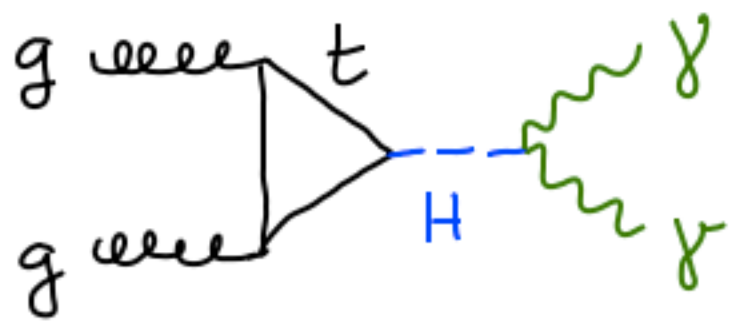
The virtues of Feynman diagrams



Easy to use.



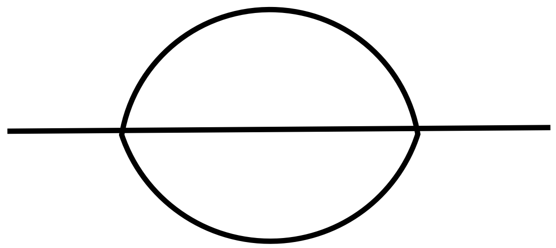
Easy to talk about.

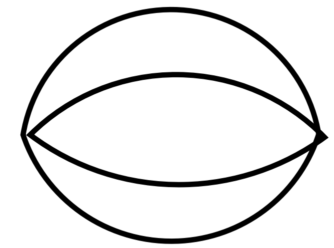
Systematic and algorithmic:

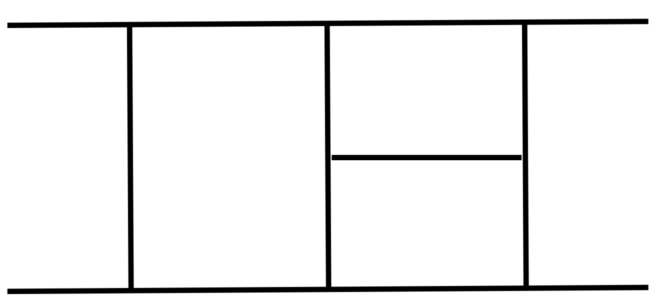
theory: (g-2)/2 = 0.001159652181643(764)

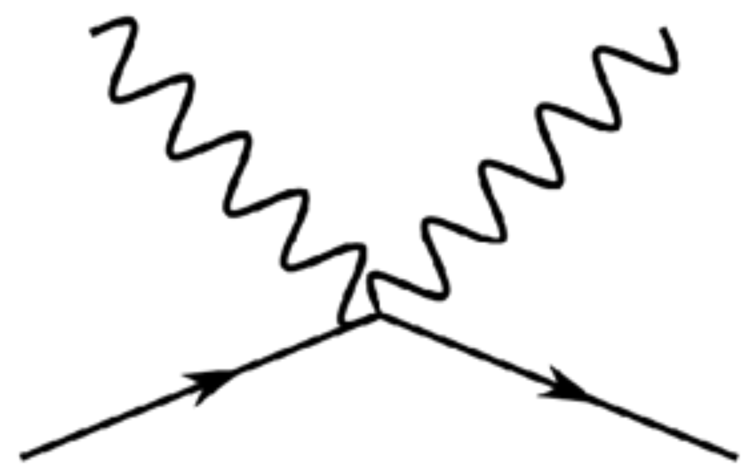
experiment: (g-2)/2 = 0.00115965218073(28)

We love them…

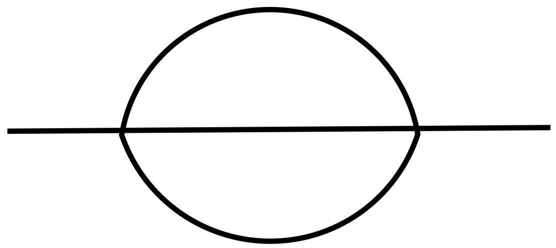




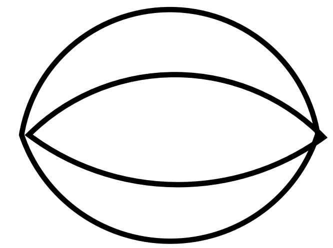


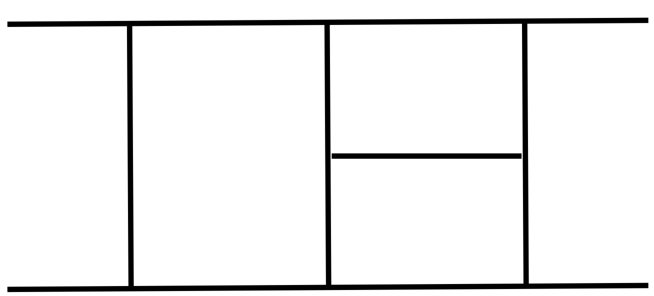


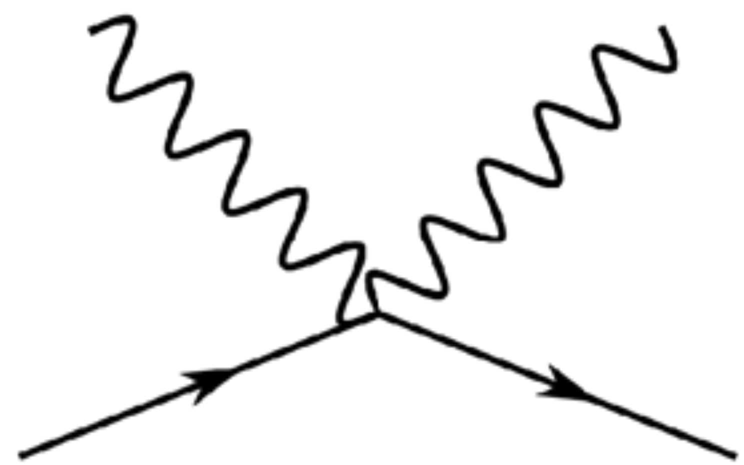
We love them…



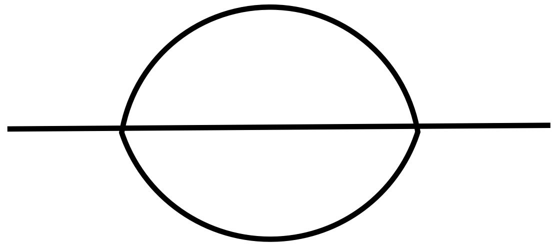
sunrise/
sunset



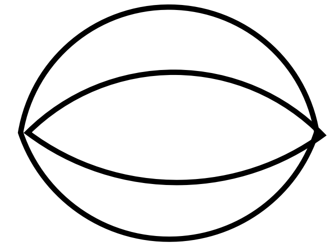




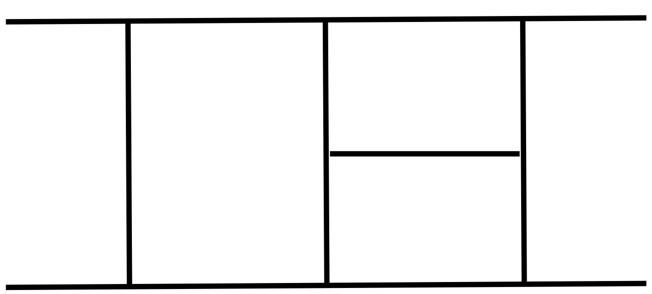
We love them…

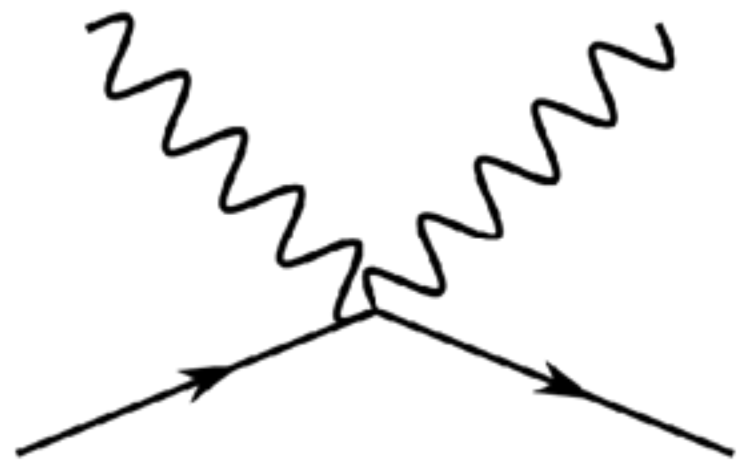


sunrise/
sunset

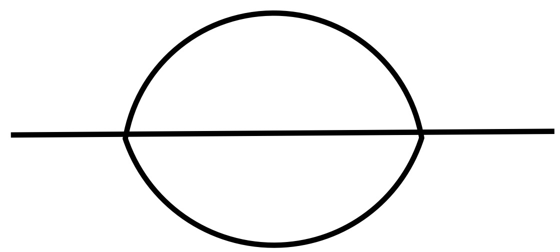


watermellon

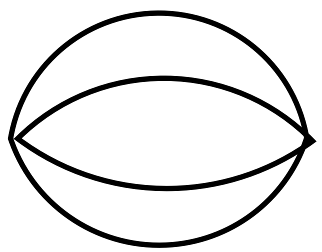




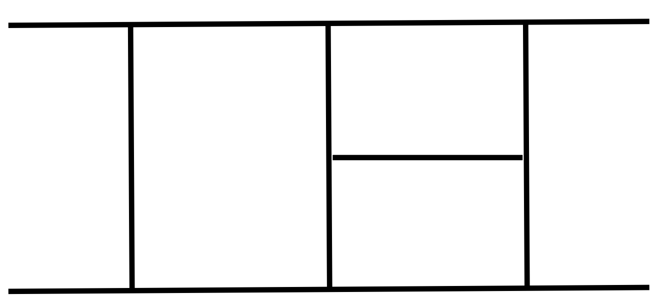
We love them…



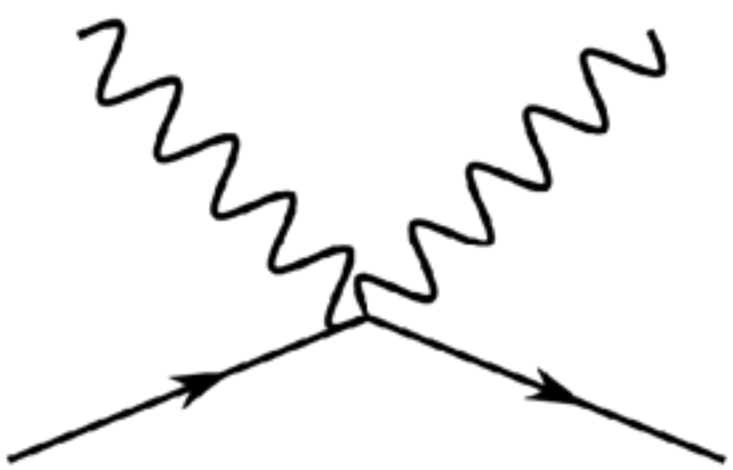
sunrise/
sunset



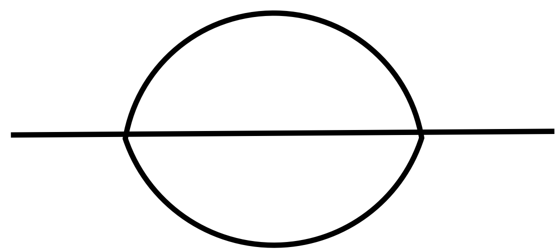
watermellon



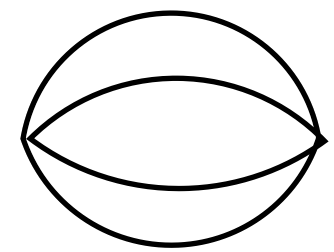
tennis court



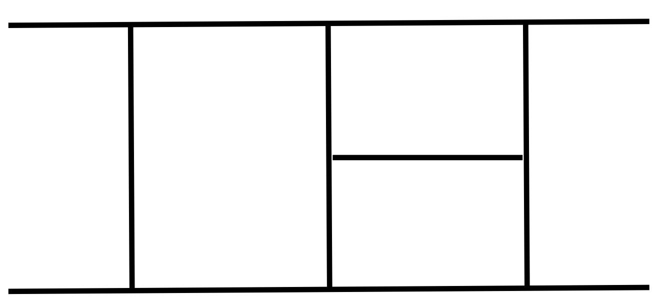
We love them…



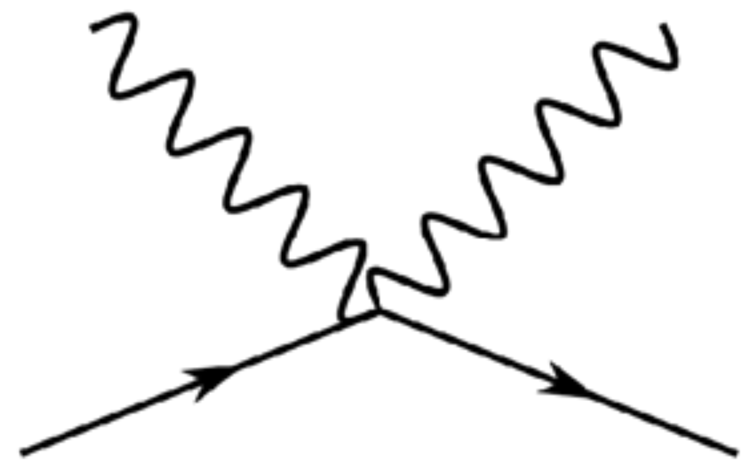
sunrise/
sunset



watermellon

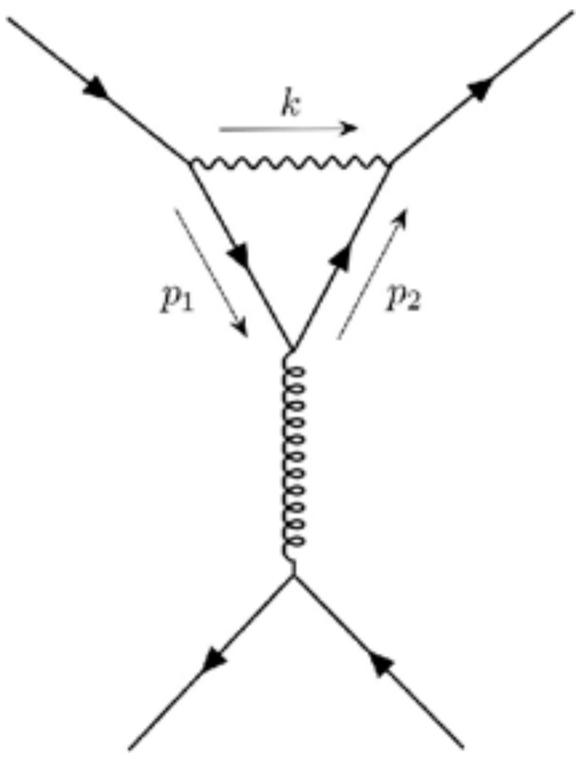


tennis court

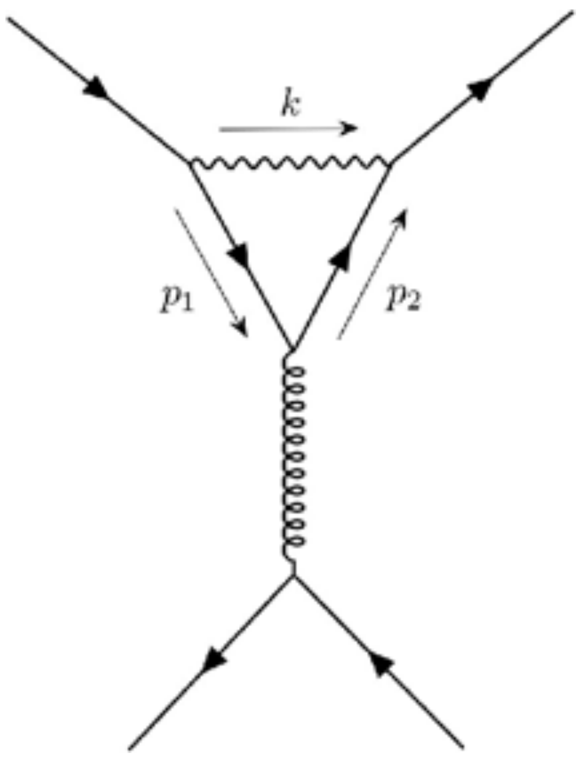


seagull

We love them…

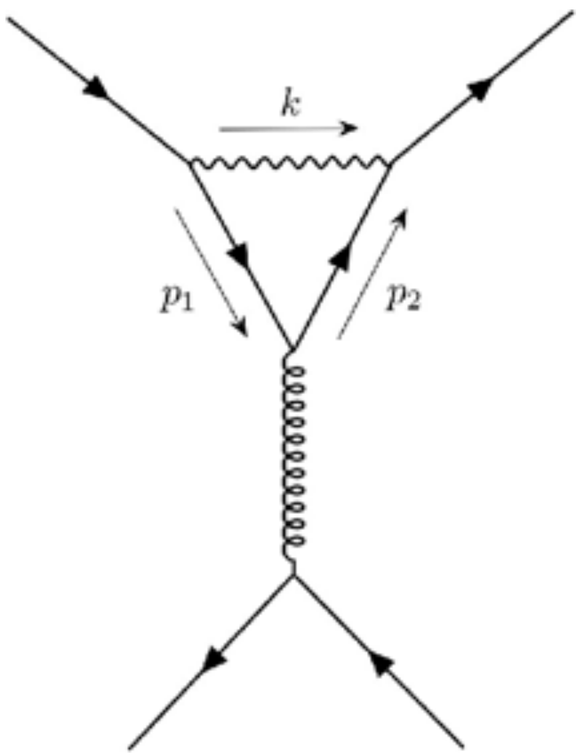


We love them…

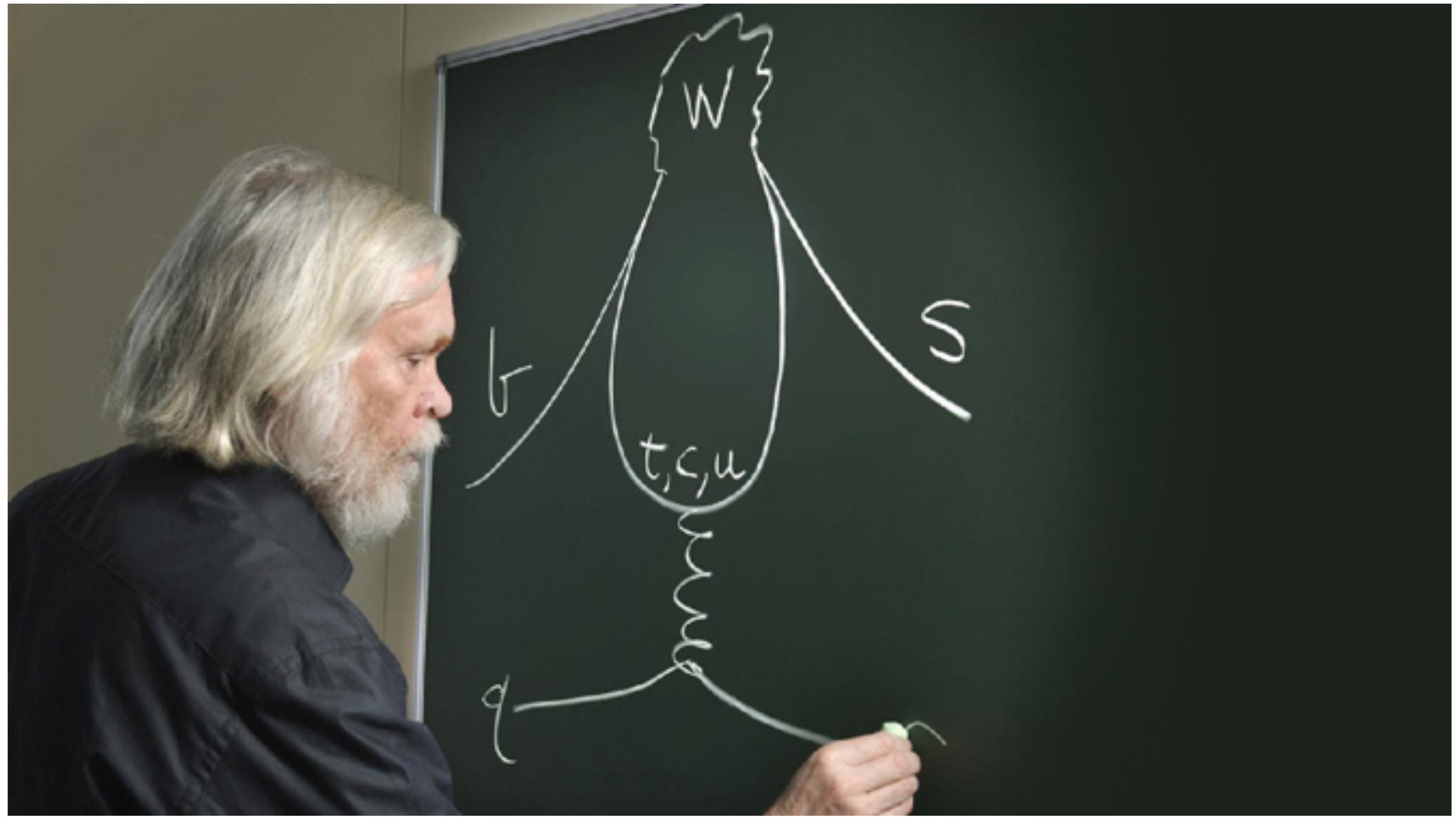


penguin

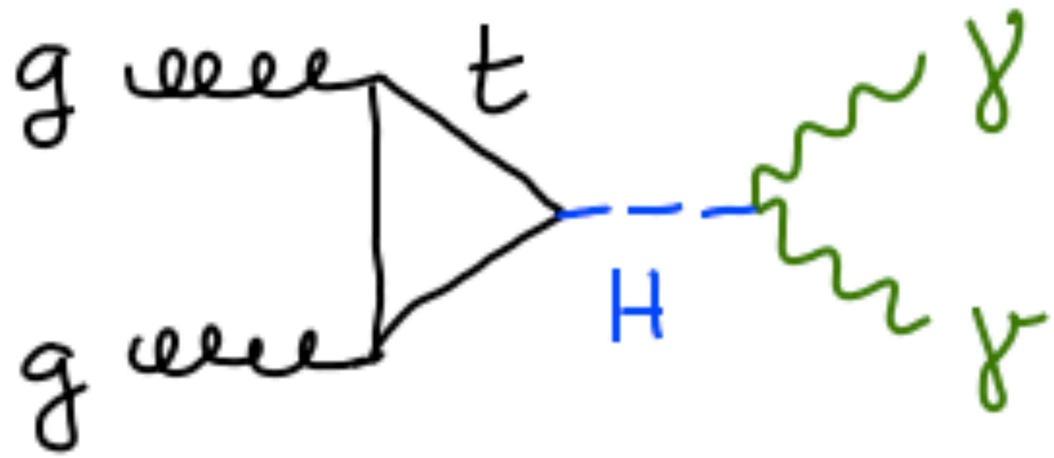
We love them…



penguin



Interpretation?

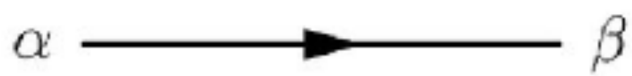


vs.

$$\int \mathscr{D}\phi \, e^{iS[\phi]}$$

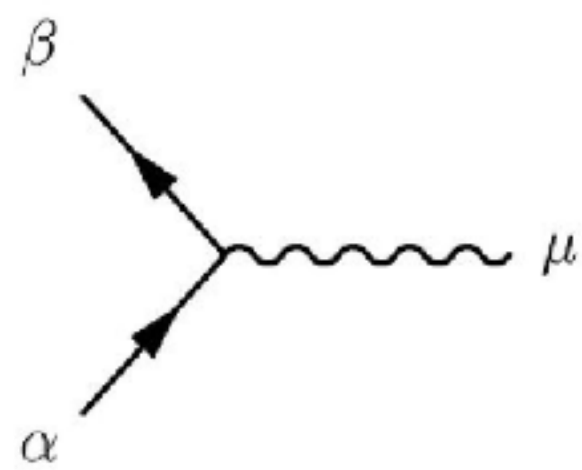
Where the simplicity begins to end:
higher orders

Higher orders

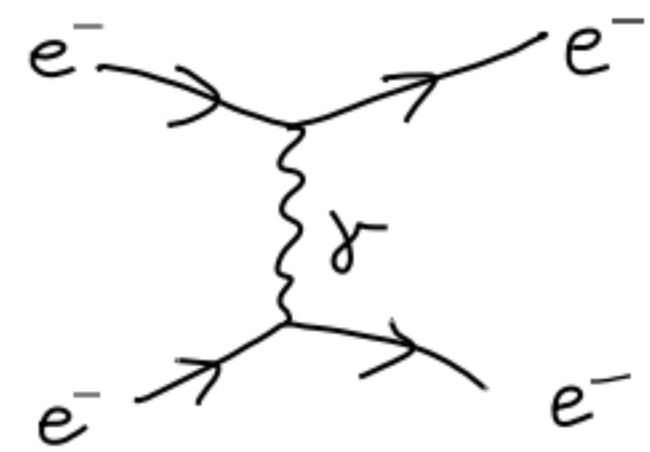


electron e^-
positron e^+

photon γ



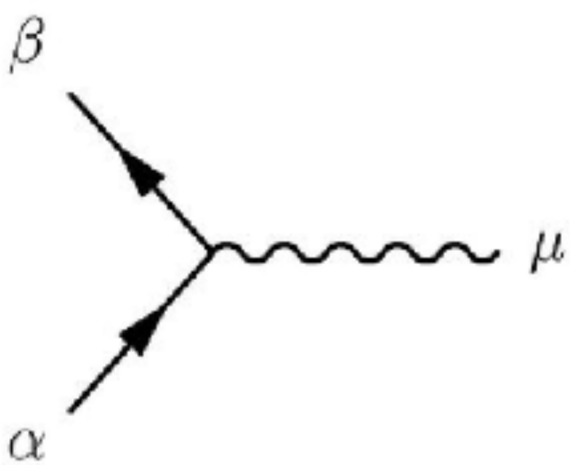
~ e (charge)



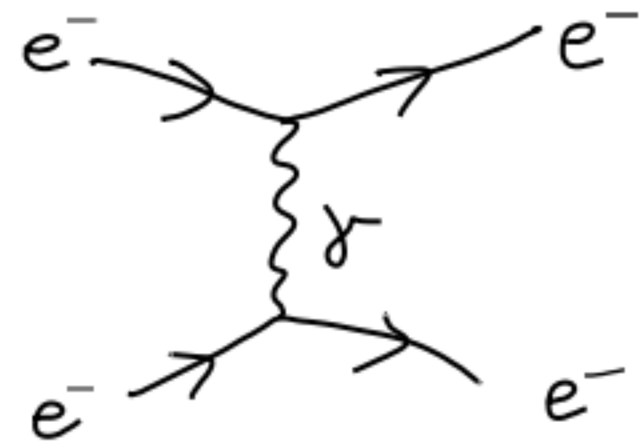
Higher orders

electron e⁻
positron e+

photon γ



~ e (charge)



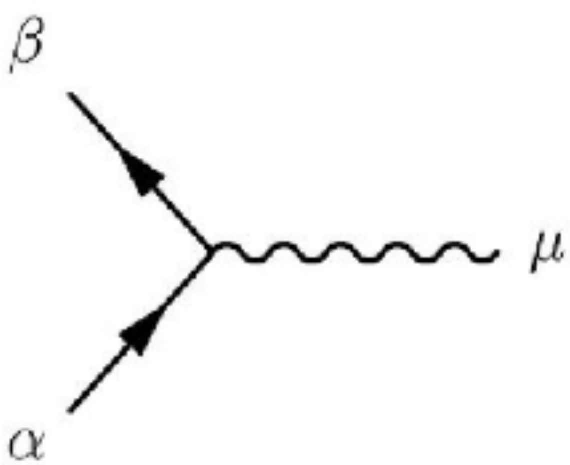
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

Higher orders

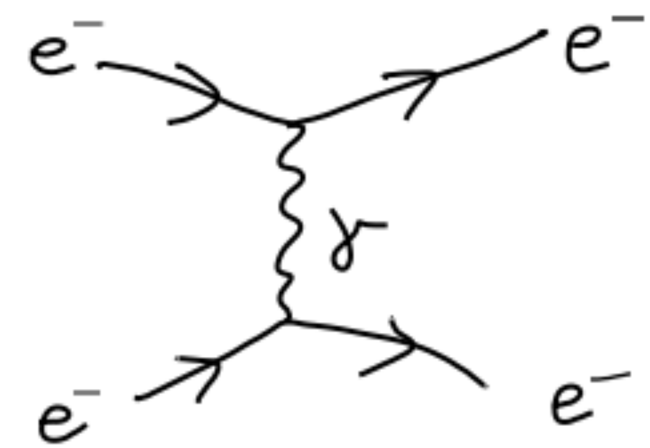
electron e⁻

positron e⁺

photon γ



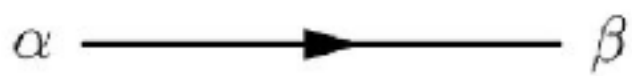
~ e (charge)



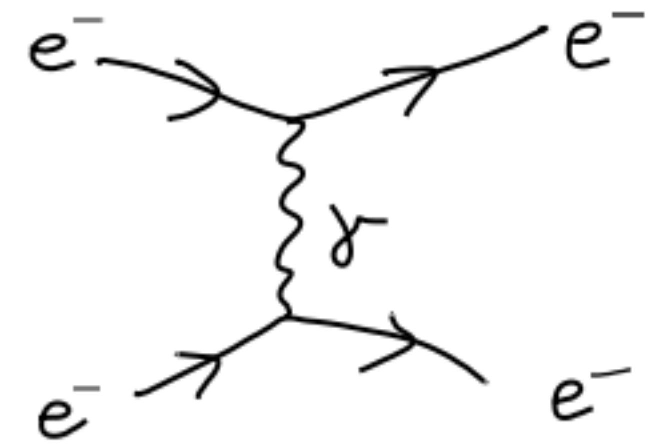
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

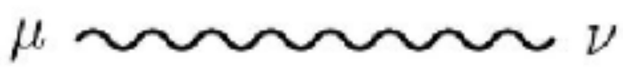
α^2

Higher orders

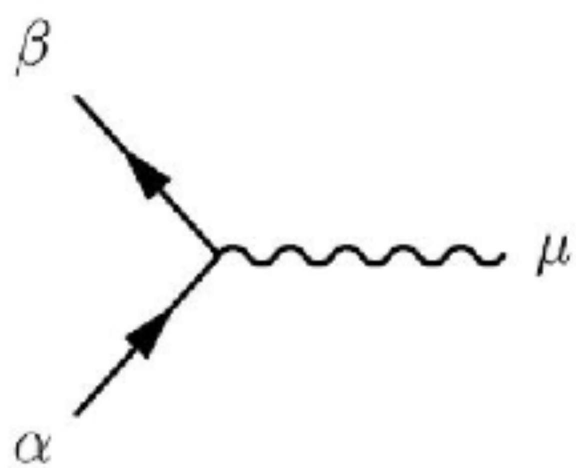


electron e⁻
positron e⁺





photon γ



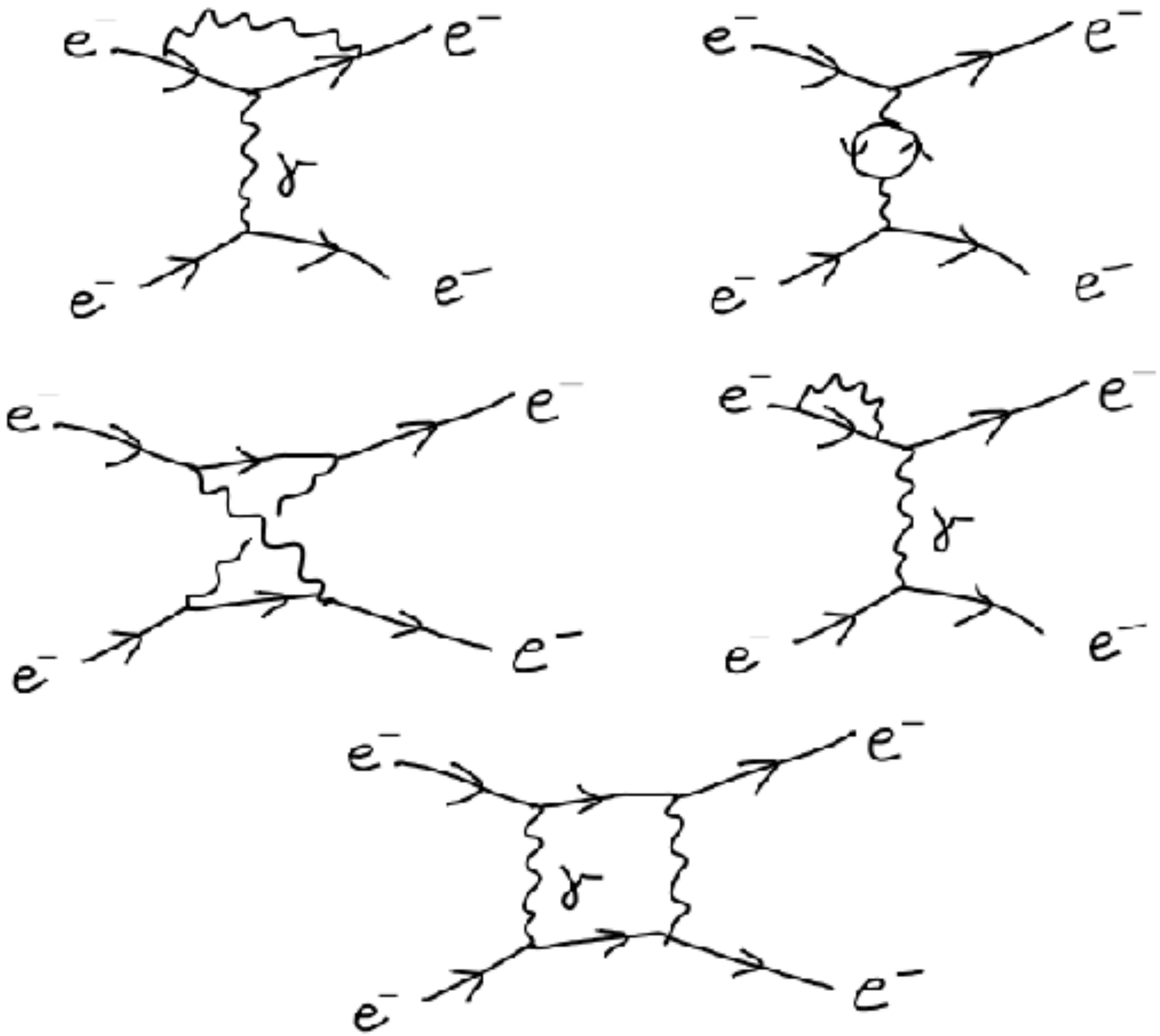
~ e (charge)

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

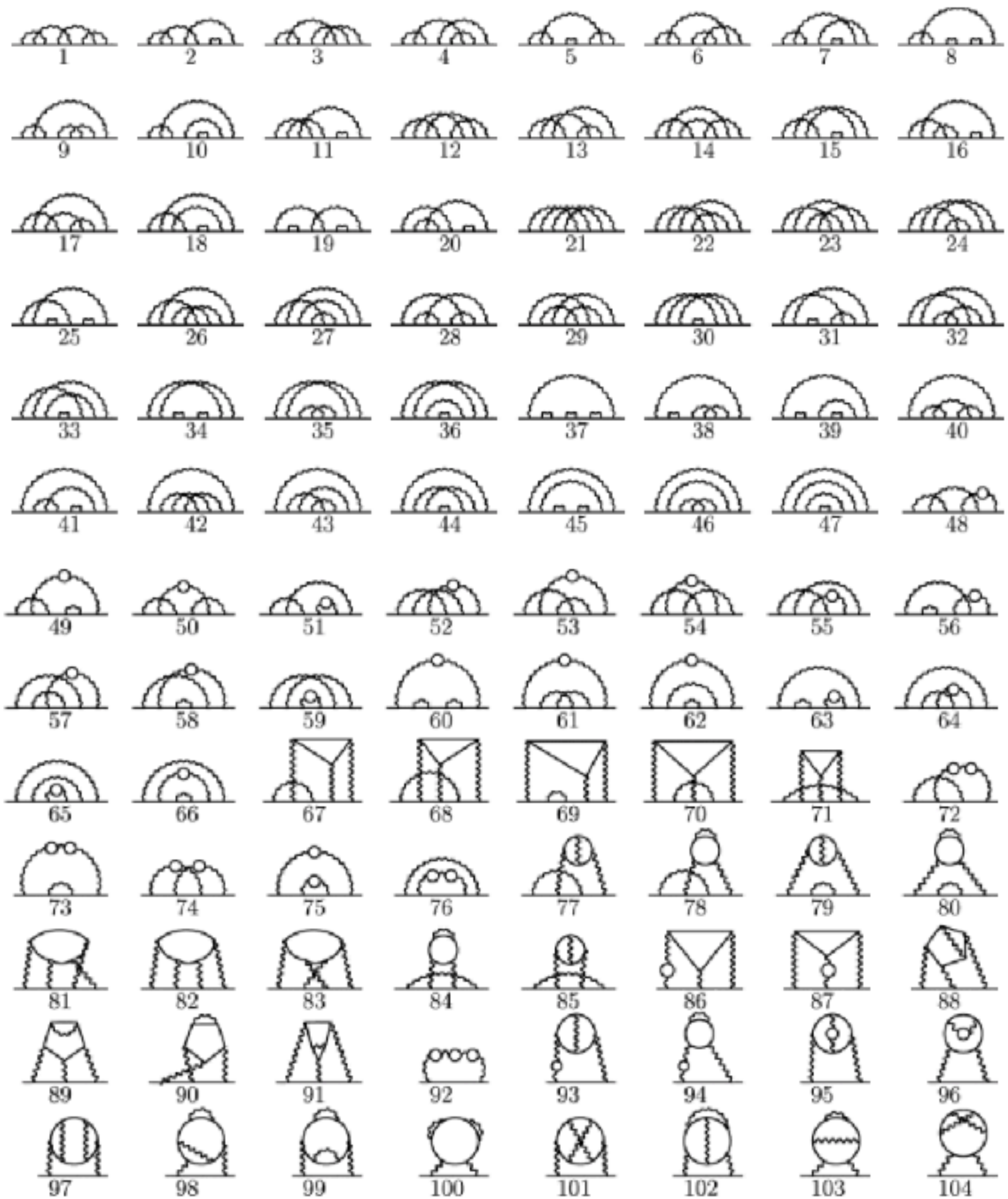
α^2

$$\int d^4k \frac{1}{(k^2 - m^2)(k - p_3)^2[(k - p_3 - p_4)^2 - m^2](k - p_1)^2}$$

Higher orders



- many diagrams
- complicated loop integrals
- divergent integrals



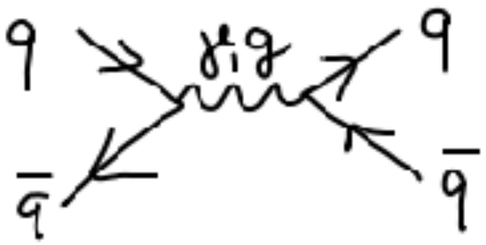
5-loop contributions to the anomalous magnetic moment of the electron

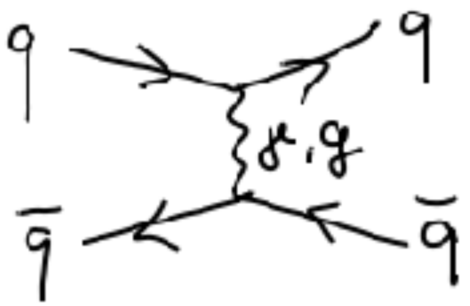
Today's calculations: O(100.000) diagrams

Require high level of automation.

source: https://www.sciencedirect.com/science/article/pii/S0370269317305324

Quark-quark scattering





```
 ------------------------------------------------------------
                        qgraf-3.1.4
 ------------------------------------------------------------


 output= 'qlist.out' ;
 style= 'form.sty' ;
 model= 'qcd';
 in= fq,fQ ;
 out= fq,fQ ;
 loops= 0;
 loop_momentum= ;
options = ;


 ------------------------------------------------------------


  3P  ---  2+  1-  ---  2N+  1C-

  3V  ---  3^3

 ------------------------------------------------------------

  3^2  ---  4 diagrams

  total =  4 diagrams
```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Quark-quark scattering

```
 ----------------------------------------------------------
                        qgraf-3.1.4
 ----------------------------------------------------------


 output= 'qlist.out' ;
 style= 'form.sty' ;
 model= 'qcd';
 in= fq,fQ ;
 out= fq,fQ ;
 loops= 1;
 loop_momentum= ;
options = ;

 ----------------------------------------------------------

  3P  ---  2+  1-  ---  2N+  1C-

  3V  ---  3^3

 ----------------------------------------------------------

  3^4  ---   130 diagrams

  total =  130 diagrams
```

Quark-quark scattering

```
 ----------------------------------------------------------
                        qgraf-3.1.4
 ----------------------------------------------------------


 output= 'qlist.out' ;
 style= 'form.sty' ;
 model= 'qcd';
 in= fq,fQ ;
 out= fq,fQ ;
 loops= 2;
 loop_momentum= ;
options = ;


 ----------------------------------------------------------


  3P   ---   2+  1-   ---   2N+  1C-

  3V   ---   3^3

 ----------------------------------------------------------


  3^6   ---   4058 diagrams

  total =   4058 diagrams
```

Quark-quark scattering

```
 ----------------------------------------------------
                    qgraf-3.1.4
 ----------------------------------------------------

 output= 'qlist.out' ;
 style= 'form.sty' ;
 model= 'qcd';
 in= fq,fQ ;
 out= fq,fQ ;
 loops= 3;
 loop_momentum= ;
options = ;

 ----------------------------------------------------

  3P  ---  2+  1-  ---  2N+  1C-

  3V  ---  3^3

 ----------------------------------------------------

  3^8  ---   134146 diagrams

  total =  134146 diagrams
```

Quark-quark scattering

```
*--#[ d222:
*

     -1
    *vx(fq(-1),fQ(-3),g(1))
    *vx(fq(-4),fQ(-2),g(2))
    *vx(fq(4),fQ(3),g(1))
    *vx(g(2),g(5),g(6))
    *vx(fq(3),fQ(7),g(5))
    *vx(fq(9),fQ(4),g(8))
    *vx(g(6),g(8),g(10))
    *vx(fq(7),fQ(9),g(10))
*
*--#] d222:
*--#[ d223:
*

     1
    *vx(fq(-1),fQ(-3),g(1))
    *vx(fq(-4),fQ(-2),g(2))
    *vx(g(1),g(3),g(4))
    *vx(fq(5),fQ(6),g(2))
    *vx(fq(7),fQ(5),g(3))
    *vx(fq(8),fQ(9),g(4))
    *vx(fq(6),fQ(8),g(10))
    *vx(fq(9),fQ(7),g(10))
*
*--#] d223:
```

Who ever looks at 134146 Feynman diagrams?

Surely you're joking…