The Gradient Flow Formalism in Perturbation Theory

Robert Harlander

RWTH Aachen University

17 April 2024

Loops and Legs in Quantum Field Theory
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No motivation

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Properties and uses of the Wilson flow in lattice QCD #8

Martin Lüscher (CERN and Geneva U.) (Jun 23, 2010)

Published in: *JHEP* 08 (2010) 071, *JHEP* 03 (2014) 092 (erratum) • e-Print: 1006.4518 [hep-lat]

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Infinite N phase transitions in continuum Wilson loop operators #5

R. Narayanan (Florida Intl. U.), H. Neuberger (Rutgers U., Piscataway) (Jan, 2006)

Published in: *JHEP* 03 (2006) 064 • e-Print: hep-th/0601210 [hep-th]

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$$\frac{\partial}{\partial t} B_\mu(t) = \mathscr{D}_\nu(t)\, G_{\nu\mu}(t) \qquad\qquad B_\mu(t = 0) = A_\mu$$

$$G_{\mu\nu}(t) = \frac{i}{g_0}[\mathscr{D}_\mu(t), \mathscr{D}_\nu(t)]$$

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Perturbative solution

flow equation: $\partial_t B \sim \partial^2 B + g_0 \partial B^2 + g_0^2 B^3$ $\qquad B_\mu(t=0) = A_\mu$

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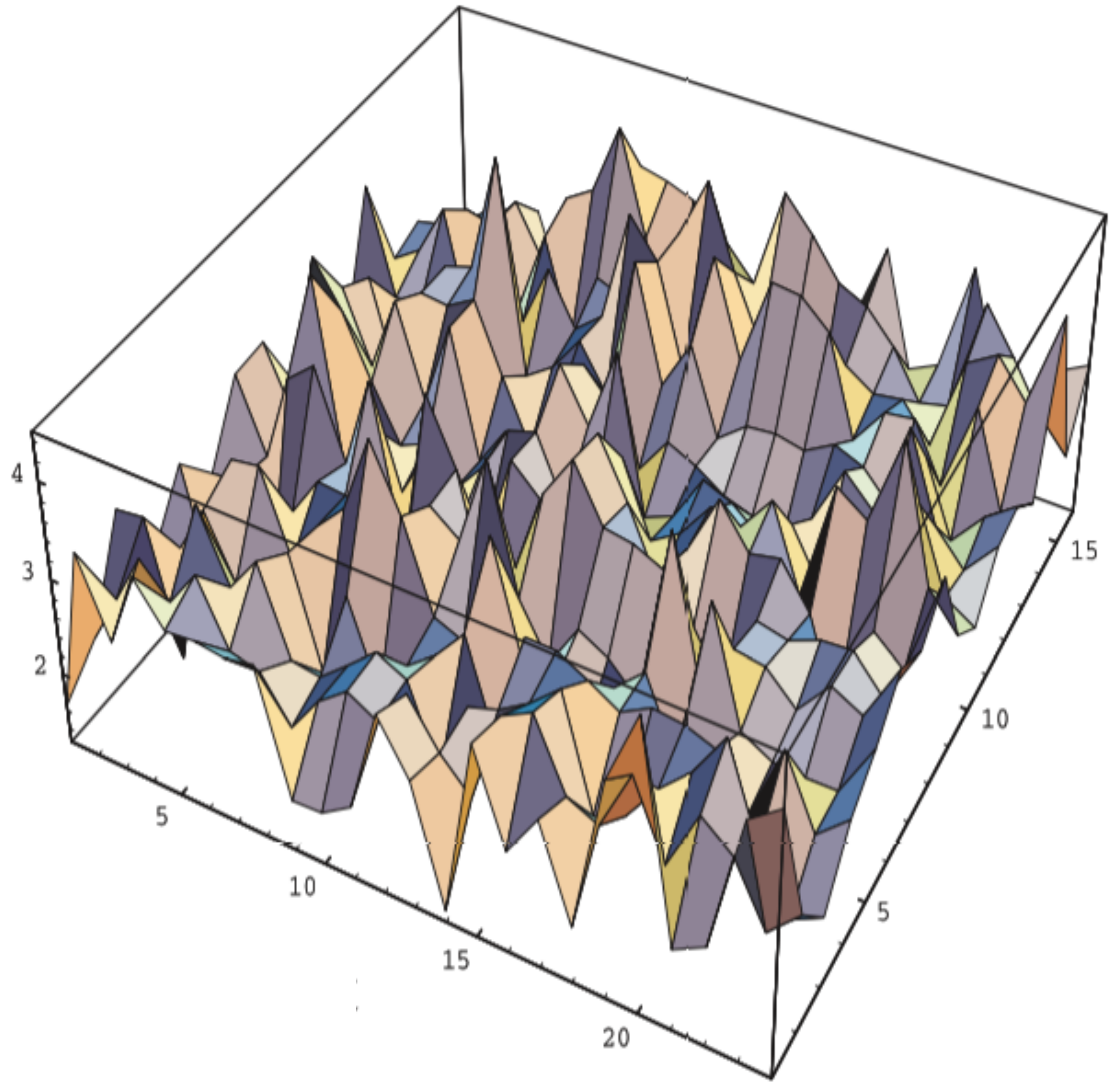

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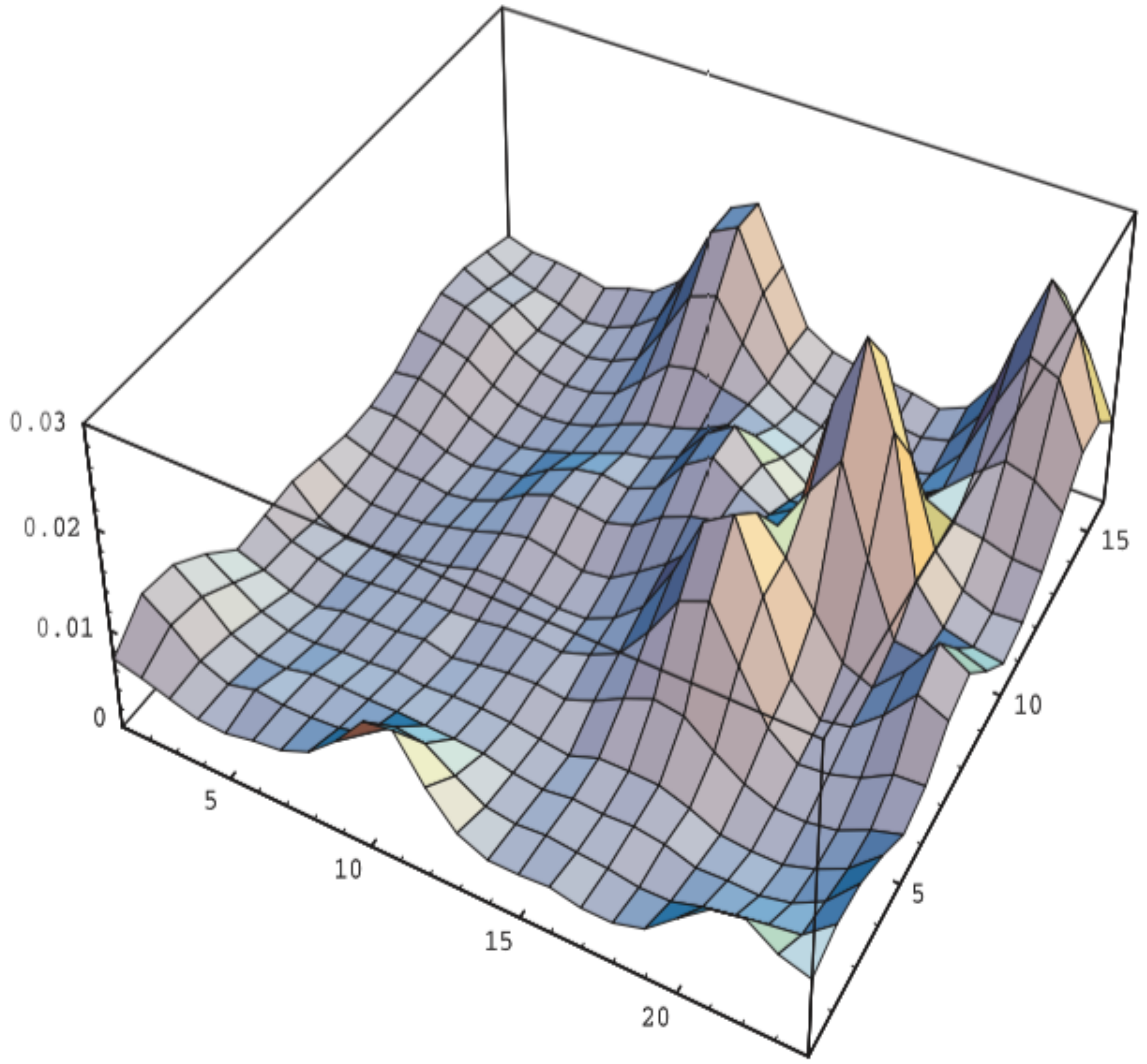

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$$\tilde{B}_1(t,p) = e^{-tp^2}\tilde{A}(p)$$

$$\tilde{B}_2(t,p) = \int_0^t ds \int d^4q\, K(t,s,p,q)\,\tilde{A}(p)\,\tilde{A}(p-q)$$

$$K(t,s,p,q) \sim \exp\left[-tp^2 - 2sq(q-p)\right]$$

etc.

Exponential damping in momentum integrals!

Quantum field theory

$$\mathscr{L} = \mathscr{L}_{\rm QCD} + \mathscr{L}_B$$

$$\mathscr{L}_B \sim \int_0^\infty dt\, L_\mu \left(\partial_t B_\mu - \mathscr{D}_\nu G_{\nu\mu}\right)$$

L_μ Lagrange multiplier field

Lüscher, Weisz 2011

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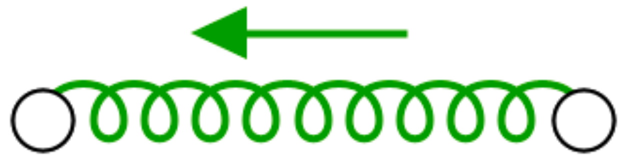

$$\delta_{ab}\delta_{\mu\nu}\,\theta(t-s)\, e^{-(t-s)p^2}$$

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“gluon flow line”

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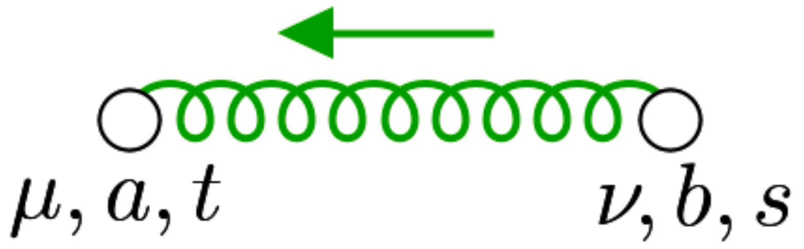

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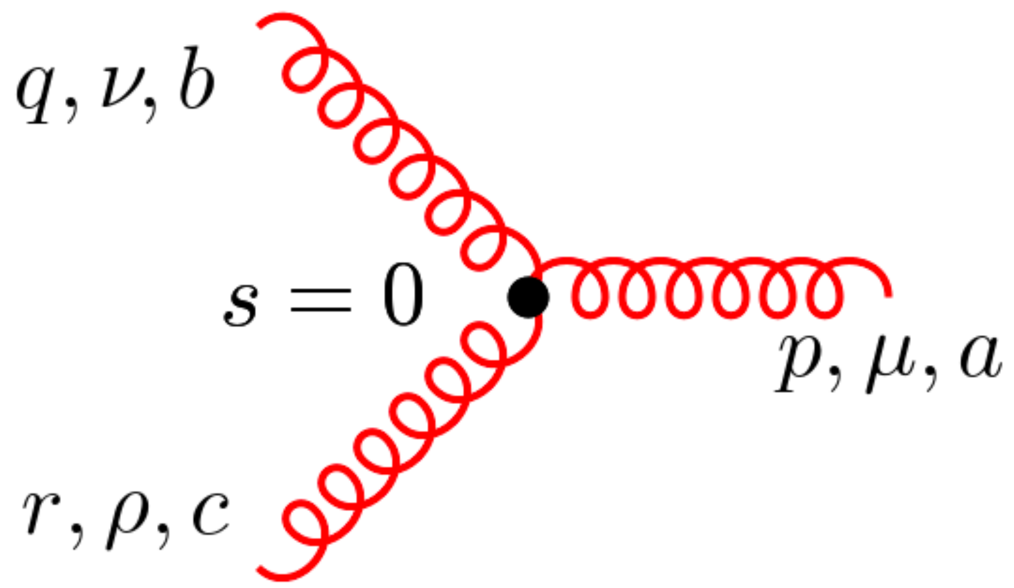
“gluon flow line”

analogously for quarks: Lüscher 2013

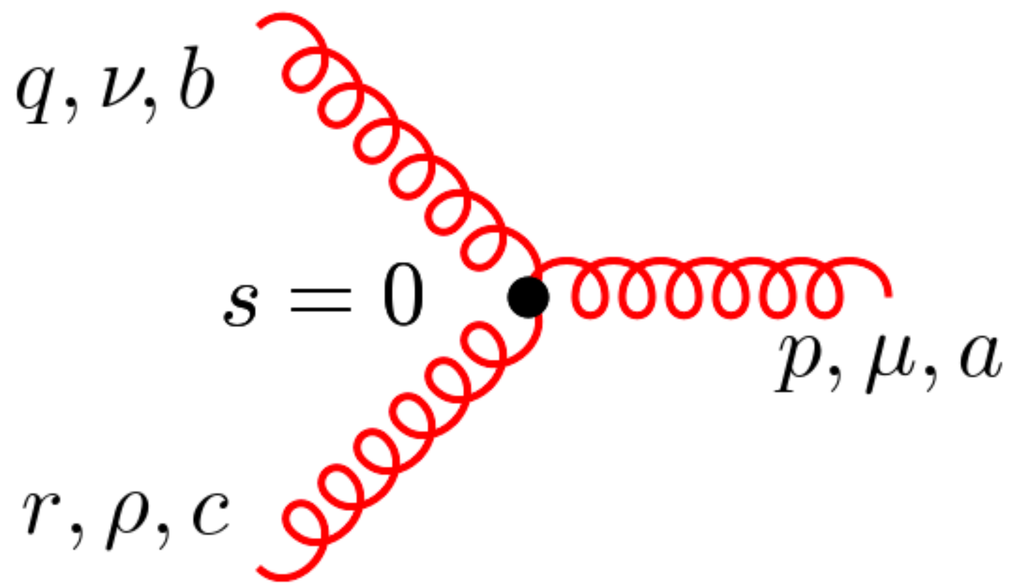
$$\mathscr{L}_\chi \sim \int_0^\infty dt\, \bar{\lambda} \left(\partial_t - \Delta\right) \chi + \mathrm{h.c.}$$

Vertices



regular 3-gluon vertex

Vertices



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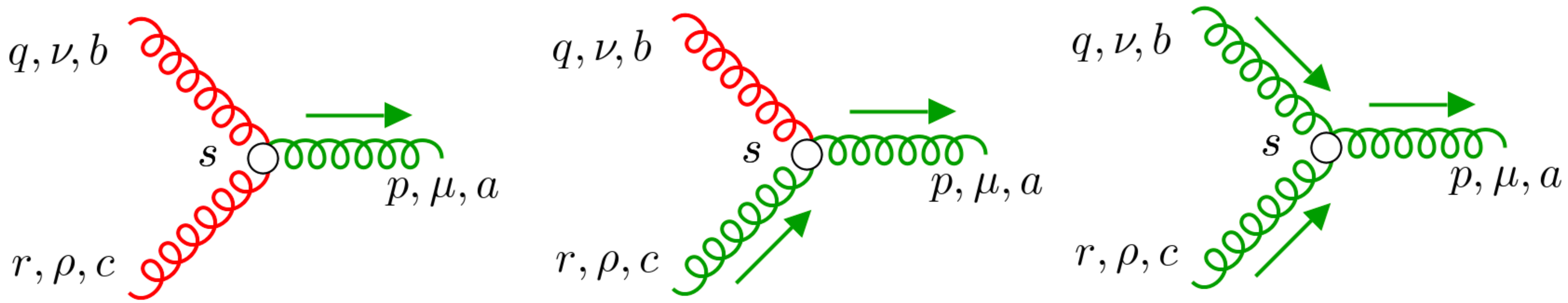

$$-\mathrm{i}gf^{abc}\int_0^\infty \mathrm{d}s\,\big(\delta_{\nu\rho}(r-q)_\mu + 2\delta_{\mu\nu}q_\rho - 2\delta_{\mu\rho}r_\nu + (\kappa-1)(\delta_{\mu\rho}q_\nu - \delta_{\mu\nu}r_\rho)\big)$$

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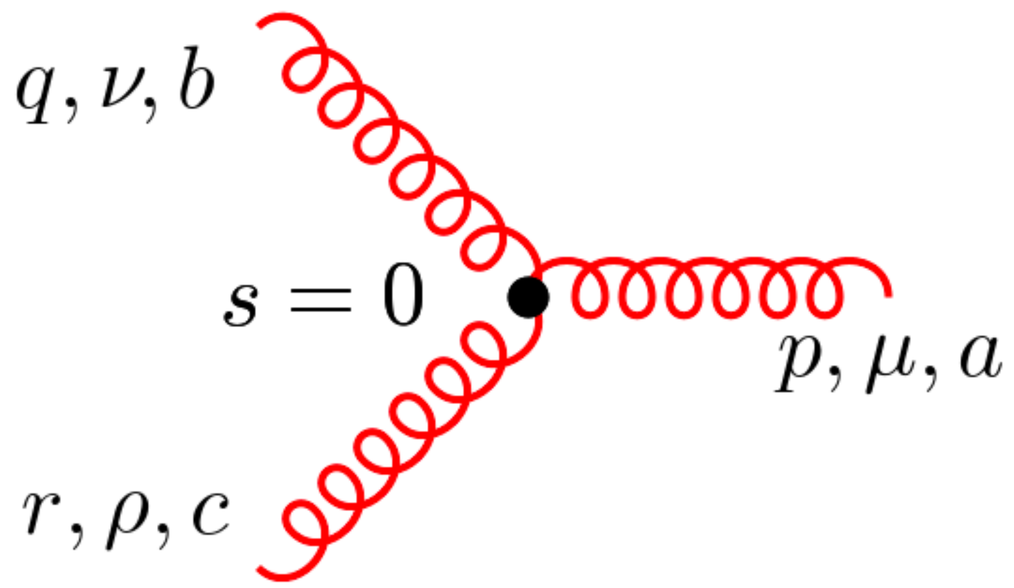

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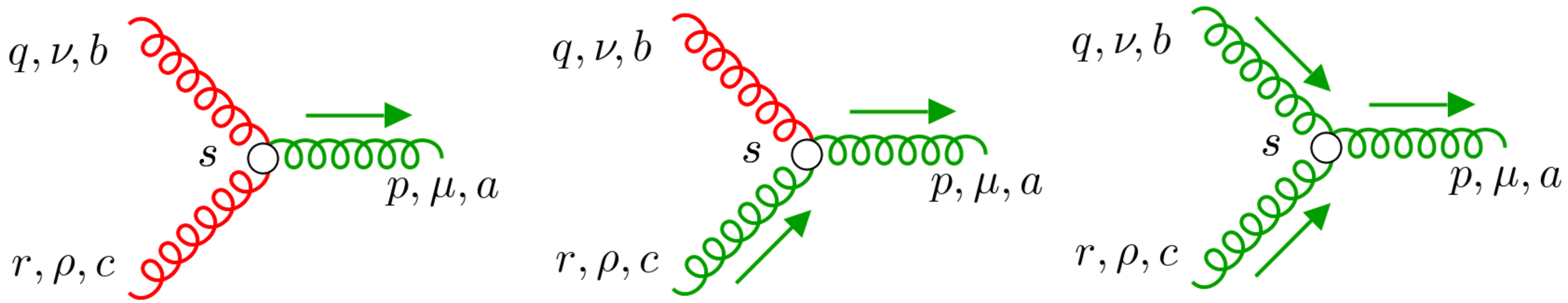

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analogously for 4-gluon vertex and quarks

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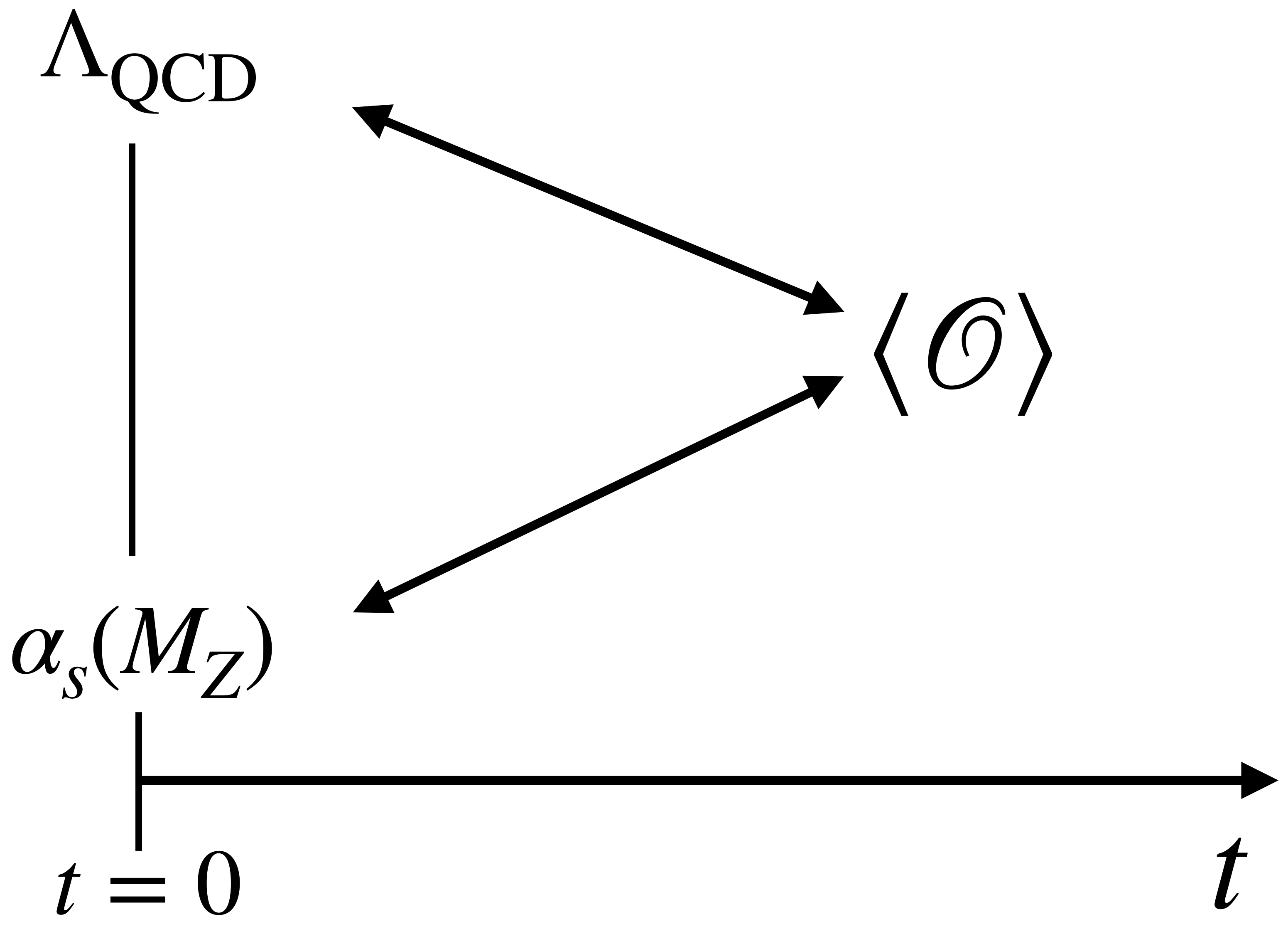

Let's calculate

$$\langle E(t)\rangle \equiv \frac{1}{4}\langle G^a_{\mu\nu}(t)G^{a,\mu\nu}(t)\rangle$$

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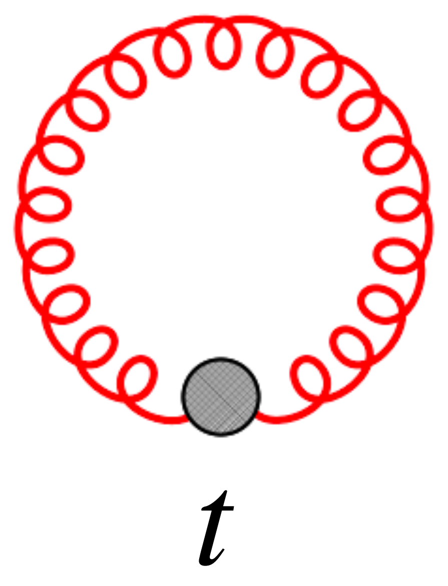
LO: 

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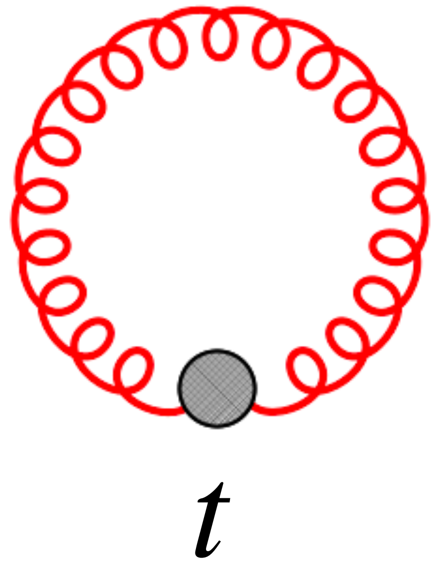

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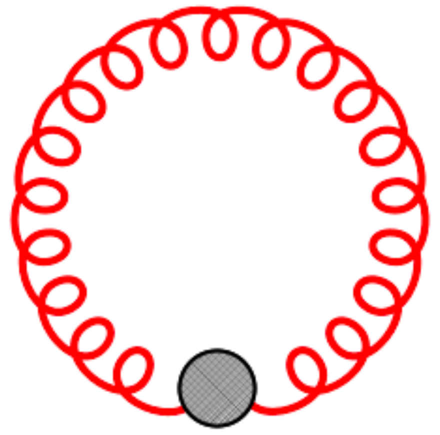

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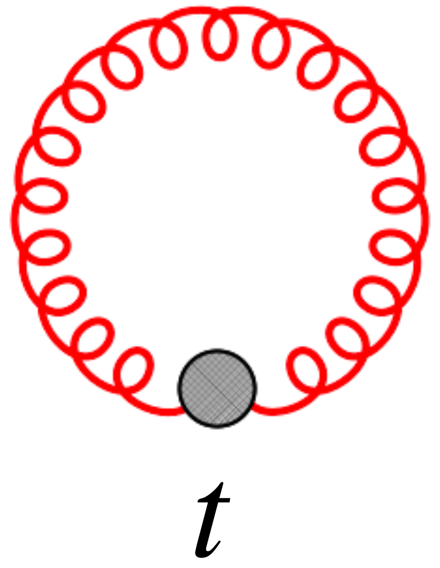

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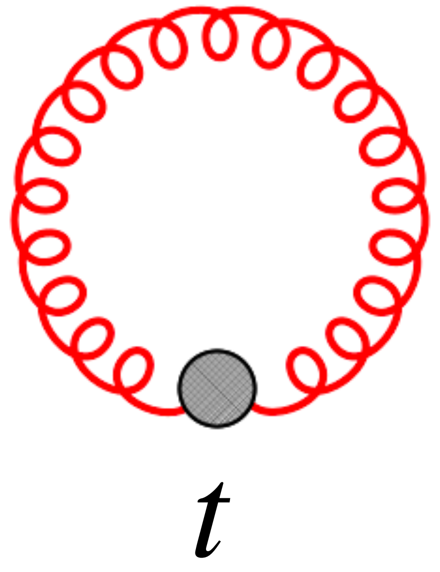

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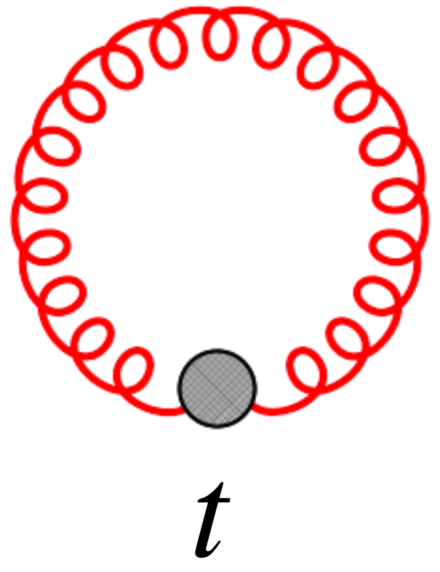

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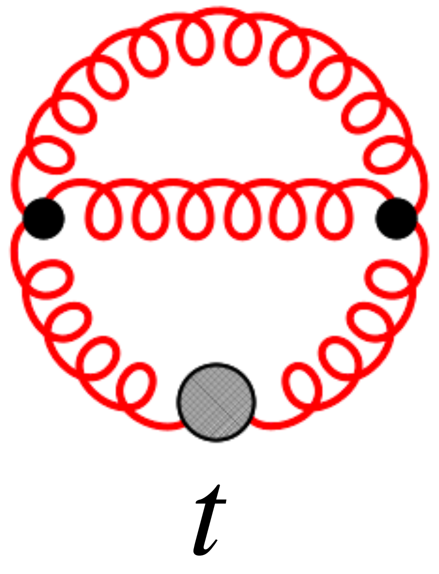
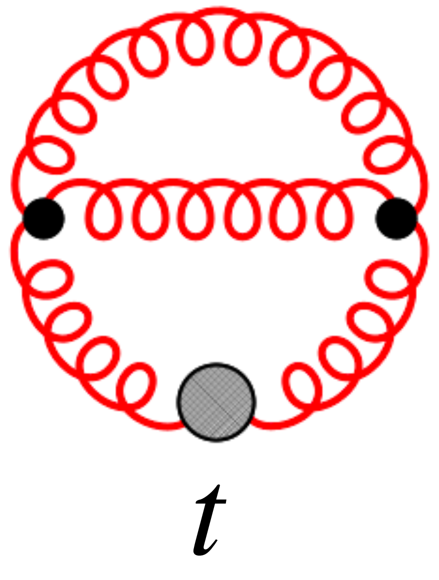
$$\alpha_s = \alpha_s(\mu)$$

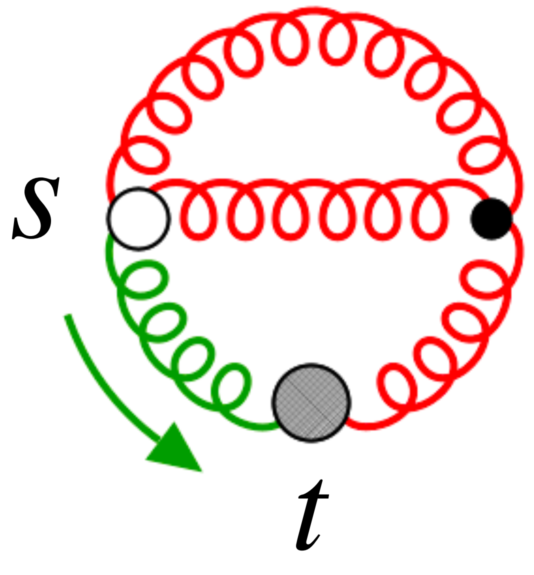
Higher orders



$$\sim \int_p \int_k \frac{e^{-2\,t\,p^2}}{p^4\,k^2\,(p-k)^2}$$

Higher orders





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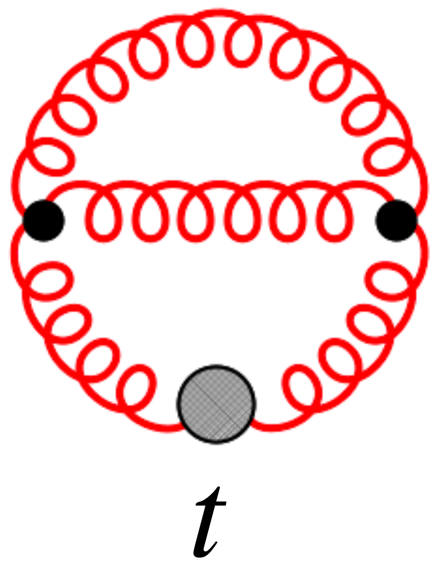

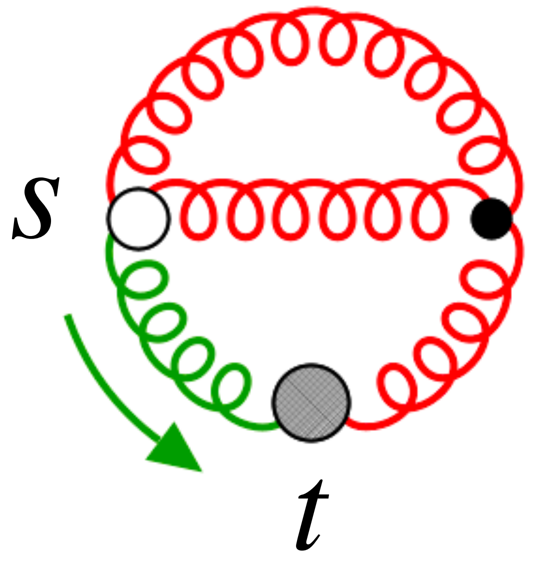

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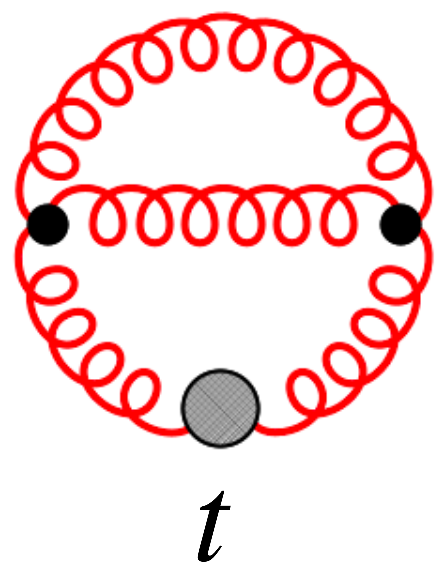
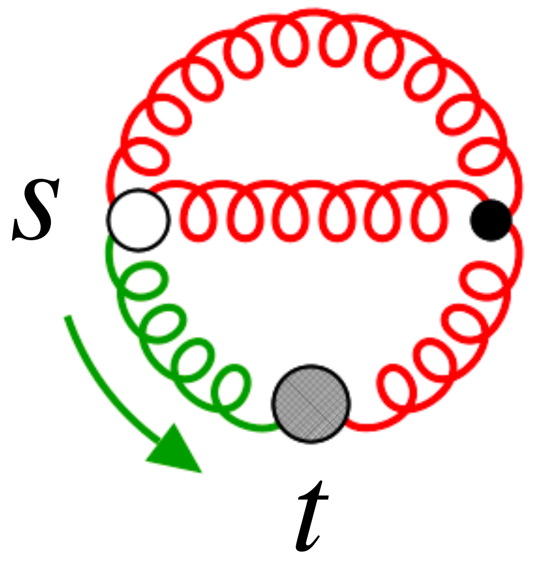
- generalized loop integrals

Higher orders





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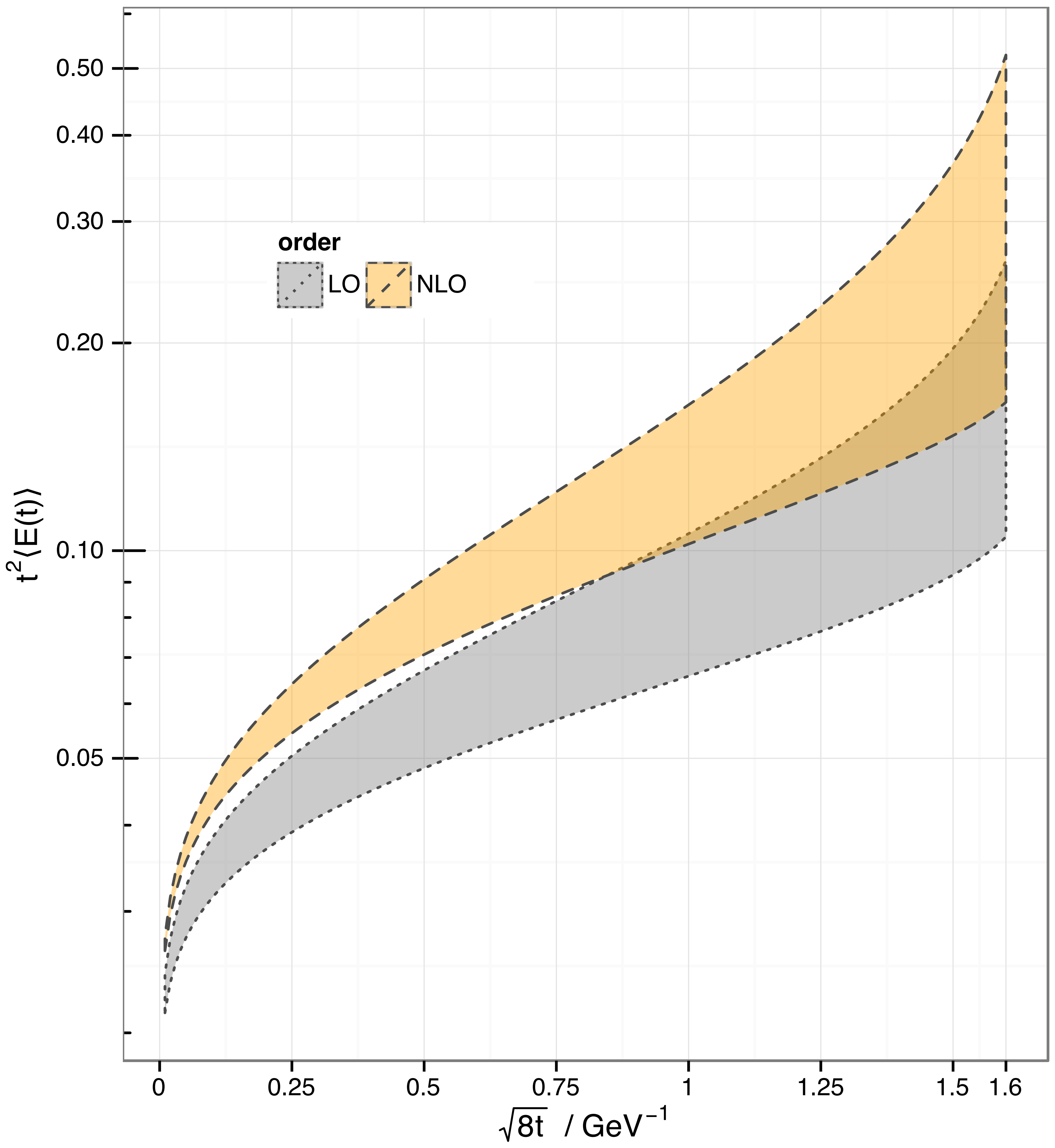
- generalized loop integrals
- integration over flow-time parameters

$$\langle t^2 E(t) \rangle = \frac{3\alpha_s(\mu)}{4\pi} \left[1 + k_1(t,\mu)\,\alpha_s(\mu)\right]$$

Lüscher 2010



$$k_1 = \left(\frac{52}{9} + \frac{22}{3}\ln 2 - 3\ln 3\right) C_A - \frac{8}{9} n_f T_R + \beta_0 L_{t\mu}$$

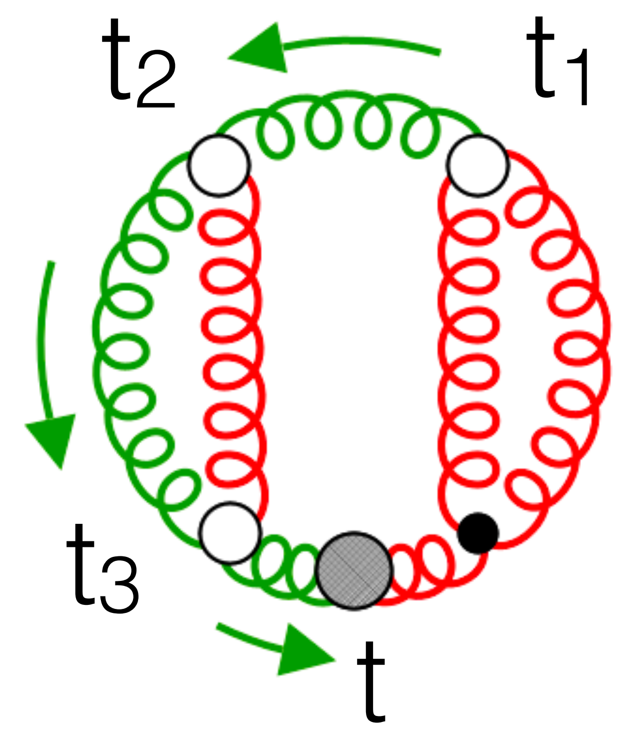
$$L_{t\mu} = \ln 2\mu^2 t + \gamma_E$$

$$\mu_0 = \frac{1}{\sqrt{8t}}$$

resulting perturbative accuracy on α_s: ± 3-5%

PDG: ± 1%

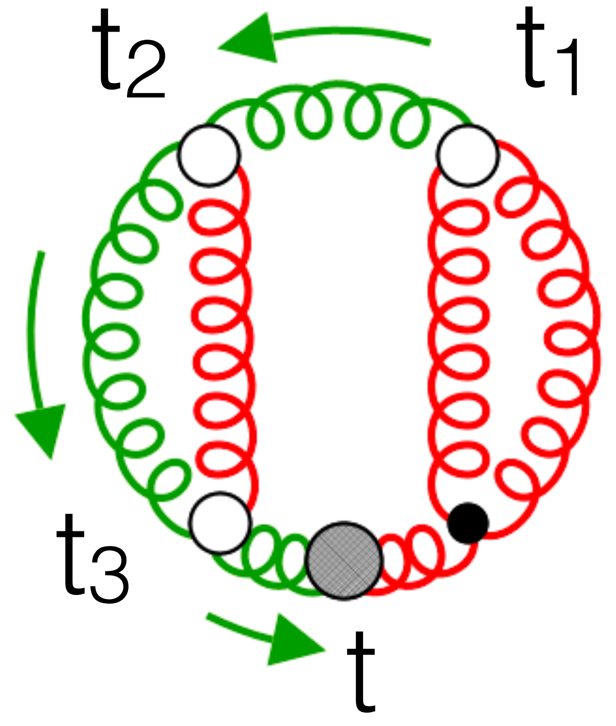
Three-loop calculation



Three-loop calculation

$$I(\{c_1,\cdots,c_f\},\{a_1(u),\cdots,a_6(u)\},\{b_1,\cdots,b_6\}) =$$

$$= \left(\prod_{i=1}^{f}\int_0^1 du_i\, u_i^{c_i}\right)\int d^Dp_1\, d^Dp_2\, d^Dp_3\, \frac{\exp\left[-t\left(a_1(u)p_1^2+\cdots+a_6(u)p_6^2\right)\right]}{(p_1^2)^{b_1}\cdots(p_6^2)^{b_6}}$$



$$u_i = t_i/t$$

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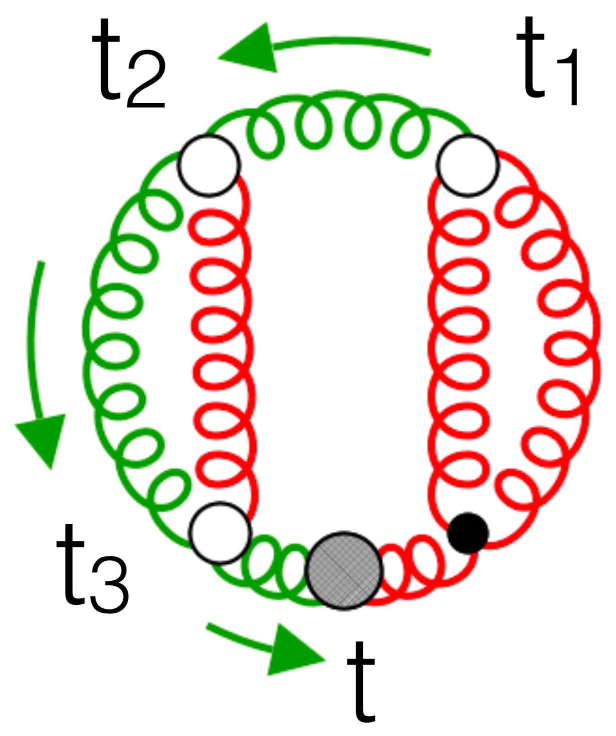

$$u_i = t_i/t$$

IbP identities: $\frac{\partial}{\partial p_i}\cdot p_j \;\; I(\cdots)$ → modifies c_k and b_k

Three-loop calculation

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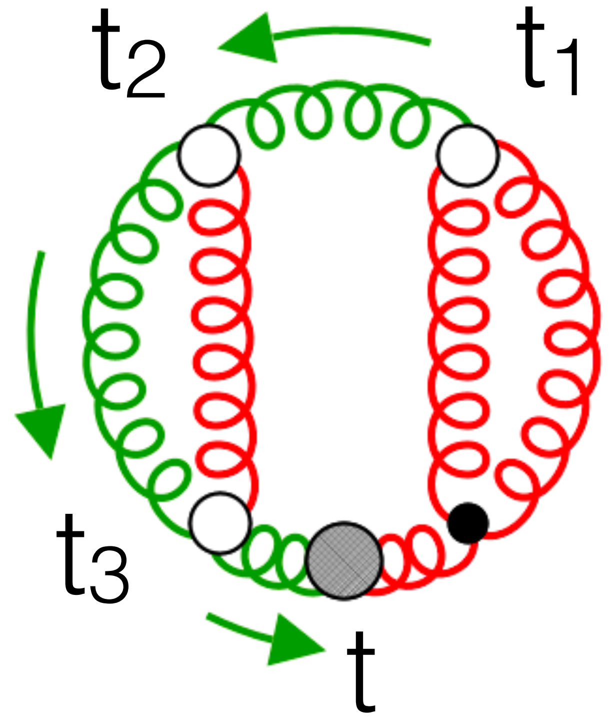
$$= \left(\prod_{i=1}^{f}\int_0^1 du_i\, u_i^{c_i}\right)\int d^Dp_1\, d^Dp_2\, d^Dp_3\, \frac{\exp\left[-t\left(a_1(u)p_1^2+\cdots+a_6(u)p_6^2\right)\right]}{(p_1^2)^{b_1}\cdots(p_6^2)^{b_6}}$$



$u_i = t_i/t$

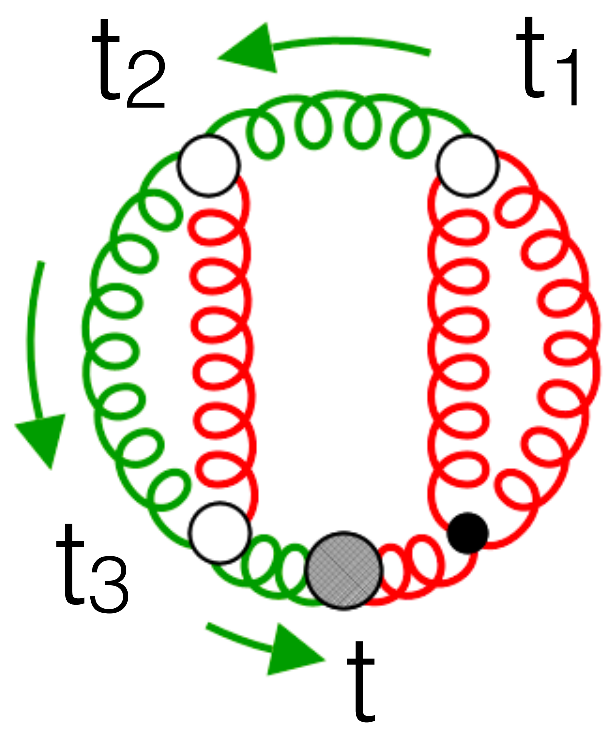
IbP identities: $\frac{\partial}{\partial p_i}\cdot p_j\ \ I(\cdots)$ → modifies c_k and b_k

$$\frac{\partial}{\partial u_i}\ I(\cdots) = I(\cdots)\Big|_{u_i=1} - I(\cdots)\Big|_{u_i=0}$$ → modifies c_k, b_k and a_k

Artz, RH, Lange, Neumann, Prausa '19

Numerical evaluation

RH, Neumann (2016)



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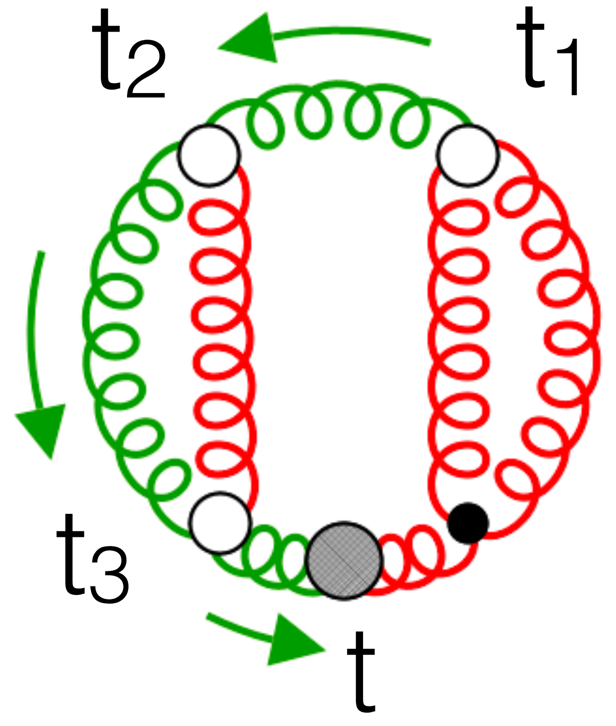

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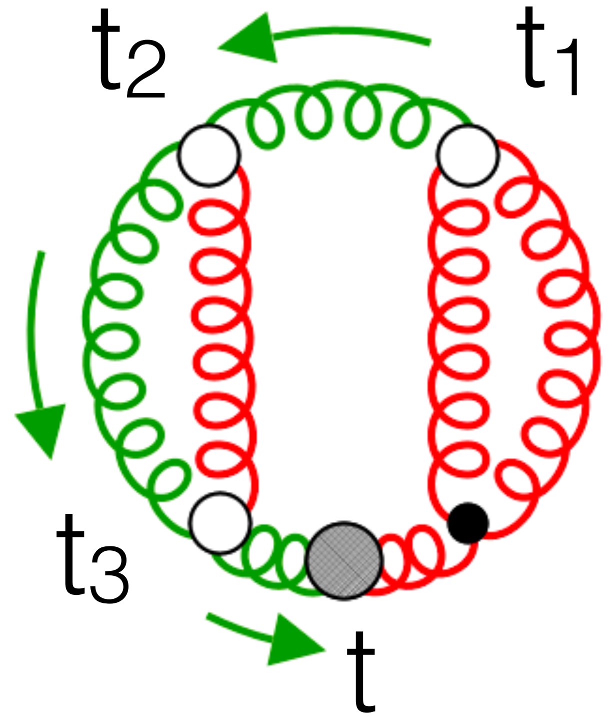

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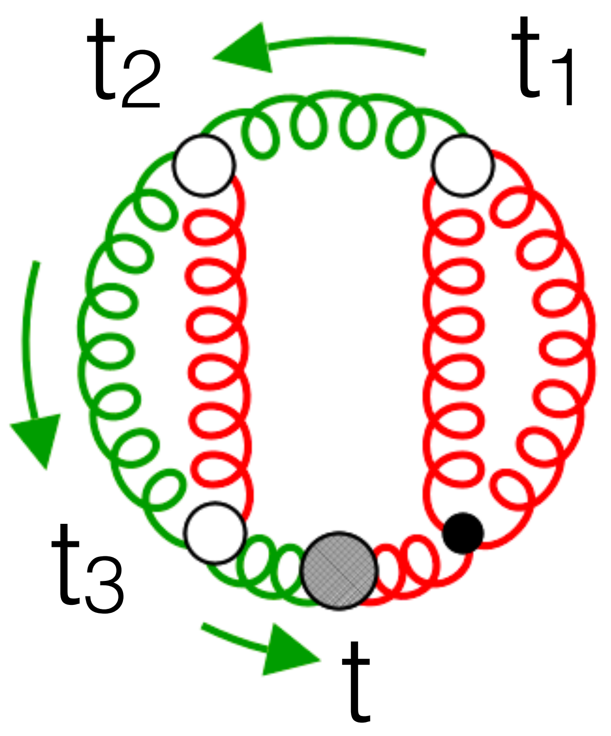

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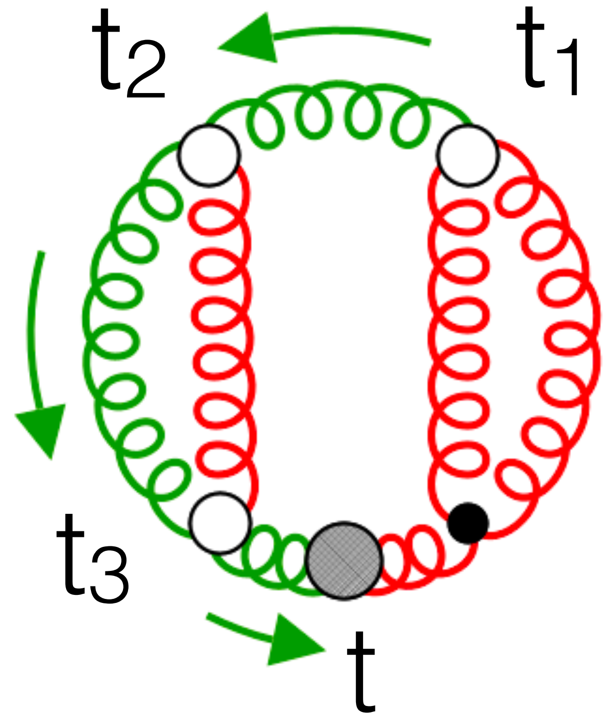

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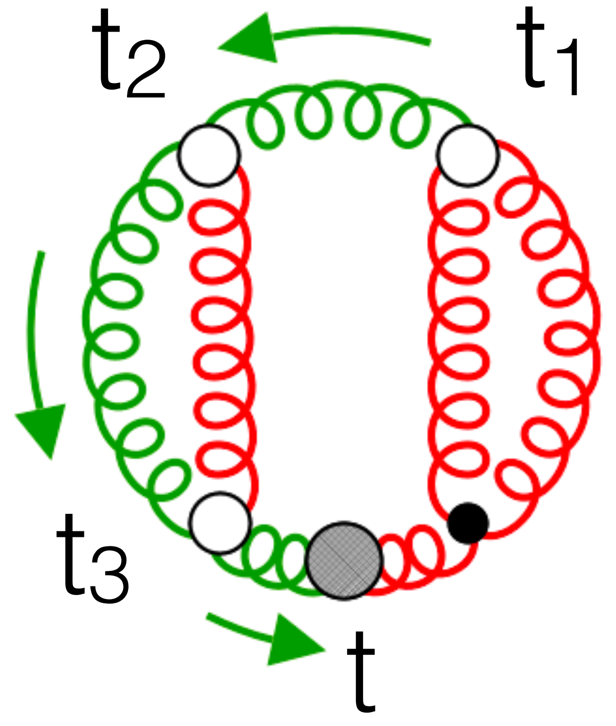

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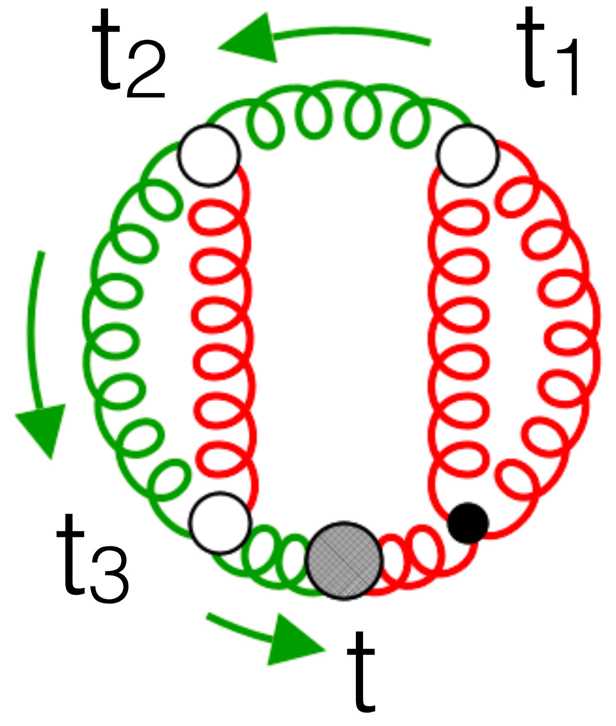

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→ sector decomposition

Binoth, Heinrich (2000)

Implementation

$$I(\{c_1, \cdots, c_f\}, \{a_1(u), \cdots, a_6(u)\}, \{b_1, \cdots, b_6\})$$

$$c_1 = c_2 = 0$$

$$a_1 = u_1 u_2\,, \quad a_2 = u_2\,, \quad a_3 = u_2 - u_1 u_2$$

$$a_4 = 1\,, \quad a_5 = 1 + u_1 u_2\,, \quad a_6 = 1 - u_2$$

$$b_1 = b_4 = 1$$

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ftint RH, Nellopoulos, Olsson (in prep)

(based on pySecDec)

Heinrich, Magerya, Kerner, Jones, …

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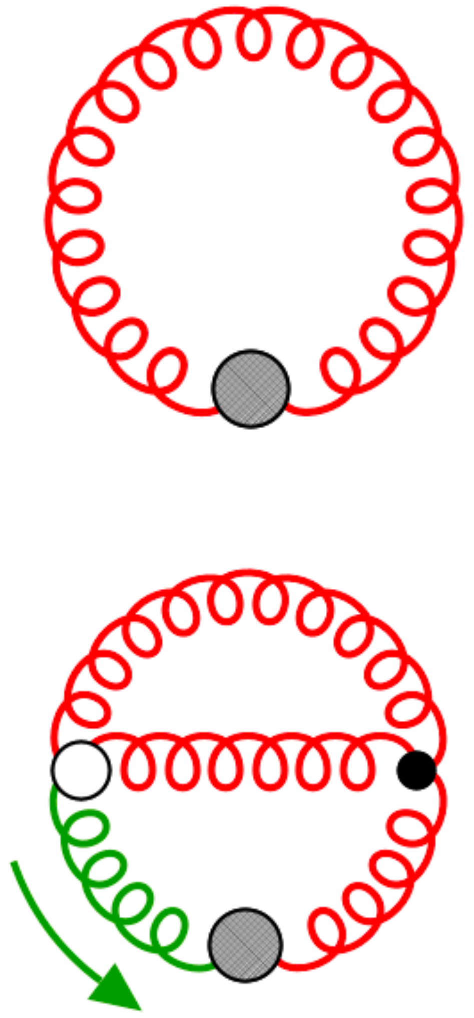
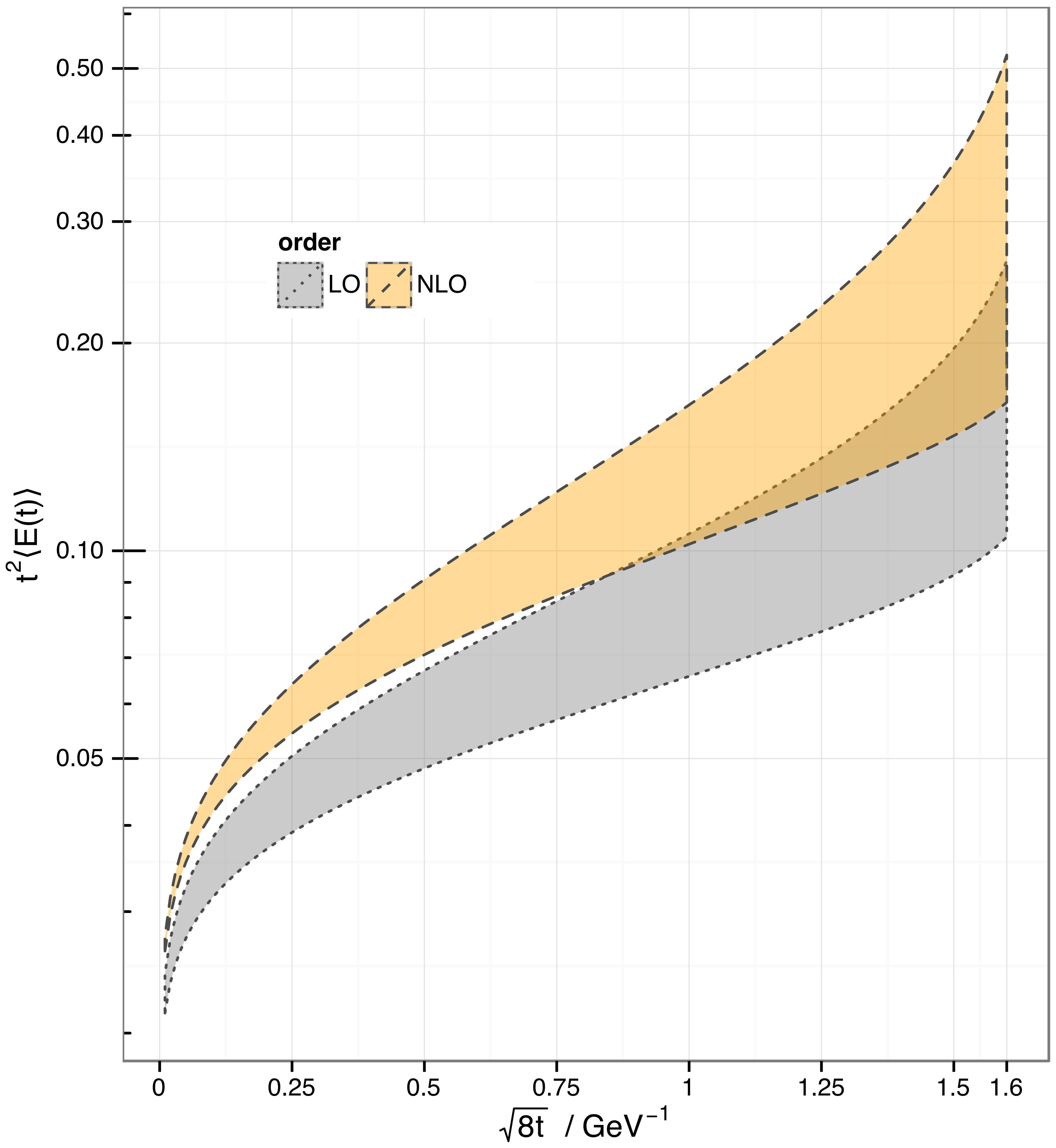
```
f[{{0,0},{u1*u2,u2,u2-u1*u2,1,1+u1*u2,1-u2}},{1,0,0,1,0,0}] -> (
    +eps^-1*(+8.333333333333343*10^-02+0.0000000000000000*10^+00*I)
    +eps^-1*(+1.4433895444086145*10^-15+0.0000000000000000*10^+00*I)*plusminus
    +eps^0*(+3.023827028456263*10^-01+0.0000000000000000*10^+00*I)
    +eps^0*(+1.6918362746499228*10^-08+0.0000000000000000*10^+00*I)*plusminus
    +eps^1*(+6.553101045801212*10^-01+0.0000000000000000*10^+00*I)
    +eps^1*(+3.7857260802916662*10^-08+0.0000000000000000*10^+00*I)*plusminus
  ),
```

$$\langle t^2 E(t)\rangle = \frac{3\alpha_s(\mu)}{4\pi}\left[1 + k_1(t,\mu)\,\alpha_s(\mu)\right]$$

Lüscher 2010





$$k_1 = \left(\frac{52}{9} + \frac{22}{3}\ln 2 - 3\ln 3\right) C_A - \frac{8}{9} n_f T_R + \beta_0 L_{t\mu}$$

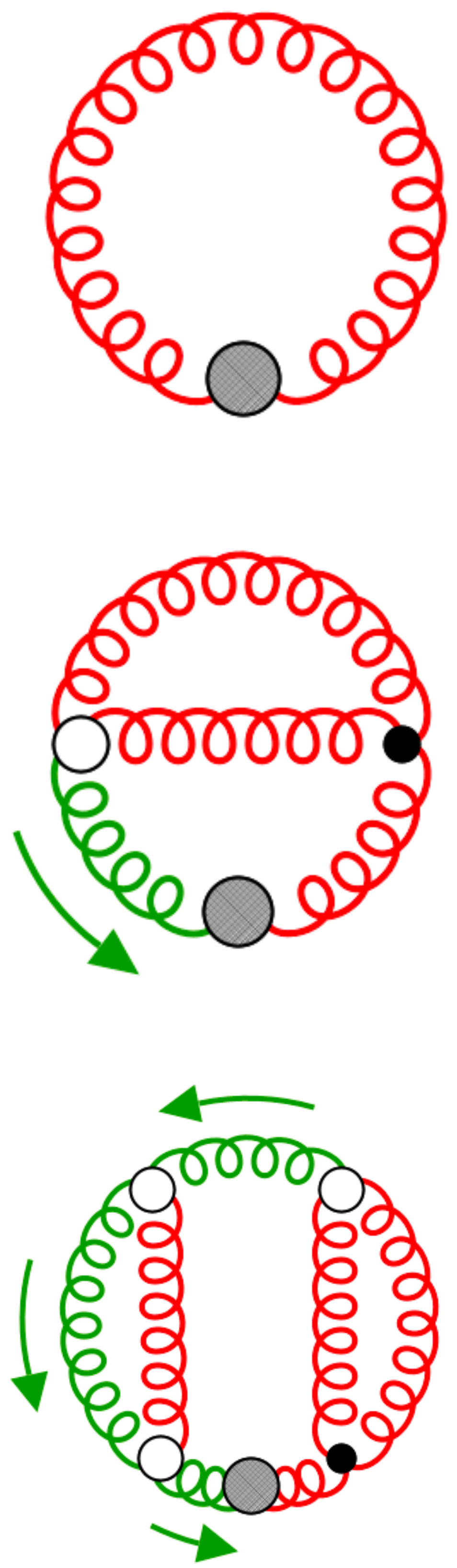
$$L_{t\mu} = \ln 2\mu^2 t + \gamma_E$$

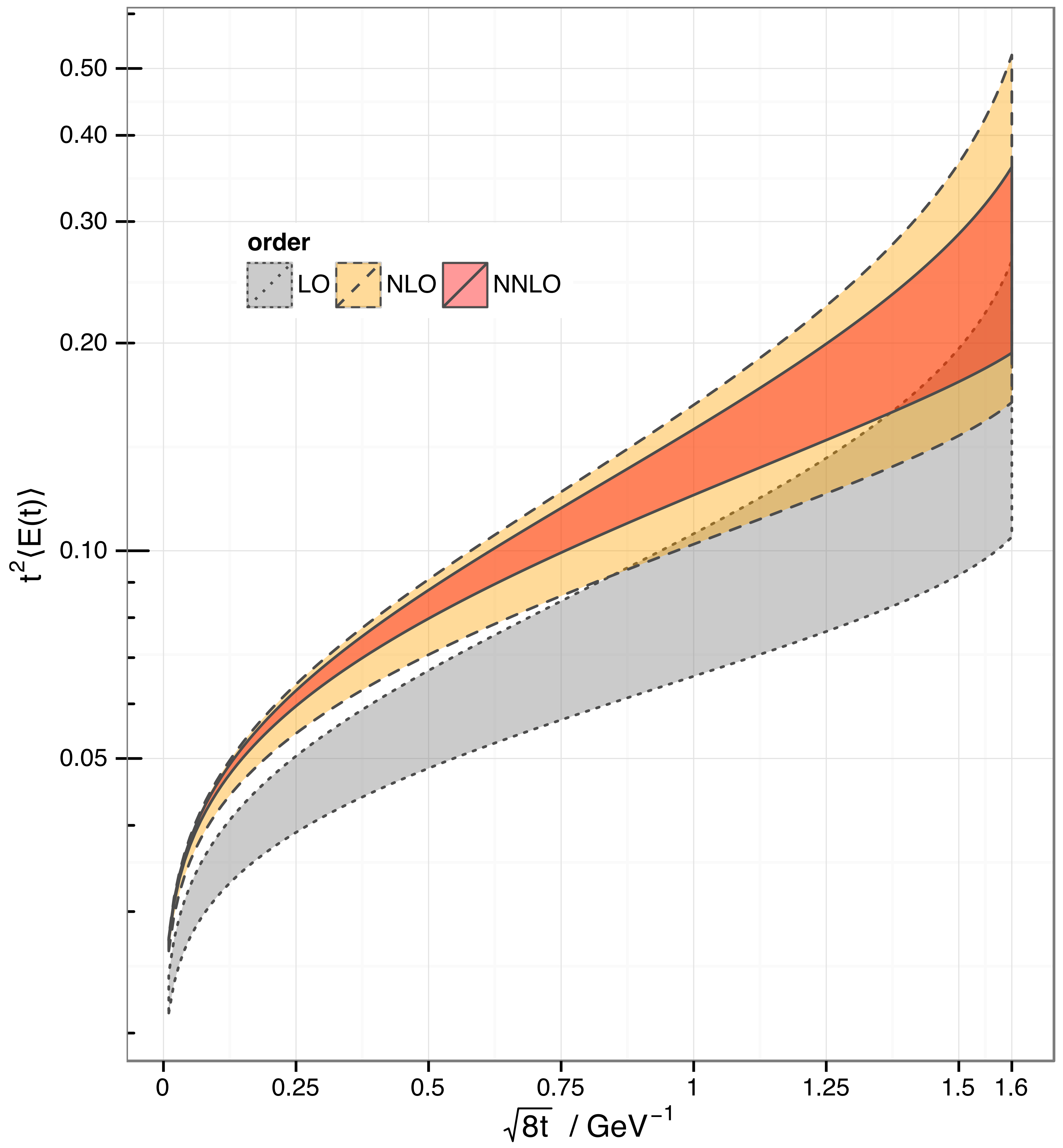
resulting perturbative accuracy on α_s: ± 3-5%

PDG: ± 1%

$$\langle t^2 E(t)\rangle = \frac{3\alpha_s(\mu)}{4\pi}\left[1 + k_1(t,\mu)\,\alpha_s(\mu) + k_2(t,\mu)\,\alpha_s^2(\mu)\right]$$

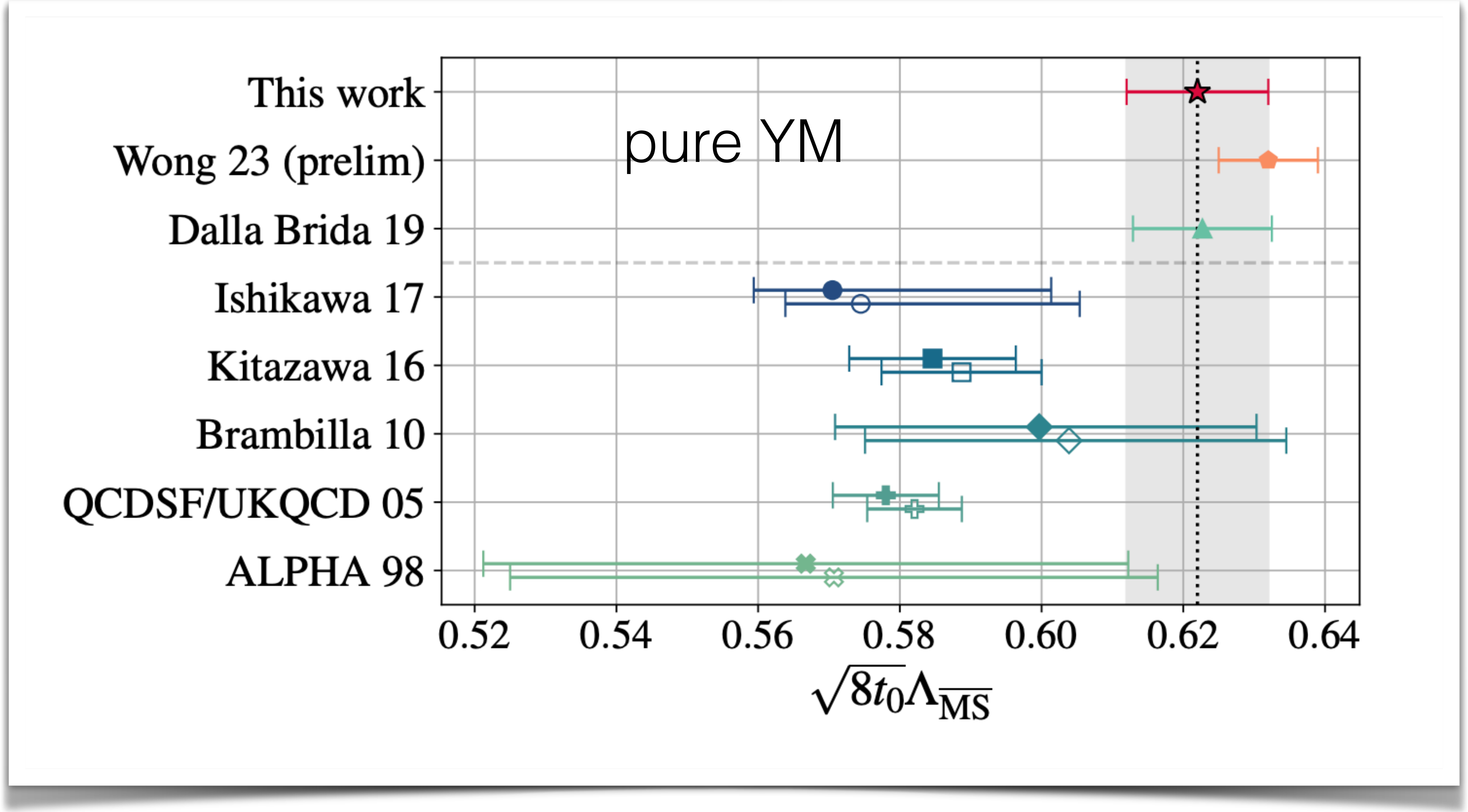
RH, Neumann 2016





resulting perturbative accuracy on α_s: O(1%)

PDG: ± 1%



A. Hasenfratz, Peterson, Sickle, Witzel (2023)

see also C.H. Wong et al.

Effective Field Theories

$$H_{\text{eff}} \sim \sum_n C_n \mathcal{O}_n$$

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"power divergences"

Example

Suzuki, Makino (2014)

energy-momentum tensor in QCD:

$$T_{\mu\nu} = \frac{1}{g_0^2}\left[\mathcal{O}_{1,\mu\nu} - \frac{1}{4}\mathcal{O}_{2,\mu\nu}\right] + \frac{1}{4}\mathcal{O}_{3,\mu\nu}$$

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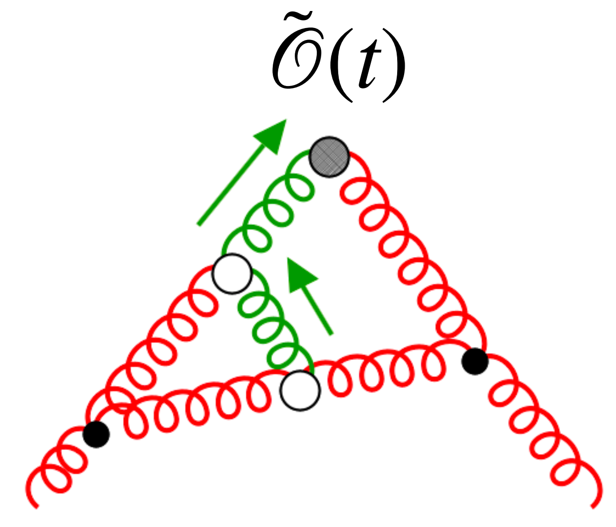
$c_n(t)$: perturbatively



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NNLO result

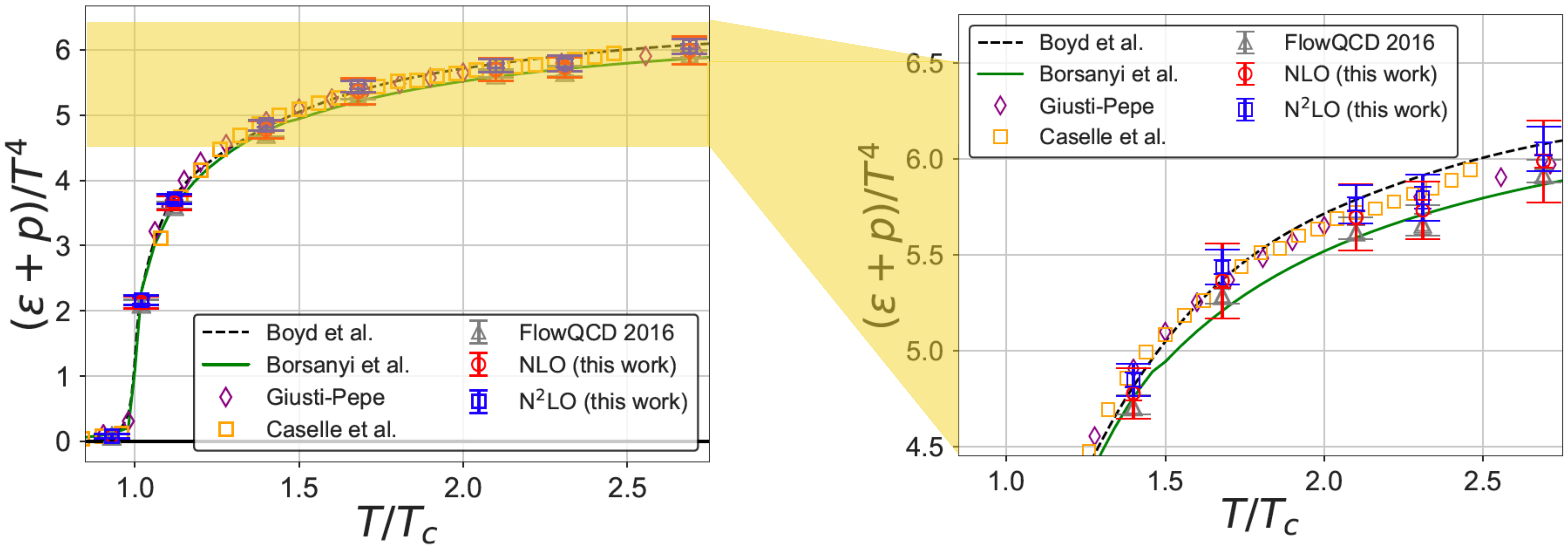
$$
\begin{aligned}
c_1(t) = \frac{1}{g^2}\Bigg\{1 &+ \frac{g^2}{(4\pi)^2}\left[-\frac{7}{3}C_A + \frac{3}{2}T_F - \beta_0\,L(\mu,t)\right] \\
&+ \frac{g^4}{(4\pi)^4}\Bigg[-\beta_1\,L(\mu,t) + C_A^2\left(-\frac{14482}{405} - \frac{16546}{135}\ln 2 + \frac{1187}{10}\ln 3\right) \\
&\quad + C_A T_F\left(\frac{59}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{10873}{810} + \frac{73}{54}\pi^2 - \frac{2773}{135}\ln 2 + \frac{302}{45}\ln 3\right) \\
&\quad + C_F T_F\left(-\frac{256}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{2587}{108} - \frac{7}{9}\pi^2 - \frac{106}{9}\ln 2 - \frac{161}{18}\ln 3\right)\Bigg] \\
&+ \mathcal{O}(g^6)\Bigg\}\,, \qquad L(\mu,t) \equiv \ln\left(2\mu^2 t\right) + \gamma_\mathrm{E}
\end{aligned}
$$

etc.

RH, Kluth, Lange '18

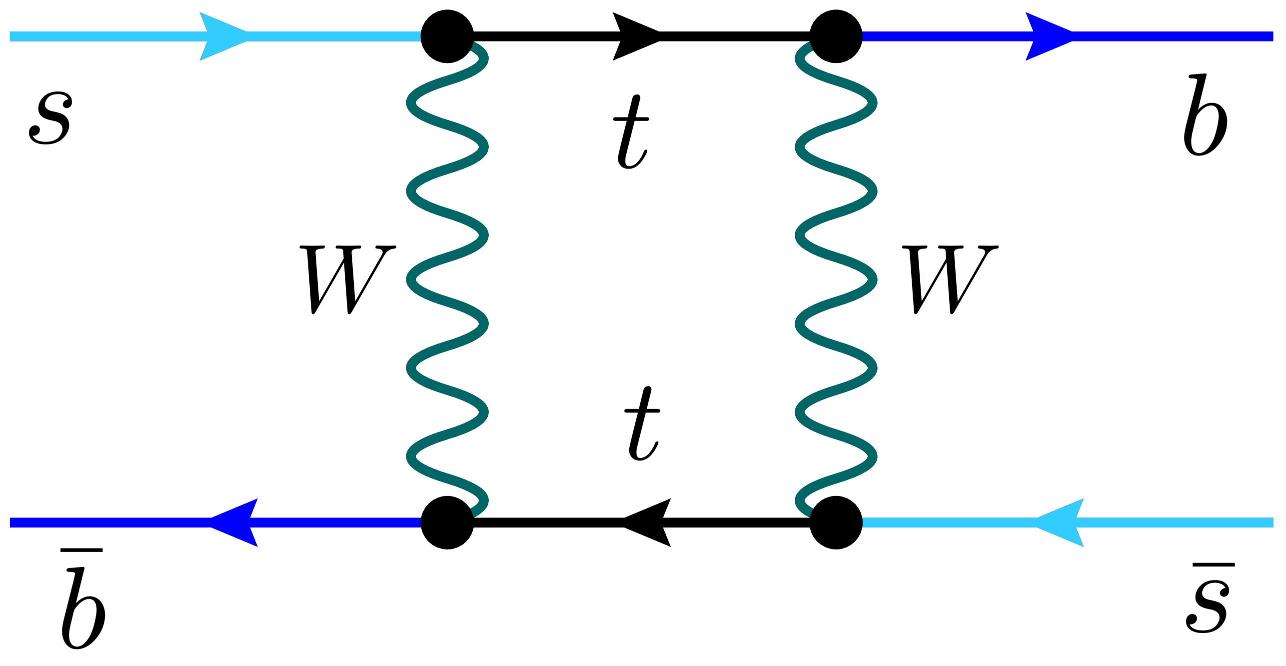
QCD Thermodynamics

Entropy density: $$\varepsilon + p = -\frac{4}{3}\left\langle T_{00}(x) - \frac{1}{4}T_{\mu\mu}(x)\right\rangle$$

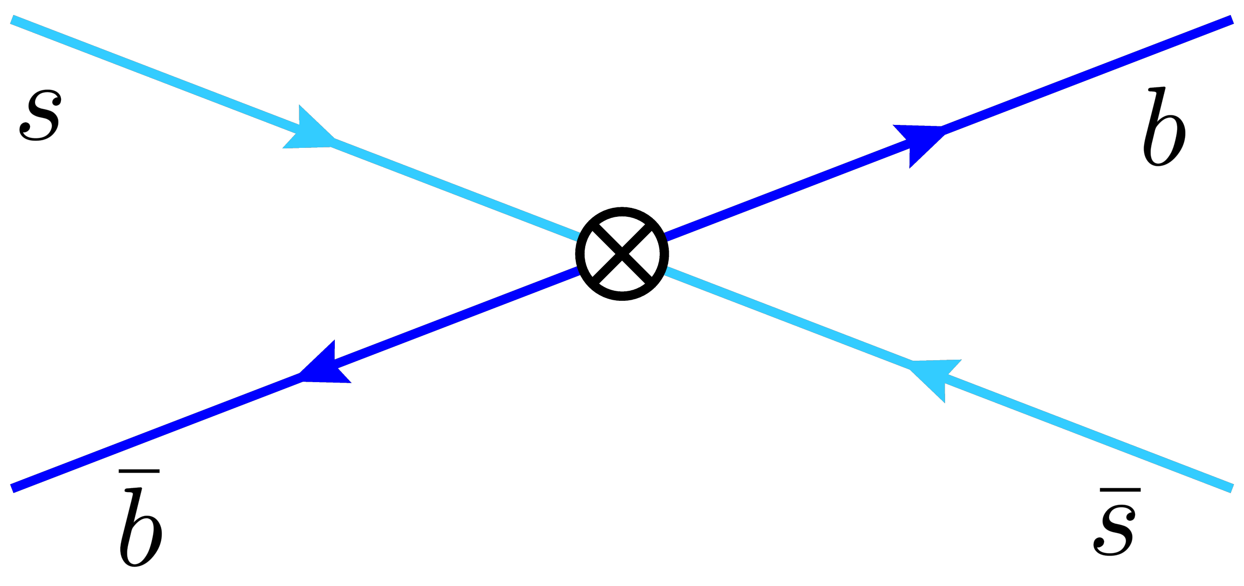


Iritani, Kitazawa, Suzuki, Takaura 2019

Flavor physics



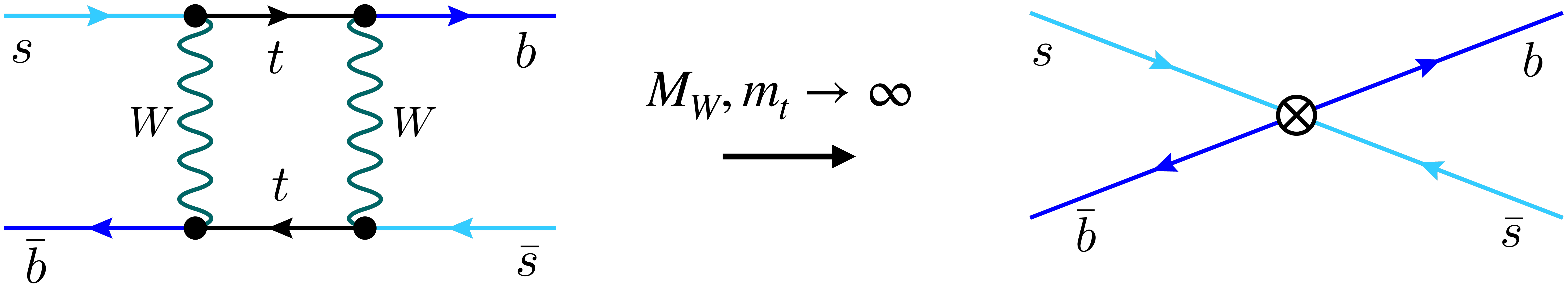
$M_W, m_t \to \infty$



$$H_{\text{eff}} \sim \sum_n C_n \mathcal{O}_n$$

leading operator for $B_s - \bar{B}_s$ mixing: $\mathcal{O}_1^s = [\bar{b}\gamma^\mu(1-\gamma_5)s][\bar{b}\gamma_\mu(1-\gamma_5)s]$

Flavor physics



$$H_{\text{eff}} \sim \sum_n C_n \mathcal{O}_n = \sum_n C_n \left(\zeta^{-1}(t)\,\zeta(t)\mathcal{O}\right)_n$$

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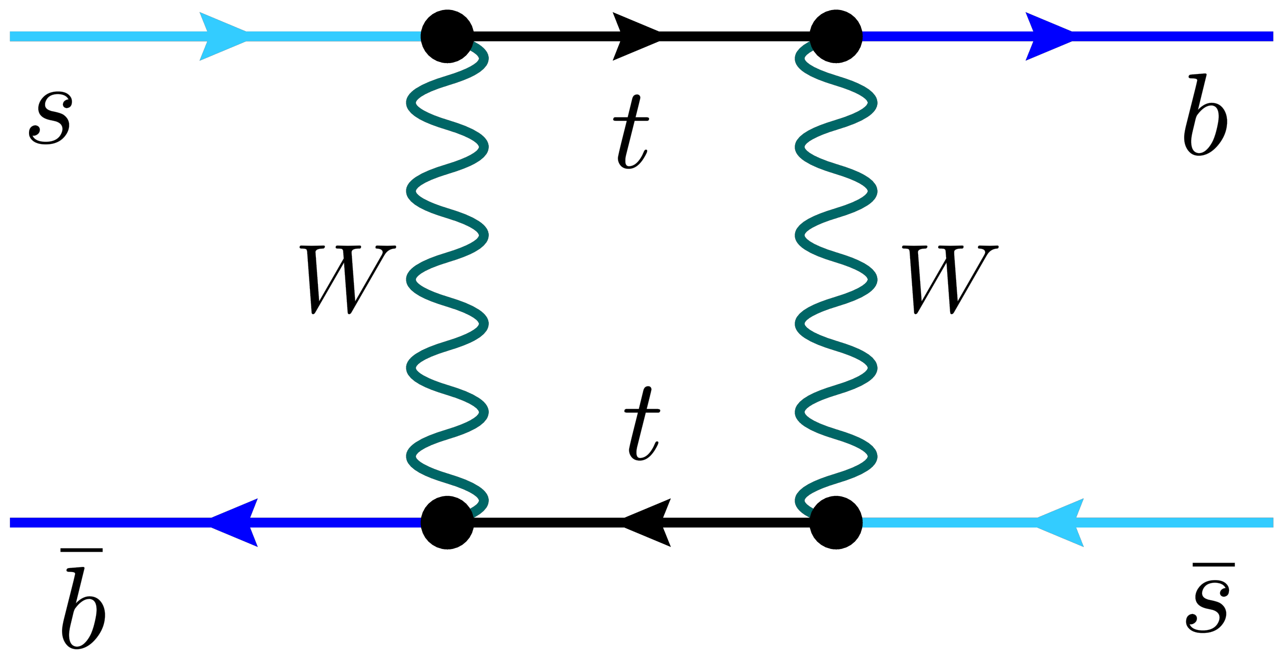

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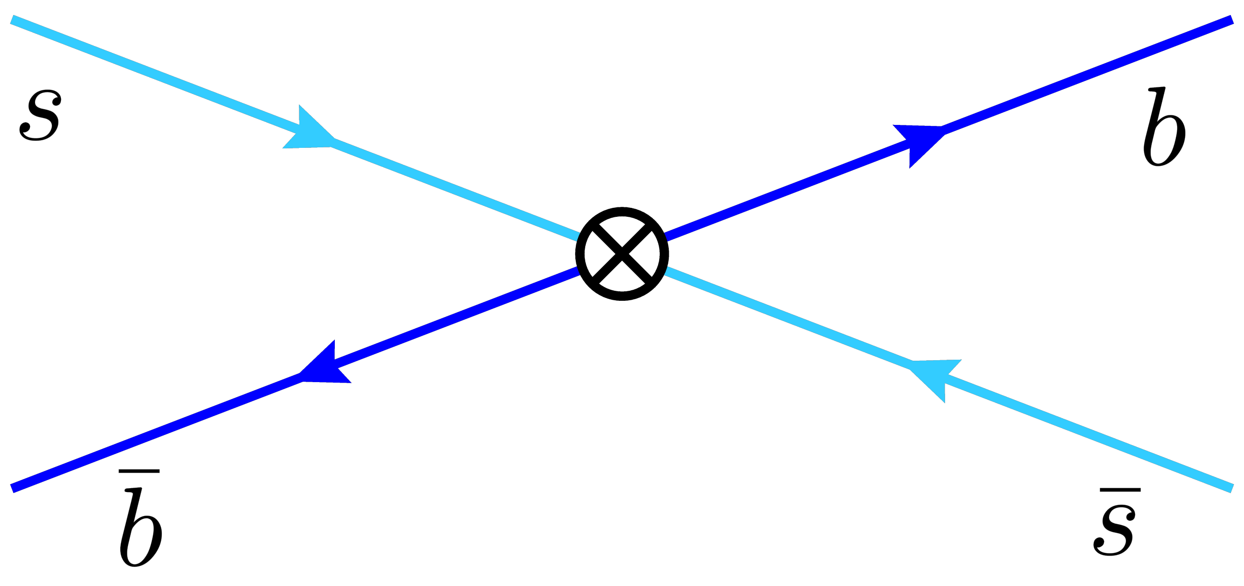

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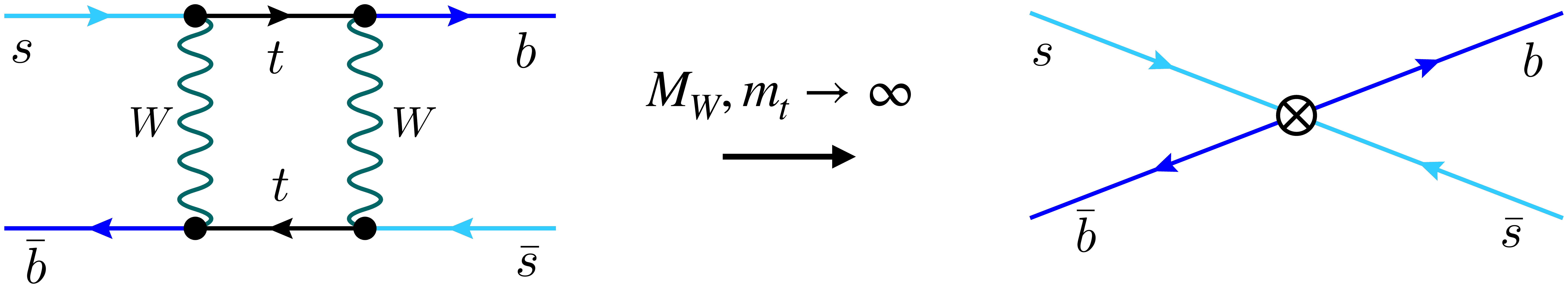

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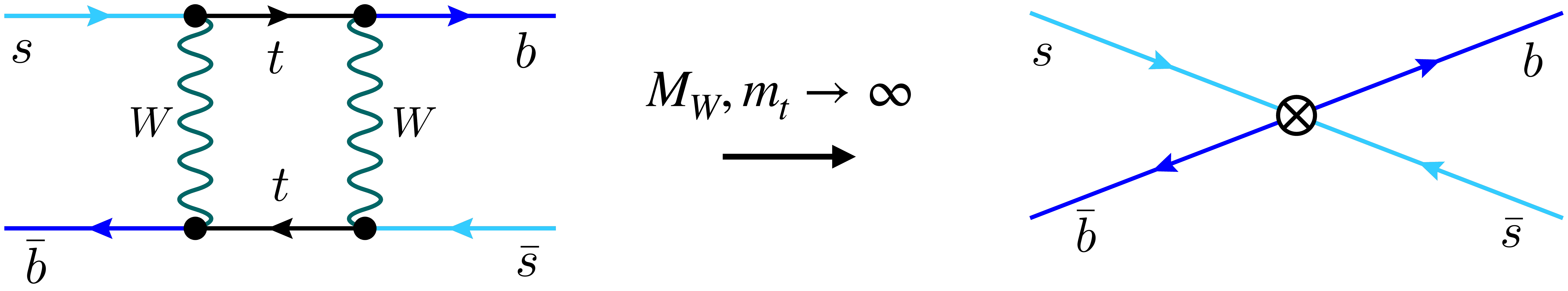

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first studies: $\zeta^{-1}(t)\,\langle B_s\,|\,\tilde{\mathcal{O}}(t)\,|\,\bar{B}_s\rangle$

Black, RH, Lange, Rago, Shindler, Witzel (2023)

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Flavor physics

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Flavor physics

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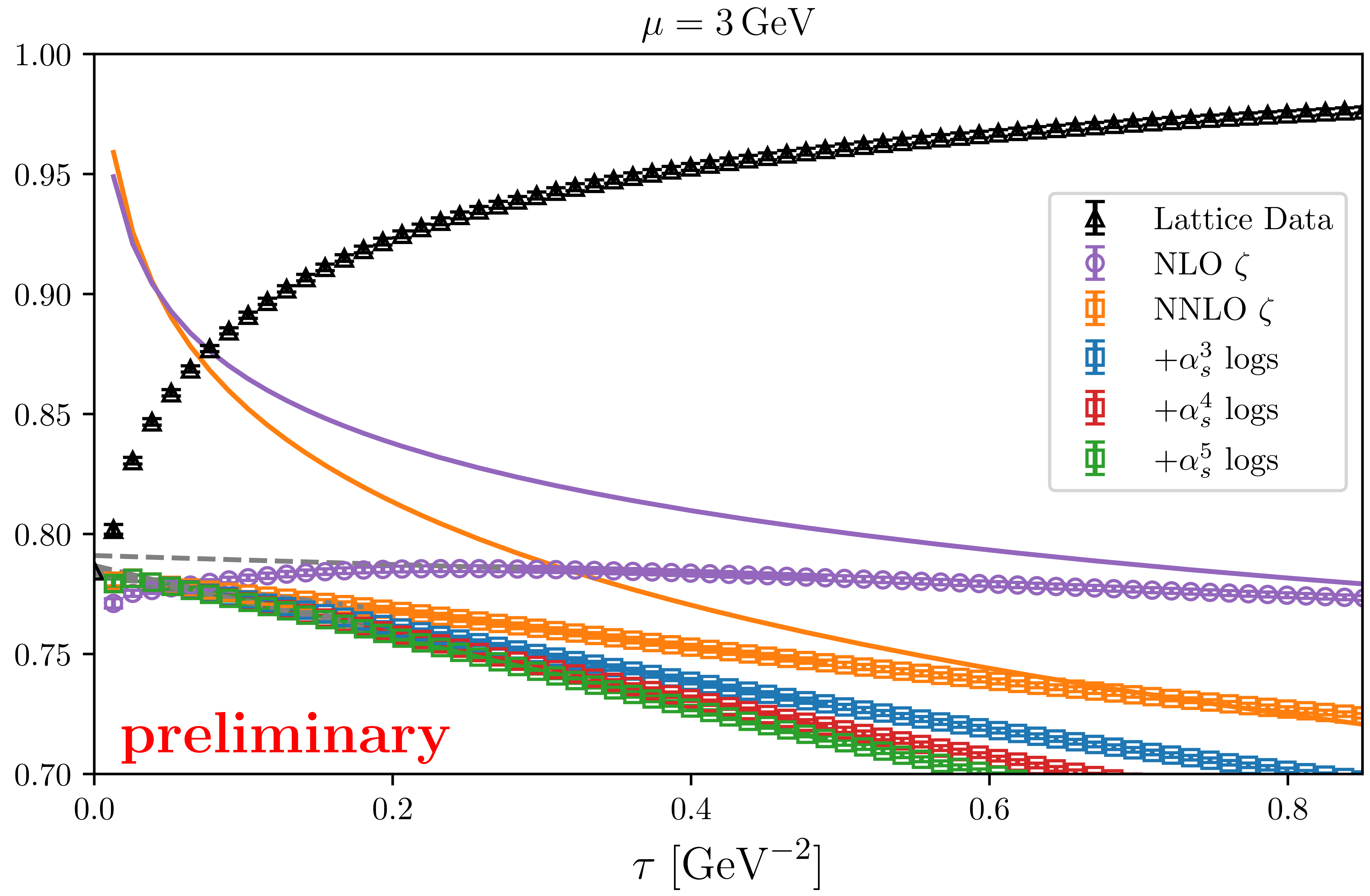

RH, Lange (2022)

Borgulat, RH, Kohnen, Lange (2023)

$\zeta^{-1}(t)$

Black, RH, Lange, Rago, Shindler, Witzel (2023)

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